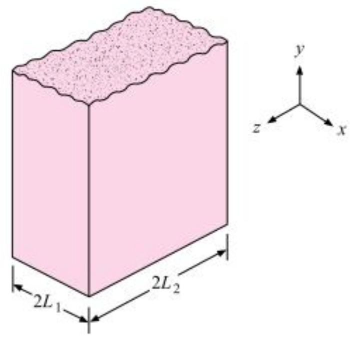
Multi-Dimensional Unsteady-state Heat Conduction

The Heisler charts are used to obtain the temperature distribution in the infinite plate of thickness 2L, in the long cylinder, or in the sphere. For the problems in which heat transferred with more than one dimensional, it is possible to combine the solutions for one-dimensional systems obtain the solutions for the multidimensional problems.

For the rectangular bar,

$$\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{\text{bar}}=\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{2L_1\ plate}\left(\frac{T-T_\infty}{T_i-T_\infty}\right)_{2L_2\ plate}$$

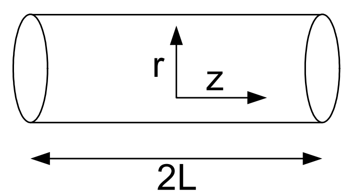


or

$$\left(\frac{\theta}{\theta_i}\right)_{bar}=\left(\frac{\theta}{\theta_i}\right)_{2L_1}\left(\frac{\theta}{\theta_i}\right)_{2L_2}$$

For short cylinder r,

$$\left(\frac{\theta}{\theta_i}\right)_{cylM}=\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{inf}\,inite\\ cyl.}}\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{inf}\,inite\\ plate}}$$



if $C(\theta)$ = solution for infinite cylinder

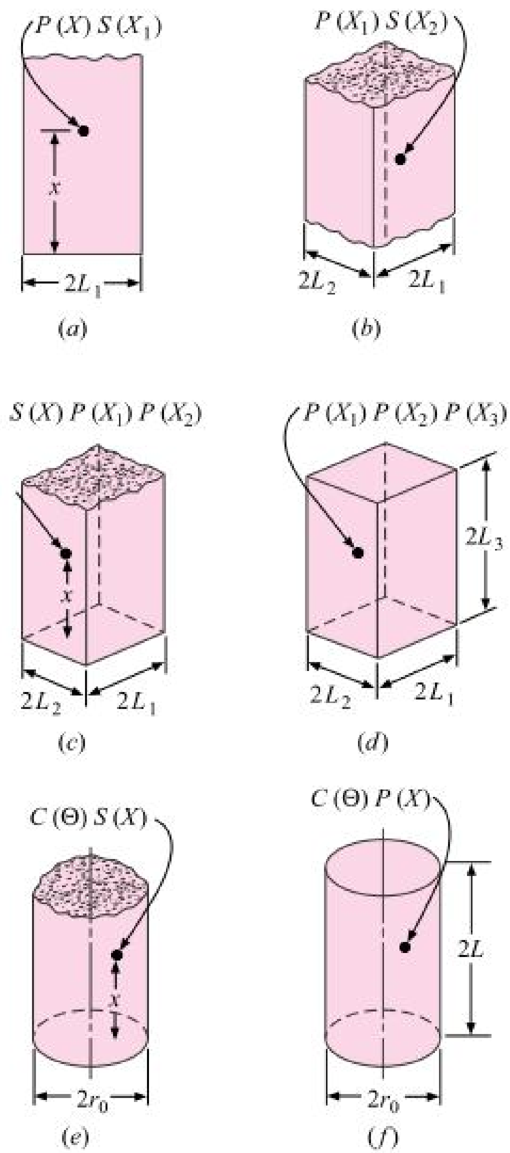
P(x) = solution for infinite plate

S(x) = solution for semi-infinite solid

The general is

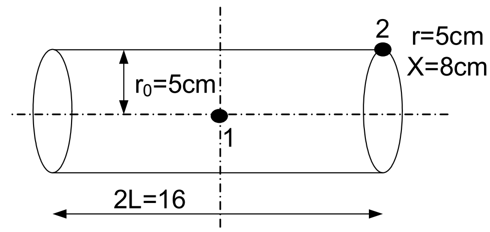
$$\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{combined}\\ \text{solid}}}=\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{intersection}\\ \text{solid 1}}}\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{intersection}\\ \text{solid 2}}}\left(\frac{\theta}{\theta_i}\right)_{\substack{\text{intersection}\\ \text{solid 3}}}$$

Figure 4-18 | Product solutions for temperatures in multidimensional systems: (*a*) semi-infinite plate; (*b*) infinite rectangular bar; (*c*) semi-infinite rectangular bar; (*d*) rectangular parallelepiped; (*e*) semi-infinite cylinder; (*f*) short cylinder.



Example: A 10cm diameter, 16cm long cylinder with properties: k=0.5 W/m.K and α=5 ×10^{-7} m^2/s is initially at a temperature of 20°C. The cylinder is placed in an oven where the ambient temperature is 500°C, *h* = 30 W/m^2.K. Determine the min and max temperature in the cylinder 30min after it has been placed in the oven.

Solution



$2L = 16 \Rightarrow L = x = 8cm, \quad t = 30 \times 60 = 1800\,\text{sec}$

$min\ temperature\ at\ x = 0, \quad r = 0$

$max\ temperature\ at\ x = L, \ r = r_0$

1) *infinite plate*

$$\frac{\alpha t}{L^2} = 0.14, \quad \frac{k}{hL} = 0.21$$

$$From\ figure\ 4.9\ \frac{\theta_0}{\theta_i} = 0.9$$

$$From\ figure\ 4.12,\ \frac{\theta}{\theta_0} = 0.27$$

$$\frac{\theta}{\theta_i} = \frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_i} = 0.27 \times 0.9 = 0.249$$

2) *infinite cylinder*

$$\frac{k}{hr_0} = 0.33, \quad \frac{\alpha t}{r_0^2} = 0.36$$

$$From\ figure\ 4.10\ \frac{\theta_0}{\theta_i} = 0.47$$

$$From\ figure\ 4.13,\ \frac{\theta}{\theta_0} = 0.33$$

$$\frac{\theta}{\theta_i} = \frac{\theta}{\theta_0} \cdot \frac{\theta_0}{\theta_i} = 0.33 \times 0.47 = 0.155$$

$$The\ min\ temperature = \left.\frac{\theta_0}{\theta_i}\right|_{combined} = \left.\frac{\theta_0}{\theta_i}\right|_{plate} \cdot \left.\frac{\theta_0}{\theta_i}\right|_{cylinder} = 0.9 \times 0.47 = 0.423$$

$$\therefore T_{min} = 500 + 0.423(20 - 500) = 297°C$$

$$The\ max\ temperature = \left.\frac{\theta}{\theta_i}\right|_{combined} = \left.\frac{\theta}{\theta_i}\right|_{plate} \cdot \left.\frac{\theta}{\theta_i}\right|_{cylinder} = 0.249 \times 0.155 = 0.039$$

$$\therefore T_{max} = 500 + 0.039(20 - 500) = 481°C$$