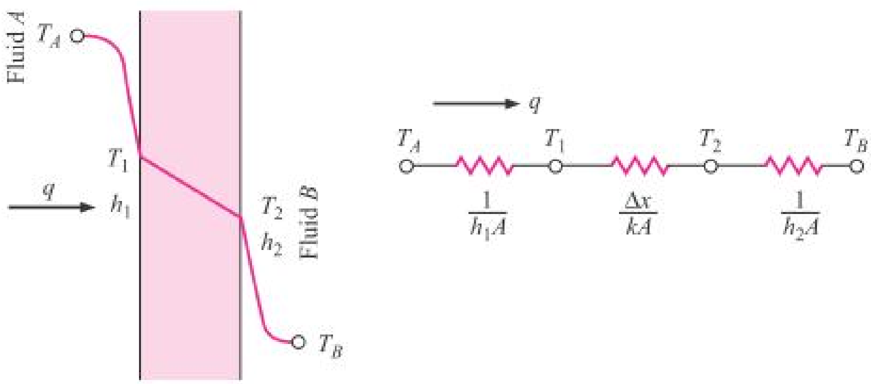
The Overall Heat Transfer Coefficient

Consider the plane wall shown. This wall is exposed to a hot fluid A on one side and a cooler fluid B on the other side. The heat transfer is expressed by

$$q = h_1 A\,(T_A - T_1) = \frac{kA}{\Delta x}(T_1 - T_2) = h_2 A\,(T_2 - T_B)$$



Since $q = \dfrac{\Delta T}{\sum R_{th}}$ $\Rightarrow$ $q = \dfrac{T_A - T_B}{1/h_1 A + \Delta x/kA + 1/h_2 A}$

The overall heat transfer by combined conduction and convection is frequently expressed in terms of an overall heat-transfer coefficient U, defined by

$$q = UA\,\Delta T_{\text{overall}}$$

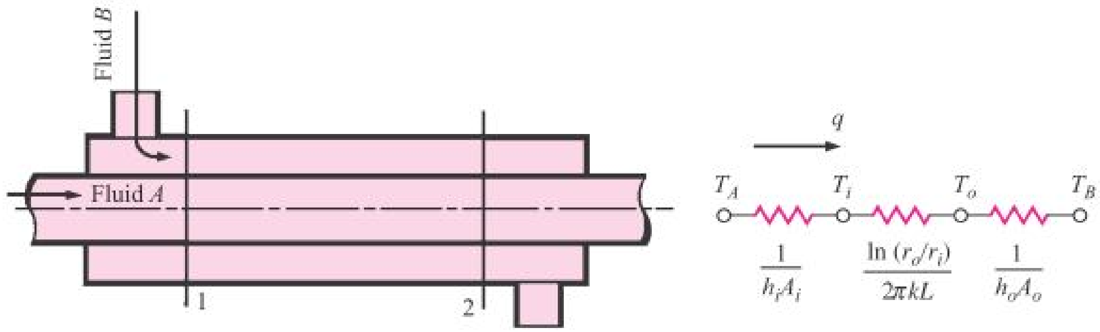
Where A is some suitable area for the heat flow. The overall heat-transfer coefficient would be

$$U = \frac{1}{1/h_1 + \Delta x/k + 1/h_2}$$

The overall heat-transfer coefficient is also related to the R value as

$$U = \frac{1}{R \text{ value}}$$

For a hollow cylinder exposed to convection on its inner and outer surfaces. TA and TB are the two fluid temperatures. Note that the area for convection is not the same for both fluids.



The overall heat transfer would be expressed by

$$q = \frac{T_A - T_B}{\dfrac{1}{h_i A_i} + \dfrac{\ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o A_o}}$$

The overall heat-transfer coefficient may be based on either the inside or the outside area of the tube. Accordingly:

$$U_i = \frac{1}{\dfrac{1}{h_i} + \dfrac{A_i \ln(r_o/r_i)}{2\pi kL} + \dfrac{A_i}{A_o}\dfrac{1}{h_o}} \quad ; \qquad q = U_i \; A_i \; \Delta T_{overall}$$

$$U_o = \frac{1}{\dfrac{A_o}{A_i}\dfrac{1}{h_i} + \dfrac{A_o \ln(r_o/r_i)}{2\pi kL} + \dfrac{1}{h_o}} \quad ; \qquad q = U_0 \; A_0 \; \Delta T_{overall}$$

$$\Rightarrow U_i \; A_i = U_0 A_0 \Rightarrow D_i U_i = D_0 U_0$$

Overall Heat-Transfer Coefficient for a Tube **EXAMPLE 2-5**

Water flows at 50°C inside a 2.5-cm-inside-diameter tube such that $h_i = 3500 \text{ W/m}^2 \cdot °\text{C}$. The tube has a wall thickness of 0.8 mm with a thermal conductivity of 16 W/m · °C. The outside of the tube loses heat by free convection with $h_o = 7.6 \text{ W/m}^2 \cdot °\text{C}$. Calculate the overall heat-transfer coefficient and heat loss per unit length to surrounding air at 20°C.

■ Solution

There are three resistances in series for this problem, as illustrated in Equation (2-14). With $L = 1.0$ m, $d_i = 0.025$ m, and $d_o = 0.025 + (2)(0.0008) = 0.0266$ m, the resistances may be calculated as

■ **Solution**

There are three resistances in series for this problem, as illustrated in Equation (2-14). With $L = 1.0$ m, $d_i = 0.025$ m, and $d_o = 0.025 + (2)(0.0008) = 0.0266$ m, the resistances may be calculated as

$$R_i = \frac{1}{h_i A_i} = \frac{1}{(3500)\pi(0.025)(1.0)} = 0.00364\ ^\circ\text{C/W}$$

$$R_t = \frac{\ln(d_o/d_i)}{2\pi k L}$$

$$= \frac{\ln(0.0266/0.025)}{2\pi(16)(1.0)} = 0.00062\ ^\circ\text{C/W}$$

$$R_o = \frac{1}{h_o A_o} = \frac{1}{(7.6)\pi(0.0266)(1.0)} = 1.575\ ^\circ\text{C/W}$$

Clearly, the outside convection resistance is the largest, and *overwhelmingly so*. This means that it is the controlling resistance for the total heat transfer because the other resistances (in series) are negligible in comparison. We shall base the overall heat-transfer coefficient on the outside tube area and write

$$q = \frac{\Delta T}{\sum R} = U A_o \Delta T \qquad [a]$$

$$U_o = \frac{1}{A_o \sum R} = \frac{1}{[\pi(0.0266)(1.0)](0.00364 + 0.00062 + 1.575)}$$

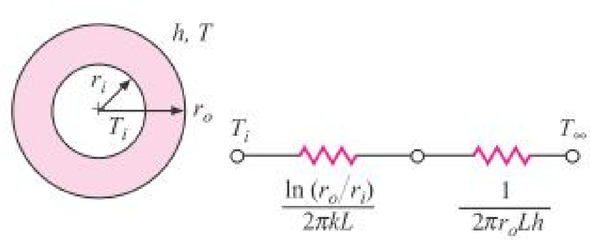
$$= 7.577\ \text{W/m}^2 \cdot {}^\circ\text{C}$$

or a value very close to the value of $h_o = 7.6$ for the outside convection coefficient. The heat transfer is obtained from Equation (a), with

$$q = U A_o\, \Delta T = (7.577)\pi(0.0266)(1.0)(50 - 20) = 19\ \text{W (for 1.0 m length)}$$

Critical Thickness of Insulation

For the layer of insulation which is installed around a circular pipe as shown in figure below. The inner temperature of the insulation is fixed at *Ti*, and the outer surface is exposed to a convection environment at *T*∞.



From the thermal network the heat transfer is:

$$q = \frac{2\pi L\,(T_i - T_\infty)}{\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}}$$

To determine the outer radius of insulation r_o that maximize the heat transfer, we may differential q to r_o as:

$$\frac{dq}{dr_o} = 0 = \frac{-2\pi L\,(T_i - T_\infty)\left(\dfrac{1}{kr_o} - \dfrac{1}{hr_o^2}\right)}{\left[\dfrac{\ln(r_o/r_i)}{k} + \dfrac{1}{r_o h}\right]^2}$$

Which gives

$$r_o = \frac{k}{h} \qquad (k = k_{insulation})$$

r_o is the critical-radius-of-insulation concept. If the outer radius is less than the value given by the last equation, the heat transfer will increase by adding more insulation. For outer radii greater than the critical value, an increase in insulation thickness will cause a decrease in heat transfer. The central concept is that for sufficiently small values of h the convection heat loss may actually increase with the addition of insulation because of increased surface area.

EXAMPLE 2-6 Critical Insulation Thickness

Calculate the critical radius of insulation for asbestos [$k = 0.17$ W/m · °C] surrounding a pipe and exposed to room air at 20°C with $h = 3.0$ W/m^2 · °C. Calculate the heat loss from a 200°C, 5.0-cm-diameter pipe when covered with the critical radius of insulation and without insulation.

Solution

From Equation (2-18) we calculate r_o as

$$r_o = \frac{k}{h} = \frac{0.17}{3.0} = 0.0567 \text{ m} = 5.67 \text{ cm}$$

The inside radius of the insulation is 5.0/2 = 2.5 cm, so the heat transfer is calculated from Equation (2-17) as

$$\frac{q}{L} = \frac{2\pi\,(200 - 20)}{\dfrac{\ln(5.67/2.5)}{0.17} + \dfrac{1}{(0.0567)(3.0)}} = 105.7 \text{ W/m}$$

Without insulation the convection from the outer surface of the pipe is

$$\frac{q}{L} = h(2\pi r)(T_i - T_o) = (3.0)(2\pi)(0.025)(200 - 20) = 84.8 \text{ W/m}$$

So, the addition of 3.17 cm (5.67 − 2.5) of insulation actually *increases* the heat transfer by 25 percent.

As an alternative, fiberglass having a thermal conductivity of 0.04 W/m · °C might be employed as the insulation material. Then, the critical radius would be

$$r_o = \frac{k}{h} = \frac{0.04}{3.0} = 0.0133 \text{ m} = 1.33 \text{ cm}$$

Now, the value of the critical radius is less than the outside radius of the pipe (2.5 cm), so addition of *any* fiberglass insulation would cause a *decrease* in the heat transfer. In a practical pipe insulation problem, the total heat loss will also be influenced by radiation as well as convection from the outer surface of the insulation.

The following table shows the concept of critical radius of insulation.

r_o (mm)	q (W/m)
25	84.8
35	98.32
45	104.1
55	105.71
56.7	**105.72**
65	105.2
75	103.7
85	101.7
105	97.3
155	82.7