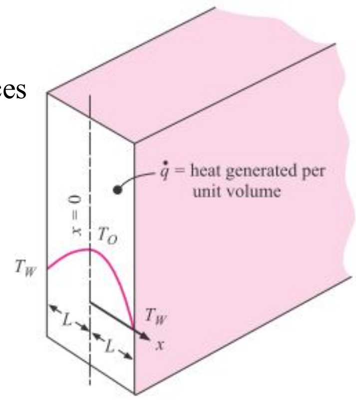
Heat Source Systems

There are systems in which heat is generated internally, such as:

1- Electrical conductors
2- Nuclear reactors
3- Chemical reactors
4- Mechanical system (viscous source)

1- Plane Wall with Heat Sources



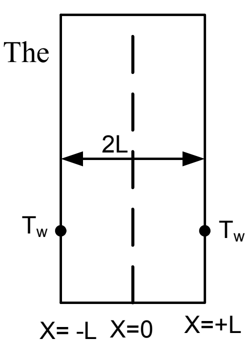
Consider the plane wall shown with uniformly distributed heat sources as shown in the figure. The heat generated per unit volume is
The general equation is $\dot{q}$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

For one-dimensional, steady state with heat generation

$$\frac{d^2T}{dx^2} + \frac{q^\bullet}{k} = 0 \;\Rightarrow\; \frac{d^2T}{dx^2} = -\frac{q^\bullet}{k} \Rightarrow by\int \Rightarrow \frac{dT}{dx} = -\frac{q^\bullet x}{k} + C_1 \Rightarrow by\int \Rightarrow$$

$$T = -\frac{q^\bullet x^2}{2k} + C_1 X + C_2 \quad \textit{(general solution)}$$



The both sides of the plane wall are subjected to a constant temperature Tw. The Boundary conditions will be

$$T = T_w \qquad \text{at } x = \pm L$$

By applying the boundary conditions above,

$C_1 = 0$

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$$T_W = -\frac{q^\bullet L^2}{2k} + C_2 \;\Rightarrow\; C_2 = T_W + \frac{q^\bullet L^2}{2k}$$

$$\therefore\; T = -\frac{q^\bullet x^2}{2k} + T_W + \frac{q^\bullet L^2}{2k}$$

$$or\;\; T - T_W = \frac{q^\bullet L^2}{2k}\left[1 - \left(\frac{x}{L}\right)^2\right]$$

i.e

$$T_{\max} = T_W + \frac{q^{\bullet} L^2}{2k}$$

$$T = -\frac{q^{\bullet} x^2}{2k} + C_1 X + C_2$$

B.C.1 $x = 0,\ \ T = T_1$

B.C. 2 $x = L,\ \ T = T_2$

From B.C.1, $T_1 = C_2$

From B.C.2, $T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + C_2$

By sub. C_2, we get

$$T_2 = -\frac{q^{\bullet} L^2}{2k} + C_1 L + T_1 \Rightarrow$$

$$C_1 = \frac{(T_2 - T_1)}{L} + \frac{q^{\bullet} L}{2k}$$

2- Radial Systems with Heat Sources

A) <u>For Solid Cylinder with Heat Source</u>.

Consider a cylinder of radius R with uniformly distributed heat sources. The general heat conduction equation in cylindrical coordinate is

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

For one dimensional steady-state with heat generation

$$\frac{d^2 T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{\dot{q}}{k} = 0$$

The boundary conditions are

B.C. 1 at $r = 0$ $dT/dr = 0$

B.C. 2 at $r = R$ $T = T_W$

$$r\frac{d^2 T}{dr^2} + \frac{dT}{dr} = \frac{-\dot{q} r}{k}$$

Since $$r\frac{d^2T}{dr^2}+\frac{dT}{dr}=\frac{d}{dr}\left(r\frac{dT}{dr}\right)$$

Subs. in eq. above, we get

$$\frac{d}{dr}\left(r\frac{dT}{dr}\right)+\frac{rq^\bullet}{k}=0 \;\Rightarrow\; \frac{d}{dr}\left(r\frac{dT}{dr}\right)=-\frac{rq^\bullet}{k} \;\Rightarrow by\int \;\Rightarrow r\frac{dT}{dr}=-\frac{q^\bullet r^2}{2k}+C_1 \;\Rightarrow$$

$$\frac{dT}{dr}=-\frac{q^\bullet r}{2k}+\frac{C_1}{r} \;\Rightarrow by\int \;\Rightarrow$$

$$T=\frac{-\dot{q}r^2}{4k}+C_1\ln r+C_2 \qquad \text{(general solution)}$$

From B.C. 1, $C_1 = 0$,

From B.C. 2,

$$T_W=-\frac{q^\bullet R^2}{4k}+C_2 \;\Rightarrow\; C_2=T_W+\frac{q^\bullet R^2}{4k}$$

$$\therefore\; T=-\frac{q^\bullet r^2}{4k}+T_W+\frac{q^\bullet R^2}{4k}$$

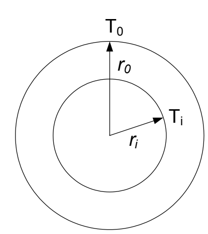
or

$$T-T_W=\frac{q^\bullet R^2}{4k}\left(1-\left(\frac{r}{R}\right)^2\right)$$

The maximum temperature at the centre, i.e at $r = 0$,

$$T_{\max}=T_W+\frac{q^\bullet R^2}{4k}$$

b) <u>For a Hollow Cylinder with Heat Source</u>.



The boundary conditions are:

$$T=T_i \quad \text{at } r=r_i \text{ (inside surface)}$$
$$T=T_o \quad \text{at } r=r_o \text{ (outside surface)}$$

The general solution is

$$T=-\frac{\dot{q}r^2}{4k}+C_1\ln r+C_2$$

From B.C.1,

$$T_0=-\frac{q^\bullet r_0}{4k}+C_1\ln r_0+C_2 \;\Rightarrow\; C_2=T_0+\frac{q^\bullet r_0}{4k}-C_1\ln r_0$$

$$\therefore\ T = \frac{\dot{q}}{4k}(r_0^2 - r^2) + C_1 \ln\frac{r}{r_0} + T_0$$

or

$$T - T_0 = \frac{\dot{q}}{4k}(r_0^2 - r^2) + C_1 \ln\frac{r}{r_0}$$

From B.C.2,

$$T_i - T_0 = \frac{\dot{q}}{4k}(r_0^2 - r_i^2) + C_1 \ln\frac{r_i}{r_0} \quad \Rightarrow$$

$$C_1 = \frac{T_i - T_o + \dot{q}\,(r_i^2 - r_o^2)/4k}{\ln\,(r_i/r_o)}$$

The temp. at r = 0 (at the center) is given by

$$T_0 = \frac{\dot{q}}{4K} + T_W$$

EXAMPLE 2-7 Heat Source with Convection

A current of 200 A is passed through a stainless-steel wire [$k = 19$ W/m · °C] 3 mm in diameter. The resistivity of the steel may be taken as 70 $\mu\Omega \cdot$ cm, and the length of the wire is 1 m. The wire is submerged in a liquid at 110 °C and experiences a convection heat-transfer coefficient of 4 k W/m^2 · °C. Calculate the center temperature of the wire.

■ Solution

All the power generated in the wire must be dissipated by convection to the liquid:

$$P = I^2R = q = hA\,(T_w - T_\infty) \qquad [a]$$

The resistance of the wire is calculated from

$$R = \rho\frac{L}{A} = \frac{(70 \times 10^{-6})(100)}{\pi(0.15)^2} = 0.099\ \Omega$$

where ρ is the resistivity of the wire. The surface area of the wire is πdL, so from Equation (a),

$$(200)^2(0.099) = 4000\pi(3 \times 10^{-3})(1)(T_w - 110) = 3960\ \text{W}$$

and

$$T_w = 215°\text{C} \quad [419°\text{F}]$$

The heat generated per unit volume $\dot{q}$ is calculated from

$$P = \dot{q}V = \dot{q}\pi r^2 L$$

so that

$$\dot{q} = \frac{3960}{\pi\,(1.5\times10^{-3})^2(1)} = 560.2\ \text{MW/m}^3 \quad [5.41\times10^7\ \text{Btu/h}\cdot\text{ft}^3]$$

Finally, the center temperature of the wire is calculated from Equation (2-26):

$$T_0 = \frac{\dot{q}r_o^2}{4k} + T_w = \frac{(5.602\times10^8)(1.5\times10^{-3})^2}{(4)(19)} + 215 = 231.6°\text{C} \quad [449°\text{F}]$$