Heat Transfer

Text book : Heat Transfer. Tenth edition by J.P. Holman.

CH.1 Introduction

Heat transfer : Is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference.

[1] Conduction Heat Transfer.

* When a temperature gradient exists in a body there is an energy transfer from the high-temperature region to the low-temperature region.
* We say that the energy is transferred by conduction and that the heat-transfer rate per unit area is proportional to the normal temperature gradient.

$$\frac{q}{A} \propto \frac{\partial T}{\partial x} \Rightarrow \frac{q}{A} = -K\frac{\partial T}{\partial x} \Rightarrow \boxed{q = -KA\frac{\partial T}{\partial x}} \rightarrow \text{Fourier's Low}$$

* q : heat-transfer rate. (W) or (J/s)
* $\frac{\partial T}{\partial x}$: Temp. gradient. $\Rightarrow \frac{\Delta T \rightarrow \text{Temp. difference}}{\Delta x \rightarrow \text{thickness}}$
* K : Thermal conductivity . (W/m. C) or (W/m.K)

نستطيع إيجاد قيم K من الجداول.

* A : H.T. area.
* ΔT : $T_2 - T_1$ ---- $T_1 > T_2$

* أي أن درجة الحرارة تنتقل من الأعلى T_1 إلى الأقل T_2.

* Note : K (for gas) < K (for liquid , solid)

* Note : K can be determain experimentally .

$$\therefore \boxed{q = -KA \frac{\Delta T}{\Delta x}}$$

Example: The front of slab of lead (k = 35 w/M.C) The front is kept @ (100 C) and the back @ (55 C) If the slab has an area of 0.45 m^2 and 3 cm thickness .

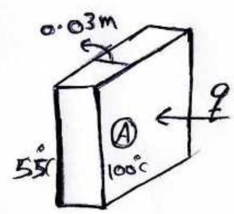
Determaine the rate of heat transfer through this slab .

Soly : $\Delta x = 0.03\ m$, $\Delta T = (55-100)$, $A = 0.45\ m^2$, $K = 35\ W/m.c$

$$q = -KA \frac{\Delta T}{\Delta x}$$

$$q = -35 * 0.45 * \frac{(55-100)}{0.03}$$

$$q = 23625\ w = 23.625\ Kw$$



EXAMPLE 1-1

One face of a copper plate 3 cm thick is maintained at 400°C, and the other face is maintained at 100°C. How much heat is transferred through the plate?

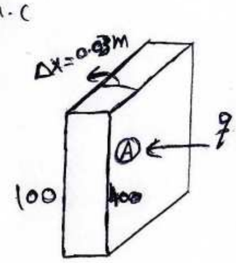
Soly : $\Delta x = 0.03\ m$, $\Delta T = (100-400)$

- $K \rightarrow$ from table A.2 $K_{copper} = 370\ W/m.c$

$$q = -KA \frac{\Delta T}{\Delta x}$$

$$\frac{q}{A} = -K \frac{\Delta T}{\Delta x} = -\frac{370 * (100-400)}{0.03}$$

$$\frac{q}{A} = 3700000\ W/m^2$$



P. 1-1 If 3 kW is conducted through a section of insulating material 0.6 m^2 in cross section and 2.5 cm thick and the thermal conductivity may be taken as 0.2 W/m · °C, compute the temperature difference across the material.

Soly:- Find ΔT ? $\quad q = 3\ KW = 3000\ W,\ \Delta x = 2.5\ cm = 0.025\ m$

$$q = -KA\frac{\Delta T}{\Delta x}$$

في هذا السؤال يصبح القانون بدون اشارة سالبة
لأنه الإشارة السالبة وجدت اصلا لعكس ناتج ما داخل ΔT.

$$q = KA\frac{\Delta T}{\Delta x} \Rightarrow \Delta T = \frac{q \cdot \Delta x}{KA} = \frac{3000 * 0.025}{0.2 * 0.6} = 625^\circ C$$

P. 1-2 A temperature difference of 85°C is impressed across a fiberglass layer of 13 cm thickness. The thermal conductivity of the fiberglass is 0.035 W/m · °C. Compute the heat transferred through the material per hour per unit area.

Soly:- Find $\frac{q}{A}$ in $J/h.m^2$. $\quad \Delta T = 85^\circ C,\ \Delta x = 13\ cm = 0.13\ m,\ K = 0.035\ \frac{W}{m.C^\circ}$

انظر للملاحظة في السؤال السابق.

$$\frac{q}{A} = K\frac{\Delta T}{\Delta x} = \frac{0.035 * 85}{0.13} = 22.885\ \frac{W}{m^2} = 22.825\ \frac{J}{S.m^2}$$

$$\frac{q}{A} = 22.885\ \frac{J}{\cancel{S}.m^2} * \frac{60\cancel{S}}{1\cancel{min}} * \frac{60\cancel{min}}{1h} \Rightarrow \boxed{\frac{q}{A} = 82386\ J/h.m^2}$$

P. 1-4 The temperatures on the faces of a plane wall 15 cm thick are 375 and 85°C. The wall is constructed of a special glass with the following properties: $k = 0.78$ W/m · °C, $\rho = 2700$ kg/m^3, $c_P = 0.84$ kJ/kg · °C. What is the heat flow through the wall at steady-state conditions?

Soly:- Find $\frac{q}{A}$? $\quad \Delta x = 15\ cm = 0.15\ m,\ K = 0.78$

$$\frac{q}{A} = -K\frac{\Delta T}{\Delta x} = -0.78 * \frac{(85 - 375)}{0.15} \Rightarrow \boxed{\frac{q}{A} = 1508\ W/m^2}$$

* General Heat conduction equation:

- $dV = A\,dx$
- $dm = \rho\,dV$
  $\Rightarrow dm = \rho A\,dx$
  $\therefore \boxed{m = \rho \Delta V}$
  $\boxed{m = \rho A dx}$

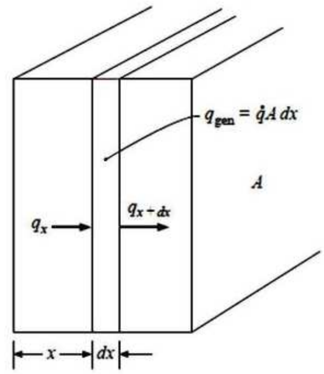


Figure 1-2 | Elemental volume for one-dimensional heat-conduction analysis.

* Energy Balance For the Element:

[1] Heat conducted through left face + [2] Heat generated within element = [3] Heat conducted from right face + [4] Heat stored within the element [Internal Energy]

$\Longrightarrow$ [1] $q_x = -KA\dfrac{\partial T}{\partial x}$

[2] $q_{gen} = \dot{q}A\,dx$

$\dot{q}$:
- rate of heat generated per unit volume
- Electric current passes through a resistance
- Exothermic reaction
- Nuclear reaction.
- (W/m^3)

[3] $q_{x+dx} = q_x + \dfrac{\partial}{\partial x}(q_x)\,dx$

$$= -KA\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(-KA\frac{\partial T}{\partial x}\right)dx$$

[4] $q_{stored} = m \cdot C \dfrac{\partial T}{\partial \tau}$ (T: Temp, τ: Time)

$$= \rho A dx\, C\frac{\partial T}{\partial \tau}$$

C: Specific heat } From tables.
ρ: Density
τ: Time

$\boxed{1} + \boxed{2} = \boxed{3} + \boxed{4}$

$$\boxed{\frac{\partial}{\partial x}\left[k\frac{\partial T}{\partial x}\right] + q^{\circ} = \rho c\frac{\partial T}{\partial \tau}}$$

or

$$\boxed{\frac{\partial^2 T}{\partial x^2} + \frac{q^{\circ}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}}$$

General one-dimensional conduction Heat transfer.

* α : Thermal Diffusivity m^2/s

$$\alpha = \frac{k}{\rho c}$$

* For three-dimensional H.T. conduction :

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q^{\circ}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Example: A 0.5m thickness wall made from a material (cp= 5000 J/kg.k , ρ=1600 kg/m³ , k= 50 w/m.k) , temp distribution is given by $T(x) = a+bx+cx^2$ a= 700 C˚ , b= -200 C˚/m , c= -20 C˚/m² , A=10m² , q˚= 2000 w/m³ Determine :

1- Rate of H.T entering and leaving the wall .

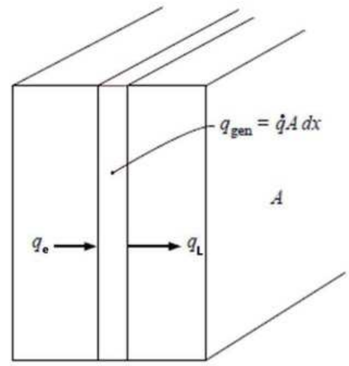
2- Rate of storing energy within the wall.

3- Rate of change of temperature @ x=0.125m

x=0.25 m

Sol:

اولا قبل ما تحل مثل هيك اسئلة اشتق المعادلة مرتين



$$T(x) = 700 - 200x - 20x^2$$

$$\frac{\partial T}{\partial x} = -200 - 40x$$

$$\frac{\partial^2 T}{\partial x^2} = -40 \qquad , \frac{\partial^3 T}{\partial x^3} = 0$$

① $$q_x \underset{x=0}{(\text{entering})} = -KA \left.\frac{\partial T}{\partial x}\right|_{x=0} = -50 * 10 * [-200 - 40(0)]$$

$$\boxed{q_x\big|_{x=0} = 100\ \text{kW}}$$

$$q_{x+dx} \underset{x=0.5}{(\text{leaving})} = -KA \left.\frac{\partial T}{\partial x}\right|_{x=0.5} + \cancel{\frac{\partial}{\partial x}\left(-KA \frac{\partial T}{\partial x}\right) dx}^{\,0.0} \leftarrow \text{المشتقة الثالثة}$$

$$q_{x+dx}\big|_{x=0.5} = -50 * 10 * [-200 - 40(0.5)] \Rightarrow \boxed{q_{x+dx} = 110\ \text{kW}}$$

② $$q_x + q_g = q_{x+dx} + q_{stored}$$

$$q_s = q_x + q_g - q_{x+dx}$$

$$q_s = 100 + 10 - 110$$

$$\boxed{q_s = 0}$$

$$\boxed{\begin{aligned} q_g &= \dot{q}\,\Delta V = \dot{q}\,\Delta x\, A \\ q_g &= 2000 * 0.5 * 10 \\ &\boxed{q_g = 10\ \text{kW}} \end{aligned}}$$

③ $$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial \tau}$$

Find. $$\left.\frac{\partial T}{\partial \tau}\right|_{\substack{x=0.125 \\ x=0.25}}$$

$$\boxed{\begin{aligned} \alpha &= \frac{k}{\rho c} = \frac{50}{1600 * 5000} \\ &\boxed{\alpha = 6.25 * 10^{-6}\ m^2/s} \end{aligned}}$$

إنتبه $\dfrac{\partial^2 T}{\partial x^2} = -40$ @ any value of x

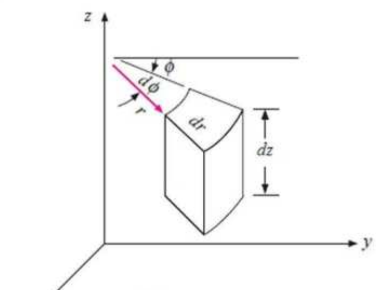
$$\frac{\partial T}{\partial \tau} = \left[-40 + \frac{2000}{50}\right] * 6.25 \times 10^{-6}$$ ثم تعويض مباشر بالمعادلة.

$$\boxed{\frac{\partial T}{\partial \tau} = 0}$$

* Steady state conduction.
* درجة الحرارة لا تتغير مع الزمن.

* Cylindrical coordinates. (3-dimension) Fig 1-3 (b)

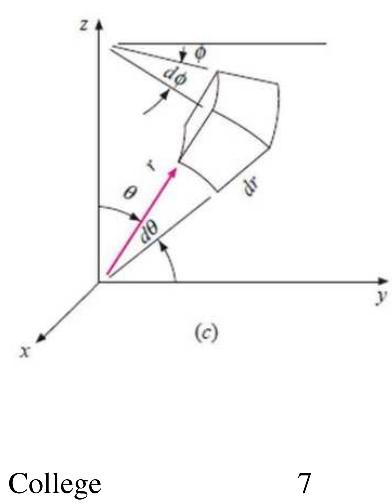
$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q^{\circ}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$



(b)

* Spherical coordinates (3 - dimension) Fig 1-3 (c)

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left[\sin\theta \frac{\partial T}{\partial \theta}\right] + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{q^{\circ}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$



(c)