Electromagnetic waves

Lecture 1

Vector

Araa Hassan

Tow stage

Department of medical physics
Al-Mustaqbal University-College

1-Vector Analysis and Vector Algebra

1-1Vector is a quantity having both magnitude and direction such as displacement, velocity, force and acceleration.

Example1

Write the vector for each of the following:

a. of the vector (1, −3, −5) to(2, −7,0).

b . of the vector (2, −7,0) to) 1, −3, −5).

c .The location vector to (4,90.(− .)

solution

a$\langle -1,4,-5 \rangle$ b$\langle 1,-4,5 \rangle$

The two vectors in a and b are different in sign only, and this shows that they have the same magnitude, but they are opposite

direction.

c$\langle -90,4 \rangle$

1-2Vector Algebra

Laws of vector algebra. If A, B and C are vectors and m and n are scalars, then

1. A+ B = B + A Commutative Law for Addition
2. A+ (B+C) = (A+B) + C Associative Law for Addition
3. mA = Am Commutative Law for Multiplication
4. m (nA) = (mn) A Associative Law for Multiplication
5. (m+ n) A = mA + nA Distributive Law
6. m (A+ B) = mA + mB Distributive Law

Example:

Let us take A = 10 and B = 5

10 + 5 = 5 + 10

15 = 15

Example:

Prove:- (3+7) = (-3)+(-7)

Proof:

-(10) = -3-7

-10 = -10

L.H.S = R.H.S

Example:

Let us take A = 2, B = 4 and C = 6

L.H.S =A+(B+C) = 2 + (4 + 6)

= 12

R.H.S = (A+B)+C = (2 + 4) + 6

=12

L.H.S = R.H.S

12 = 12

Example:

Let us take A = 2, B = 3 and C = 5

L.H.S =A × (B + C)= 2 × (3+5)

= 2 × 8

= 16

R.H.S = A × B + A × C = 2 × 3 + 2 × 5

=6+ 10

=16

L.H.S = R.H.S

16 = 16

Example

A=4,m=5

mA = Am

5x4=4x5

20=20

Example

A=5

m=3

n=2

m (nA) = (mn) A

3(2x5)=(3x2)5

3x10=6x5

30=30

1-3 Scalar product

The process of multiplying a vector quantity by another vector quantity, the product of which is a non-vector scalar quantity, which has only an amount..

- The dot product of two vectors is given by the formula $\vec{a} \cdot \vec{b} = |a||b| \cos(\theta)$.

1-3-1 The following laws are valid:

1. A . B = B . A Commutative Law for Dot Products
2. A . (B + C) = A . B + A . C Distributive Law
3. m(A . B) = (mA) . B = A . (m B) = (A . B)m, where mis a scalar.
4. i . i = j . j = k . k = 1, i . j = j . k = k . i = 0
5. If A = Al i + A2 j + A3 k and B = Bl i + B2 j + B3 k, then
A . B = A1B1+A2 B2+A3 B3
A . A = A2 = A1 2 + A2 2 + A3 2
B . B = B 2 = B1 2 + B2 2 + B3 2
6. If A . B = 0 and A and B are not null vectors, then A and B are perpendicular.

Example: Find the scalar product of the vectors a = 2i + 3j - 6k and b = i + 9k.

Solution: To find the scalar product of the given vectors a and b, we will multiply their corresponding components.

$a.b = (2i + 3j - 6k).(i + 0j + 9k)$

$= 2.1 + 3.0 + (-6).9$

$= 2 + 3 - 54$

$= -49$

Example: Calculate the scalar product of vectors a and b when the modulus of a is 9, modulus of b is 7 and the angle between the two vectors is 60°.

Solution: To determine the scalar product of vectors a and b, we will use the scalar product formula.

$a.b = |a|\ |b| \cos\theta$

$= 9 \times 7 \cos 60°$

$= 63 \times 1/2$

$= 31.5$

Example 1: Find the angle between the two vectors 2i + 3j + k, and 5i -2j + 3k.

Solution:

The two given vectors are:

$\vec{a}$ = 2i + 3i + k, and $\vec{b}$ = 5i -2j + 3k

$|a| = \sqrt{2^2 + 3^2 + 1^2} = \sqrt{4 + 9 + 1} = \sqrt{14}$

$|b| = \sqrt{5^2 + (-2)^2 + 3^2} = \sqrt{25 + 4 + 9} = \sqrt{38}$

Using the dot product we have $\vec{a} \cdot \vec{b}$ = 2.(5) + 3.(-2) + 1.(3) = 10 - 6 + 3 = 7

$$\text{Cos}\theta = \frac{a.b}{|a|.|b|}$$

$$= \frac{7}{\sqrt{14}.\sqrt{38}}$$

$$= \frac{7}{2.\sqrt{7 \times 19}}$$

$$= \frac{7}{2\sqrt{133}}$$

$$\theta = \text{Cos}^{-1}\frac{7}{2\sqrt{133}}$$

$\theta = \text{Cos}^{-1}\, 0.304 = 72.3°$

1-4 Vector products

The process of multiplying a vector quantity by another vector quantity, the product of which is a vector quantity with magnitude and direction.

- The cross product of two vectors is given by the formula $\vec{a} \times \vec{b} = |a||b| \sin(\theta)$.

Cross or vector product

1-4-1 The following laws are valid:

1. $A \times B = - B \times A$ Commutative Law for Cross Products Fails
2. $A \times (B + C) = A \times B + A \times C$ Distributive Law
3. $m(A \times B) = (mA) \times B = A \times (mB) = (A \times B)m$, where m is a scalar.
4. $i \times i = j \times j = k \times k = 0$,
$i \times j = -j \times i = k$,
$j \times k = -k \times j = i$,
$k \times i = -i \times k = j$.

5. If $A = Al\ i + A2\ j + A3\ k$ and $B = Bl\ i + B2\ j + B3\ k$, then
6. The magnitude of $A \times B$ is the same as the area of a parallelogram with sides A and B.
7. If $A \times B = 0$ and A and B are not null vectors, then A and B are parallel.

Example: Find the cross product of two vectors $\vec{a} = (3,4,5)$ and $\vec{b} = (7,8,9)$

Solution:

The cross product is given as,

$$a \times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 5 \\ 7 & 8 & 9 \end{vmatrix}$$

$= [(4\times9)-(5\times8)]\,\hat{i} - [(3\times9)-(5\times7)]\,\hat{j} + [(3\times8)-(4\times7)]\,\hat{k}$

$= (36-40)\hat{i} - (27-35)\hat{j} + (24-28)\,\hat{k} = -4\hat{i} + 8\hat{j} - 4\hat{k}$

Example: Two vectors have their scalar magnitude as $|a|=2\sqrt{3}$ and $|b| = 4$, while the angle between the two vectors is 60°.

Calculate the cross product of two vectors.

Solution:

We know that $\sin 60^\circ = \sqrt{3}/2$

The cross product of the two vectors is given by, $\vec{a} \times \vec{b} = |a||b|\sin(\theta)\hat{n} = 2\sqrt{3}\times4\times\sqrt{3}/2 = 12\hat{n}$

Example If $\vec{a}$ = (2, -4, 4) and $\vec{b}$ = (4, 0,3), find the angle between them.

Solution

$\vec{a} = 2i - 4j + 4k$

$\vec{b} = 4i + 0j + 3k$

The magnitude of $\vec{a}$ is

$|a| = \sqrt{(2^2+4^2+4^2)} = \sqrt{36} = 6$

The magnitude of $\vec{b}$ is

$|b| = \sqrt{(4^2+0^2+3^2)} = \sqrt{25} = 5$

As per the cross product formula, we have

$$\vec{a} \times \vec{b} = \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 4 \\ 4 & 0 & 3 \end{matrix}$$

$$= [(-4 \times 3) - (4 \times 0)]\hat{i}$$
$$-[(3 \times 2) - (4 \times 4)]\hat{j}$$

$$+[(2 \times 0) - (-4 \times 4)]\hat{k}$$

$$= -12\hat{i} + 10\hat{j} + 16\hat{k}$$

$$\vec{a} \times \vec{b} = (-12, 10, 16)$$

The length of the $\vec{c}$ is

$|c|=\sqrt{(-(12)^2+10^2+16^2)}$

$=\sqrt{(144+100+256)}$

$=\sqrt{500}$

$=10\sqrt{5}$

$$\vec{a} \times \vec{b} = |a||b|\sin\theta$$

$$\sin\theta = \frac{\vec{a} \times \vec{b}}{|a||b|}$$

$\sin\theta = 10\sqrt{5}/(5\times6)$

$\sin\theta = \sqrt{5}/3$

$\theta = \sin^{-1}(\sqrt{5}/3)$

$\theta = \sin^{-1}(0.74)$

$\theta = 48°$