المحاضرة السادسة

* Condection - convection systems (fins) الزعانف

الهدف من الزعانف هو زيادة نقل الحرارة

fins extended surface that are designed to increase heat transfer

cas

cas 1: the fin is very long, and the temp at the end of the fins is essentially that of the surrounding fluid.

$$q = \sqrt{hpkA}\,\theta_o$$

T_o: درجة حرارة (fin) او base Temp

$\theta_o = T_o - T_\infty$

T_∞ = fluid temp

A = مساحة fin

P = المحيط

$$\theta = T - T_\infty \quad , \quad \theta_o = T_o - T_\infty \quad \frac{\theta}{\theta_o} = e^{-mx}$$

$$m = \sqrt{\frac{hP}{kA}}$$

2-71 A long, thin copper rod 5 mm in diameter is exposed to an environment at 20 ∘ C.The base temperature of the rod is 120 ∘ C. The heat-transfer coefficient between the rod and the environment is 20 W/m2 · ∘ C. Calculate the heat given up by the rod.

P.2-71) copper rod, $d = 5\,mm$, $h = 20\,W/m^2 \cdot {}^\circ C$

$T_\infty = 20^\circ C$

$T_o = 120^\circ C$

$K_{copper} = 372$

Find q:

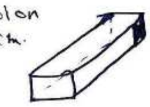
$$q = \sqrt{hPKA}\,\theta_o \qquad \begin{cases} P = 2\pi r = \pi d = \pi \times 0.005 = 0.0157\ m \\ A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.005)^2 = 1.96349 \times 10^{-5}\ m^2 \end{cases}$$

$$q = \sqrt{20 \times 0.0157 \times 372 \times 1.96 \times 10^{-5}} \times (120 - 20)$$

$$\boxed{q = 4.7848\ W}$$

2-85 A long stainless-steel rod [k =16 W/m・ ∘ C] has a square cross section 12.5 by 12.5 mm and has one end maintained at 250 ∘ C. The heat-transfer coefficient is 40 W/m2 C, and the environment temperature is 90 ∘ C. Calculate the heat lost by the rod.

stainless-steel rod. ($K = 16$) square cross section 12.5 cm × 12.5 cm.



$T_o = 250°C$
$T_\infty = 90°C$
$h = 40$
find q.

بالكتاب 12.5mm
انا اعتبرتها 12.5 cm

$$q = \sqrt{hPKA}\,\theta_o \qquad \begin{cases} P = 12.5 \times 4 = 50\ cm = 0.5\ m \\ A = (0.125)^2 = 0.015625\ m^2 \end{cases}$$

$$q = \sqrt{40 \times 0.5 \times 16 \times 0.015625} \times (250 - 90)$$

$$\boxed{q = 357.77\ W}$$

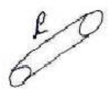
Case 2) The fin is of finite length and loses heat by convection from it's end.

$$q = \sqrt{hPKA}\,\theta_o \frac{\sinh(mL) + \left(\frac{h}{mK}\right)\cosh(mL)}{\cosh(mL) + \left(\frac{h}{mK}\right)\sinh(mL)} \qquad m = \sqrt{\frac{hP}{KA}}$$

$$\frac{\theta}{\theta_o} = \frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh[m(L-x)] + (h/mK)\sinh[m(L-x)]}{\cosh(mL) + \frac{h}{mK}\sinh(mL)}$$

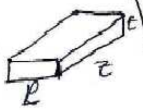
$L \rightarrow$ fin طول الـ

$x \rightarrow$ البعد المطلوب ايجاد T عنده



$A = \frac{\pi}{4} d^2$
$P = \pi d$ ← rod ← case 2 يحسبه حله على ← إذا كانت له طول معين

أو نحوله الى case3



$P = 2z + 2t$
$A = zt$ ← rectangular

If $z >>> t$ ...

$\frac{P}{A} = \frac{2z + 2t}{zt} = \frac{2}{t}$

2-68 An aluminum rod 2.0 cm in diameter and 12 cm long protrudes from a wall that is maintained at 250 ◦ C. The rod is exposed to an environment at 15 ◦ C. The convection heat-transfer coefficient is 12 W/m2 · ◦ C. Calculate the heat lost by the rod.

Aluminum rod $d = 2\,cm$, $L = 12\,cm$

$T_{wall} = 250\,°C$
$T_\infty = 15\,°C$
$h = 12\ w/m^2\text{-}c$
$k = 204\ w/m\text{-}c$

find $q = ?$

$$q = \sqrt{hpKA}\ \theta_o\ \frac{\sinh(mL) + \frac{h}{mk}\cosh(mL)}{\cosh(mL) + \frac{h}{mk}\sinh(mL)}$$

rod.

$p = \pi d = \pi * 0.02 = 0.0628\ m$

$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.02)^2 = 3.1415 * 10^{-4}\ m^2$

$m = \sqrt{\frac{hp}{KA}} = \sqrt{\frac{12 * 0.0628}{204 * 3.1415 * 10^{-4}}} = 3.429$

$mL = 3.429 * 0.12 = 0.41149$

$\frac{h}{mk} = 0.01715$

$\theta_o = 250 - 15$

$$q = \sqrt{12 * 0.628 * 204 * 3.1415 * 10^{-4}}\ (250 - 15) * \frac{\sinh(0.41149) + 0.01715 \cosh(0.41149)}{\cosh(0.41149) + 0.01715 \sinh(0.41149)}$$

$q = 20.8735\ W$

general $L_c = L + \frac{A}{P}$ للكرسي بالتشريح

$L_c \rightarrow$ corrected length.

rectangular

$A = Zt$

$P = 2Z + 2t$

$L_c = L + \frac{A}{P}$

case3 ثم حل

. تحول case 2 إلى case3

rod. (circular fin)

$L_c = L + \frac{d}{4}$

$\frac{A}{P} = \frac{\frac{\pi}{4} d^2}{\pi d}$

$\frac{A}{P} = \frac{d}{4}$

$m L_c$

ثم حل case3

قانون رسم

* case 3 The end of the fin is insulated so that $\frac{dT}{dx} = 0$ at $x = L$

$$q = \sqrt{hPKA}\ \theta_o \tanh(mL_c)$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\theta}{\theta_o} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}$$

at the end of fin $x = L_c$

هنا يطلب درجة الحرارة عند

طول معين هو X إذا طلب عند

آخر ال fin تكون $L_c = X$

$m = \sqrt{\frac{hP}{KA}}$

L_c ... rod

rectangular

P.2-68 Aluminum rod. $d = 2\,cm$, $L = 12\,cm$, $T_w = 250°C$, $T_\infty = 15°C$

$h = 12$, $K = 204$

sol:-

$L_c = L + \frac{d}{4} = 12 + \frac{2}{4} = 12.5\,cm$

$A = \frac{\pi}{4} d^2 = 3.1415 \times 10^{-4}\,m^2$

$P = 0.0628\,m$

$m = \sqrt{\frac{hP}{KA}} = 3.429$

$m L_c = 3.429 \times 0.125 = 0.428625$

$q = \sqrt{hPKA}\ \theta_o \tanh(mL_c) = \sqrt{12 * 0.0628 * 204 * 3.1415 \times 10^{-4}}\ (250 - 15) * \tanh(0.428625)$

$q = 20.87313\,W$

Case 3

Rectangular.

L → length
t → thickness.

① نجد L_c $\quad L_c = L + \frac{t}{2}$

$\boxed{A_m = t L_c}$

② نجد $L_c^{3/2}\left(\frac{h}{k A_m}\right)^{1/2}$

③ عن طريق الرسمة ⑪ Fig 2-11 نجد η_f

④ نعوض في المعادلة لنجد q

$\boxed{q = \eta_f h A \theta_0}$

$\boxed{A = 2 L_c W}$

2-74 A straight fin of rectangular profile has a thermal conductivity of 14 W/m · ∘C, thickness of 2.0 mm, and length of 23 mm. The base of the fin is maintained at a temperature of 220 ∘C while the fin is exposed to a convection environment at 23 ∘C with h = 25 W/m2 · ∘C. Calculate the heat lost per meter of fin depth.

$k = 14$ W/m.c , $t = 2$ mm $\quad L = 23$ mm

$T_0 = 220°c \quad T_\infty = 23°c \quad h = 25 \quad$ find q ?

Solve

$$L_c = L + \frac{t}{2} = 23 + \frac{2}{2} = 24 \text{ mm} = \boxed{0.024 \text{ m}}$$

$$L_c^{\frac{3}{2}}\left(\frac{h}{kA_m}\right)^{\frac{1}{2}} = (0.024)^{\frac{3}{2}}\left(\frac{25}{14 * 0.002 * 0.024}\right)^{\frac{1}{2}} = 0.717$$

from fig 2-11 $\quad \eta_f = 0.75$

$$q = \eta_f h A \theta_0 = \eta_f h \, 2 L_c W \theta_0$$

$$\frac{q}{W} = 0.75 * 25 * 2 * 0.024 * (220 - 23)$$

$$= \boxed{177.3 \text{ W/m}}$$

Case 3 triangular.

$L_c = L$ ①

$$A_m = \frac{t}{2} L_c$$

$L_c^{3/2} \left(\frac{h}{KA_m} \right)^{\frac{1}{2}}$ ②

③ عن طريق الرسمة ⑪ Fig 2-11 نجد η_f

$q = \eta_f h A \theta_o$ ④

$$A = 2w \sqrt{L^2 + \frac{t^2}{4}}$$

2-77 A triangular fin of stainless steel (18% Cr, 8% Ni) is attached to a plane wall maintained at 460 ∘ C. The fin thickness is 6.4 mm, and the length is 2.5 cm. The environment is at 93 ∘ C, and the convection heat-transfer coefficient is 28 W/m2 ・ ∘ C. Calculate the heat lost from the fin.

P. 2-77) مثال Stainless steel (18% Cr, 8% Ni) $\Rightarrow K = 16.3$, $T_o = 460°C$

$t = 6.4$ mm, $L = 2.5$ cm, $T_\infty = 93°C$, $h = 28$, find q:

Sol: Assume $w = 1\,m$

$L_c = L = 0.025\ m$

$$A_m = \frac{t}{2} L_c = \frac{0.0064}{2} * 0.025 = 8 * 10^{-5}\ m^2$$

$$L_c^{3/2} \left(\frac{h}{KA_m} \right)^{\frac{1}{2}} = 0.579$$

from fig 2-11 $\rightarrow \eta_f = 0.85$

$$q = \eta_f h A \theta_o$$

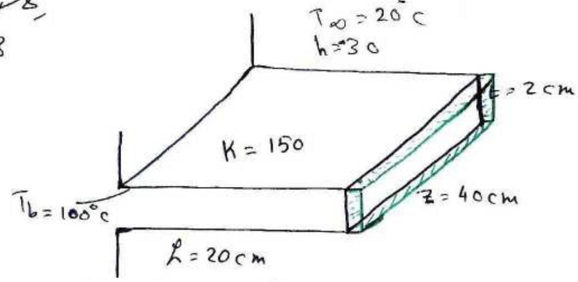
$$q = \eta_f h \left(2w \sqrt{L^2 + \frac{t^2}{4}}\right) (T_o - T_\infty) \Rightarrow q = 440.29\ W$$

* لاحظ أنه المساحة مضروبة في 2 في القانون النهائي لأنه انتقال الحرارة يكون على سطحي الـ fin

Example:- Determain the heat transfer rate from the rectangular fin. The tip of the fin is not insulated, and the fin has thermal conductivity of 150 w/m.c, The Base temp is 100°c and the the fluid is @ 20°c, the heat transfer coefficient between the fin and air is 30 w/m².c°

يمكن حله مباشرة على case 2 أو تحويله إلى case3 .

هنا سنحوّله ونحله على case3 .



Sol/y:-

①

corrected length

$$L_c = L + \frac{A}{p} = L + \frac{Zt}{2Z + 2t}$$

$$L_c = 0.2 + \frac{0.4 * 0.02}{2 * 0.4 + 2 * 0.02} = 0.2095$$

$p = 2Z + 2t = 0.84$

$A = Zt = 0.008$

$m = \sqrt{\frac{hp}{kA}} = \sqrt{\frac{30 * 0.84}{150 * 0.008}} = 4.58$

$mL_c = 4.58 * 0.2095 = 0.96$

$$q = \sqrt{hpkA}\ \theta_0 \tanh(mL_c)$$

$$\Rightarrow q = \sqrt{30 * 0.84 * 150 * 0.008} * (100 - 20) \tanh(0.96)$$

$$= 327.43\,W$$

② Determain the temp of the end of the fin. $\boxed{x = L_c}$

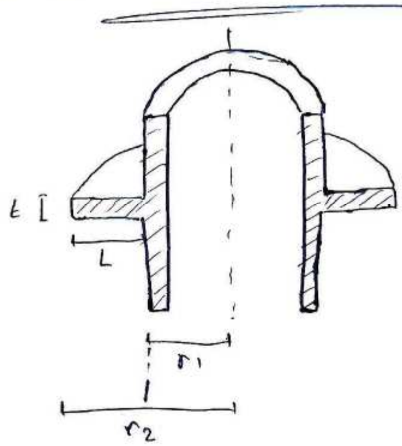
$$\frac{\theta}{\theta_o} = \frac{\cosh[m(L_c - x)]}{\cosh(mL_c)}$$

$$\frac{T - T_\infty}{T_o - T_\infty} = \frac{\cosh m(0)}{\cosh(0.96)} \Rightarrow \frac{T - 20}{100 - 20} = \frac{1}{1.49729}$$

$$\boxed{T_{x=L_c} = 73.43^\circ C}$$

* ملاحظة: بإمكانك حل السؤال على <u>case2</u>

<u>case 3</u> circumferential fin of rectangular profile.



① $r_1 = r_2 - L$

② $L_c = L + \frac{t}{2}$

③ $r_{2c} = r_1 + L_c$

④ $A_m = t L_c$

⑤ r_{2c}/r_1

⑥ $L_c^{3/2} \left(\frac{h}{k A_m}\right)^{1/2}$

⑦ نجد η_f ونعوض في ② عن Fig 2-12 من

$$q = \eta_f h A \theta_o$$

$$\boxed{A = 2\pi(r_{2c}^2 - r_1^2)}$$

2-75 A circumferential fin of rectangular profile is constructed of a material having $k = 55 \text{W/m} \cdot {}^\circ\text{C}$ and is installed on a tube having a diameter of 3 cm. The length of fin is 3 cm and the thickness is 2 mm. If the fin is exposed to a convection environment at 20 ∘ C with a convection coefficient of 68 W/m2 · ∘ C and the tube wall temperature is 100 ∘ C, calculate the heat lost by the fin.

$k = 55$, $d = 3\text{cm}$, $r_1 = 1.5\text{ cm}$, $L = 3\text{ cm}$, $t = 2\text{mm}$
$T_\infty = 20^\circ C$, $h = 68$, $T_w = 100^\circ C$, find q.

- $L_c = L + \frac{t}{2} = 0.03 + \frac{0.002}{2} = 0.031\ m$
- $r_{2c} = r_1 + L_c = 0.015 + 0.031 = \boxed{0.046\ m}$
- $A_m = t L_c = 0.002 * 0.031 = \boxed{6.2 * 10^{-5}\ m^2}$
- $r_{2c}/r_1 = \boxed{3.0667}$
- $L_c^{3/2}\left(\frac{h}{K A_m}\right)^{\frac{1}{2}} = (0.031)^{3/2}\left(\frac{68}{55 * 6.2 * 10^{-5}}\right)^{\frac{1}{2}} = \boxed{0.77}$

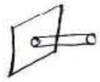
$\eta_f \rightarrow$ from fig 2-12 ② $\Rightarrow \boxed{\eta_f = 0.6}$

$$q = \eta_f\ h A\ \theta_o = \eta_f h\ (2\pi(r_{2c}^2 - r_1^2))\ (T_o - T_\infty)$$

$$\boxed{q = 38.78\ W}$$

2-89 An aluminum block is cast with an array of pin fins protruding like that shown in Figure 2-10*d* and subjected to room air at 20 ∘ C. The convection coefficient between the pins and the surrounding air may be assumed to be *h* = 13.2 W/m2 ⋅ ∘ C. The pin diameters are 2 mm and their length is 25 mm. The base of the aluminum block may be assumed constant at 70 ∘ C. Calculate the total heat lost by an array of 15 by 15, that is, 225 fins

Pin fins (rod)



, $T_\infty = 20°C$, $h = 13.2$, $d = 2mm$, $L = 25mm$, $T_o = 70°C$
find q, q for 225 fins

$\boxed{K = 204}$ Aluminum

$$q = \sqrt{hPKA}\ \theta_o\ \tanh(m L_c)$$

Sol:

$L_c = L + \frac{d}{4} = 0.025 + 0.002/4 = 0.0255$

$$m = \sqrt{\frac{hP}{KA}} = \sqrt{\frac{h \pi d}{K \frac{\pi}{4} d^2}} = \sqrt{\frac{4h}{Kd}} = \sqrt{\frac{4 * 13.2}{204 * 0.002}} = \boxed{11.38}$$

$$q_{(one\ pin)} = \sqrt{13.2 * \pi * 0.002 * 204 * \frac{\pi}{4}(0.002)^2}\ (70-20)\ \tanh(11.38 * 0.0255)$$

$\boxed{q = 0.10291\ W}$ for one pin.

$q_{(225\ pin)} = 225 * 0.10291 = \boxed{23.15\ W}$