CH.2 steady - state Conduction - one dimension.

$$* \quad \frac{\partial^2 (KT)}{\partial x^2} + \cancelto{0.0}{\frac{\partial^2 (KT)}{\partial y^2}} + \cancelto{0.0}{\frac{\partial^2 (KT)}{\partial z^2}} + \cancelto{0.0}{q} = \rho c \cancelto{0.0}{\frac{\partial T}{\partial \tau}}$$

Assumption :-

1. one dimensional H.T.
2. No Heat generation
3. steady. (لا يوجد تغير في درجة الحرارة مع الزمن)

$$\frac{\partial^2 (KT)}{\partial x^2} = 0 \quad \Rightarrow \quad \frac{d(KT)}{dx} = \text{const.} = -\frac{q}{A}$$

If K constant.

$$q = -KA\frac{\Delta T}{\Delta x}$$

تم الحل علاقة في CH.1

If K not constant.

$$K = K_o(1+\beta T)$$

$$q = -\frac{K_o A}{\Delta x}(T_2 - T_1)\left[1 + \frac{\beta}{2}(T_1 + T_2)\right]$$

اخراج عامل مشترك $(T_2 - T_1)$

or

$$q = \frac{-K_o A}{\Delta x}\left[(T_2 - T_1) + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

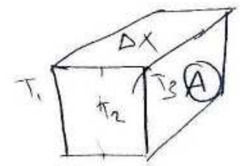
$T \rightarrow$ in $^\circ C$

2.2) $\Delta x = 2.5$ cm

$A = 0.1\ m^2$

$T_1 = 35^\circ C$, $T_2 = 62^\circ C$, $T_3 = 95^\circ C$

$q = 1000$ W



find → K as function of temp. $k = k_o(1+\beta T)$

find K ?

Sol :-

لنجد K نجد β و K_o

(1) $$q = -\frac{K_o A}{\Delta x}\left[T_3 - T_1 + \frac{\beta}{2}(T_3^2 - T_1^2)\right] = -\frac{K_o A}{\Delta x/2}\left[T_2 - T_1 + \frac{\beta}{2}(T_2^2 - T_1^2)\right]$$

$$95-35+\frac{\beta}{2}(95^2-35^2) = 2\left(62-35+\frac{\beta}{2}(62^2-35^2)\right)$$

$$6 = \beta(62^2-35^2) - \frac{\beta}{2}(95^2-35^2)$$

$$\boxed{\beta = -4.6838 * 10^{-3}}$$

(2) To find K_o.

$$q = -\frac{K_o A}{\Delta x}\left(T_3 - T_1 + \frac{\beta}{2}(T_3^2 - T_1^2)\right)$$

$$\boxed{K_o = 5.988}$$

2-2 A certain material 2.5 cm thick, with a cross-sectional area of 0.1 m2, has one side maintained at 35 ∘ C and the other at 95 ∘ C. The temperature at the center plane of the material is 62 ∘ C, and the heat flow through the material is 1 kW. Obtain an expression for the thermal conductivity of the material as a function of temperature.

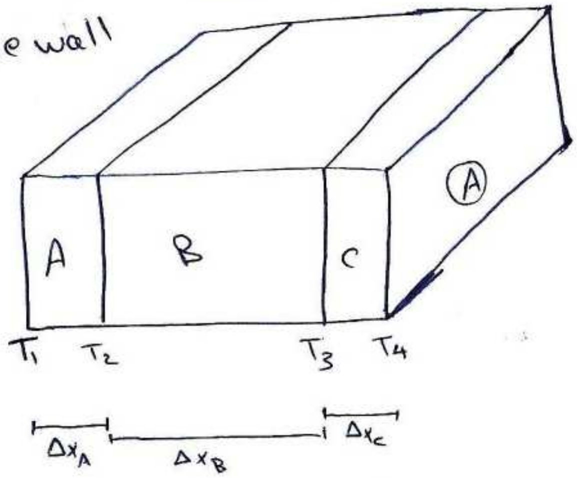
$\Rightarrow$ $K = K_o(1+\beta T)$

$$K = 5.988(1 - 4.6838 * 10^{-3}\, T)$$

* Multilayer wall.

ثلاث مواد مختلفة

plane wall



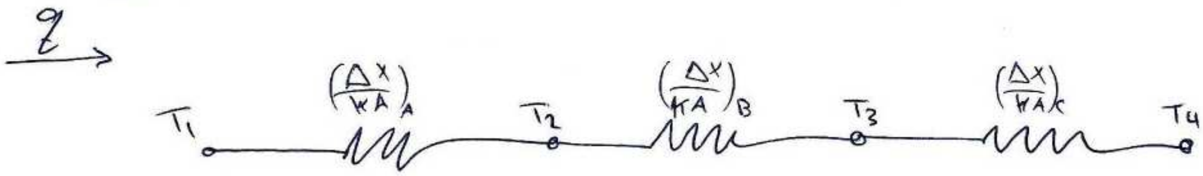
$$q_A = -K_A A \frac{(T_2 - T_1)}{\Delta x_A}$$

$$q_B = -K_B A \frac{(T_3 - T_2)}{\Delta x_B}$$

$$q_C = -K_C A \frac{(T_4 - T_3)}{\Delta x_C}$$

* Electric Analogy.

لتسهيل الموضوع..



$$q = \frac{\Delta T}{\sum R_{th}}$$

$R_{th} \rightarrow$ thermal resistance.

in sereise.

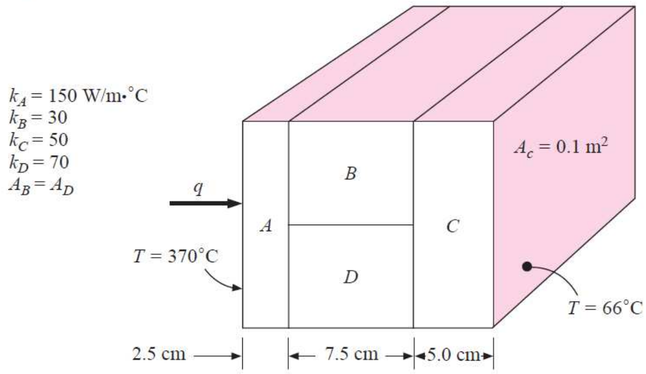
$$\sum R_{th} = \left(\frac{\Delta x}{KA}\right)_A + \left(\frac{\Delta x}{KA}\right)_B + \left(\frac{\Delta x}{KA}\right)_C$$

$$q = \frac{T_1 - T_4}{\left(\frac{\Delta x}{KA}\right)_A + \left(\frac{\Delta x}{KA}\right)_B + \left(\frac{\Delta x}{KA}\right)_C}$$

Note. IF Δx increas. R_{th} will increas.

2-4 Find the heat transfer per unit area through the composite wall in Figure P2-4. Assume one-dimensional heat flow.

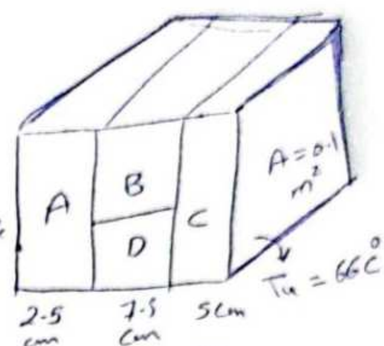
Figure P2-4



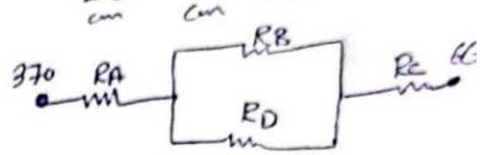
$k_A = 150$, $k_B = 30$

$k_C = 50$ $k_D = 70$

$A_B = A_D$



$$R_A = \frac{\Delta x}{kA} = \frac{0.025}{150 \times 0.1} = 1.667 \times 10^{-3}$$



$$R_B = \frac{0.075}{30 \times \frac{0.1}{2}} = 0.05$$

$$R_D = \frac{0.075}{70 \times \frac{0.1}{2}} = 0.02143$$

$$R_C = \frac{0.05}{50 \times 0.1} = 0.01$$

$$R_{BD} = \frac{1}{R_B} + \frac{1}{R_D}$$
$$= 0.015$$

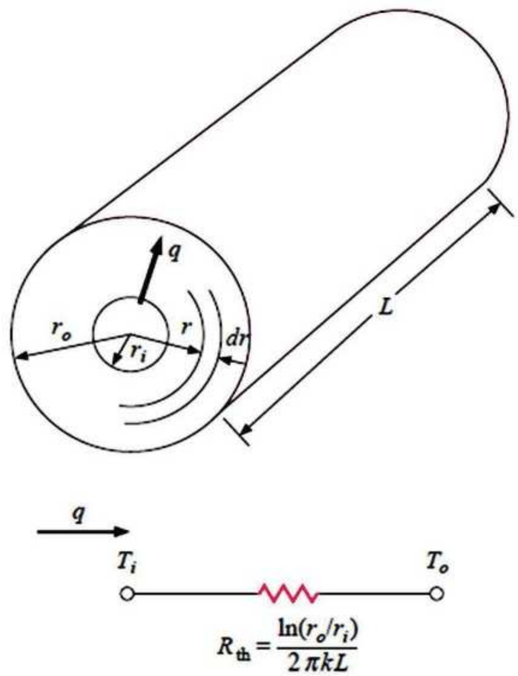
$$\Sigma R_{th} = R_A + R_C + R_{BD}$$
$$= 1.667 \times 10^{-3} + 0.01 + 0.015$$
$$= 2.667 \times 10^{-2}$$

$$q = \frac{\Delta T}{\Sigma R_{th}} = \frac{370 - 66}{2.667 \times 10^{-2}} = \boxed{11400 W}$$

* Radial systems

1 Cylinders.

Figure 2-3 | One-dimensional heat flow through a hollow cylinder and electrical analog.



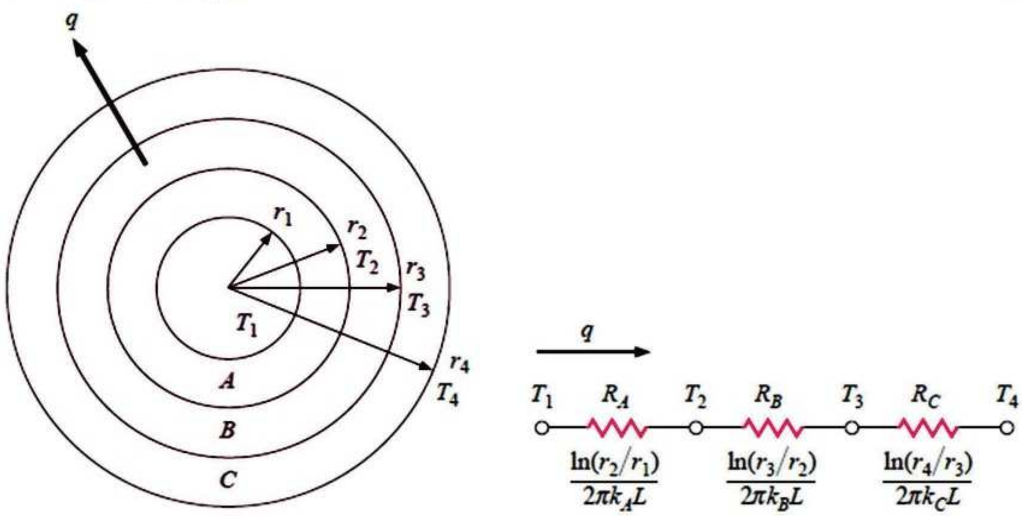
one dimensional heat transfer in the radial direction

$$q = -K A_r \frac{dT}{dr}$$

$$q = -K(2\pi r L)\frac{dT}{dr}$$

$$\frac{dr}{r}\, q = -2\pi K L\, dT$$

$$q = \frac{(T_i - T_o)}{\frac{\ln(r_o/r_i)}{2\pi K L}} = \frac{\Delta T}{R_{th.}}$$

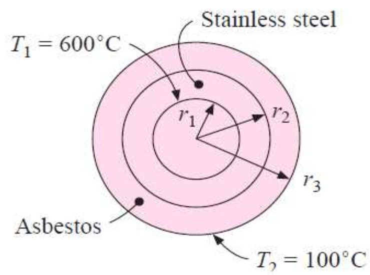


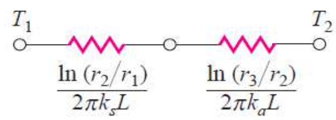
$$q = \frac{\Delta T}{R} = \frac{T_1 - T_4}{\frac{\ln(r_2/r_1)}{2\pi K_A L} + \frac{\ln(r_3/r_2)}{2\pi K_B L} + \frac{\ln(r_4/r_3)}{2\pi K_C L}}$$

EXAMPLE 2-2 Multilayer Cylindrical System

A thick-walled tube of stainless steel [18% Cr, 8% Ni, $k = 19$ W/m · °C] with 2-cm inner diameter (ID) and 4-cm outer diameter (OD) is covered with a 3-cm layer of asbestos insulation [$k = 0.2$ W/m · °C]. If the inside wall temperature of the pipe is maintained at 600°C, calculate the heat loss per meter of length. Also calculate the tube–insulation interface temperature.

Figure Example 2-2





$\boxed{\text{example 2-2}}$

نثبت درجة حرارة عازلة

$d_1 = 2cm$ $\quad k = 19$ C $\quad k = 0.2$ مادة عازلة asbestos

$d_2 = 4cm$

$\frac{q}{A} = ?$ $\quad T_1 = 600°C$ $\quad$ 3cm سمك

$T_2 = 100°C$

$$q = \frac{T_1 - T_2}{\frac{\ln(r_2/r_1)}{2\pi k_s L} + \frac{\ln(r_3/r_2)}{2\pi k_a L}}$$

$$\frac{q}{L} = \frac{2\pi\,(T_1 - T_2)}{\frac{\ln(r_2/r_1)}{k_s} + \frac{\ln(r_3/r_2)}{k_a}} = \frac{2\pi\,(600 - 100)}{\frac{\ln(2/1)}{19} + \frac{\ln(5/2)}{0.2}}$$

$$= 680 \text{ w/m}$$

$$\frac{q}{L} = \frac{T_a - T_2}{\frac{\ln(r_3/r_2)}{2\pi k_a}} = 680 = \frac{T_a - 100}{\frac{\ln(5/2)}{2\pi * 0.2}}$$

$$T_a = 595.8\ °C$$

2 spheres.

- one-dimensional
- temp is a function of radius only.

$$q = \frac{4\pi K (T_i - T_o)}{\frac{1}{r_i} - \frac{1}{r_o}}$$

P.2.17) sphere.

$$\boxed{R_{th} = \frac{\frac{1}{r_i} - \frac{1}{r_o}}{4\pi K}}$$

Aluminum

$d_i = 4\,cm$ $r_i = 2\,cm$

$d_o = 8\,cm$ $r_o = 4\,cm$.

$T_i = 100^\circ c$, $T_o = 50^\circ c$.

Find q.

$K \rightarrow$ from Table A-2. Aluminum pure

$K = 204\ w/m.^\circ c$

$$q = \frac{4\pi * 204\,(100-50)}{\frac{1}{0.02} - \frac{1}{0.04}} = \boxed{5127\ W}$$

* Convection Boundary conditions.

$$q_{conv} = hA(T_w - T_\infty)$$

صورة أخرى $q_{conv} = \frac{(T_w - T_\infty)}{\frac{1}{hA}} = \frac{\Delta T}{R_{th}}$

$R_{th} = \frac{1}{hA} \rightarrow$ convection resistance.