Multilayer Conduction — EXAMPLE 2-1

An exterior wall of a house may be approximated by a 4-in layer of common brick [$k = 0.7\ \text{W/m}\cdot{}^\circ\text{C}$] followed by a 1.5-in layer of gypsum plaster [$k = 0.48\ \text{W/m}\cdot{}^\circ\text{C}$]. What thickness of loosely packed rock-wool insulation [$k = 0.065\ \text{W/m}\cdot{}^\circ\text{C}$] should be added to reduce the heat loss (or gain) through the wall by 80 percent?

Solution

The overall heat loss will be given by

$$q = \frac{\Delta T}{\sum R_{\text{th}}}$$

Because the heat loss with the rock-wool insulation will be only 20 percent (80 percent reduction) of that before insulation

$$\frac{q \text{ with insulation}}{q \text{ without insulation}} = 0.2 = \frac{\sum R_{\text{th}} \text{ without insulation}}{\sum R_{\text{th}} \text{ with insulation}}$$

We have for the brick and plaster, for unit area,

$$R_b = \frac{\Delta x}{k} = \frac{(4)(0.0254)}{0.7} = 0.145\ \text{m}^2\cdot{}^\circ\text{C/W}$$

$$R_p = \frac{\Delta x}{k} = \frac{(1.5)(0.0254)}{0.48} = 0.079\ \text{m}^2\cdot{}^\circ\text{C/W}$$

so that the thermal resistance without insulation is

$$R = 0.145 + 0.079 = 0.224\ \text{m}^2\cdot{}^\circ\text{C/W}$$

Then

$$R \text{ with insulation} = \frac{0.224}{0.2} = 1.122\ \text{m}^2\cdot{}^\circ\text{C/W}$$

and this represents the sum of our previous value and the resistance for the rock wool

$$1.122 = 0.224 + R_{rw}$$

$$R_{rw} = 0.898 = \frac{\Delta x}{k} = \frac{\Delta x}{0.065}$$

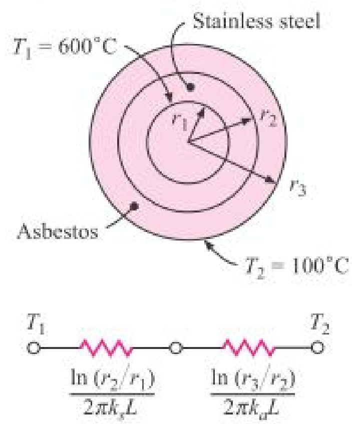
so that

$$\Delta x_{rw} = 0.0584\ \text{m} = 2.30\ \text{in}$$

EXAMPLE 2-2 — Multilayer Cylindrical System

A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19\ \text{W/m}\cdot{}^\circ\text{C}$] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2\ \text{W/m}\cdot{}^\circ\text{C}$]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

Figure Example 2-2



Solution

Figure Example 2-2 shows the thermal network for this problem. The heat flow is given by

$$\frac{q}{L} = \frac{2\pi(T_1 - T_2)}{\ln(r_2/r_1)/k_s + \ln(r_3/r_2)/k_a} = \frac{2\pi(600 - 100)}{(\ln 2)/19 + (\ln \frac{5}{2})/0.2} = 680 \text{ W/m}$$

This heat flow may be used to calculate the interface temperature between the outside tube wall and the insulation. We have

$$\frac{q}{L} = \frac{T_a - T_2}{\ln(r_3/r_2)/2\pi k_a} = 680 \text{ W/m}$$

where T_a is the interface temperature, which may be obtained as

$$T_a = 595.8°\text{C}$$

The largest thermal resistance clearly results from the insulation, and thus the major portion of the temperature drop is through that material.

Convection Boundary Conditions

The heat transfer by convection can be calculated as:

$$q_{conv} = hA(T_w - T_\infty)$$

An electric-resistance analogy can also be drawn for the convection process by

$$q_{conv} = \frac{T_w - T_\infty}{1/hA}$$

Where *1/hA* becomes the convection thermal resistance.