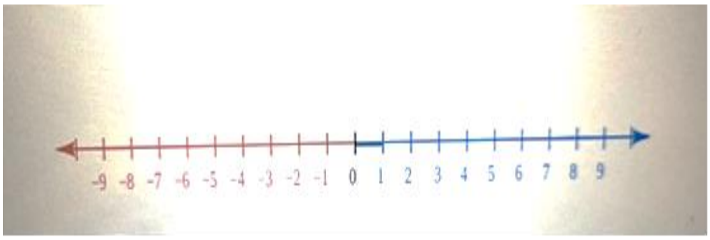
❖ Coordinate System

- Used to describe the position of a point in space
- Coordinate System consists of
  - A fixed reference point called the origin
  - Specific axes with scales and labels
  - Instruction on how to label a point relative to the origin and the axes

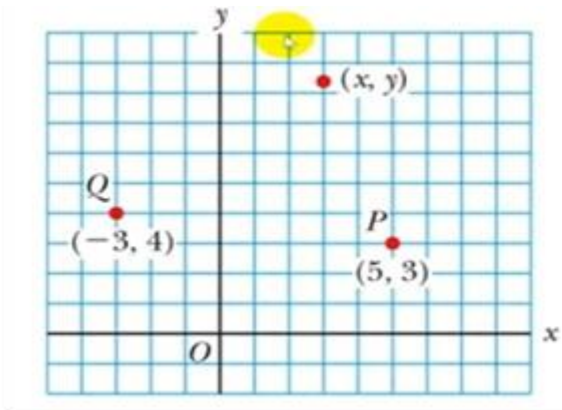
❖ Types of Coordinate Systems

- Number line.
- Cartesian coordinate system.
- Polar coordinate system.
- Cylindrical and spherical coordinate systems.
- Homogeneous coordinate system.
- Other commonly used systems.
- Relativistic coordinate systems.



❖ Cartesian Coordinate System

- Also called rectangular Coordinate System
- x- and y- axes intersect at the origin
- Points are labeled (x , y)
- The plural of axis is axes
- Ordered pair (x , y) with x-value first



Example /Determine the following points

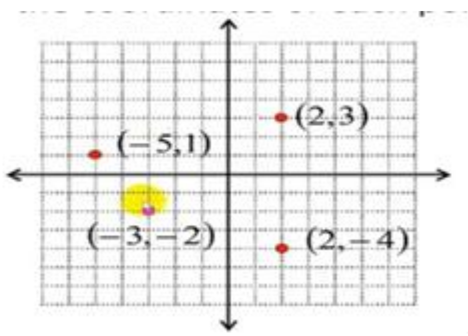
a.(2 ,3)

b.(-5 ,1)

c.(-3,-2)

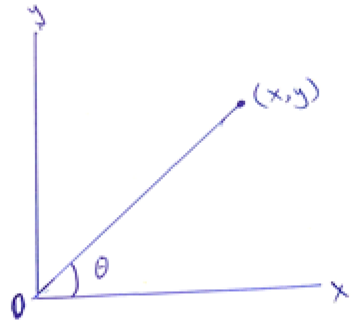
d.(2,-4)

solution :



❖ Polar coordinate system

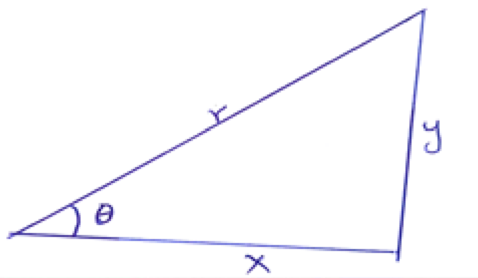
- Origin and reference line are noted.
- Point is distance r from the origin in the direction of angle θ , ccw from reference line .
- Points are labeled (r , θ) .



❖ Polar to Cartesian coordinates

$sin\theta = \frac{y}{r}$, $cos\theta = \frac{x}{r}$, $tan\theta = \frac{y}{x}$

- Based on forming a right triangle from r and θ .
- $x = r\cos\theta$
- $y = r\sin\theta$



❖ Cartesian to Polar coordinates

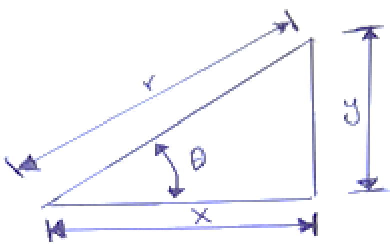
- If the Cartesian coordinates are known :
  - r is the hypotenuse and θ an angle

    $$tan\theta = \frac{y}{x}$$

    $$r = \sqrt{x^2 + y^2}$$

  - θ must be ccw from positive x axis for these equations to be valid

Example : The Cartesian coordinates of a point in the xy Plane are (x,y)=(12 , 5) . Fined the polar coordinates of this point .



Solution : from equation

$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{12^2 + 5^2}$$

$$r = 13$$

And from equation ,

$$tan\theta = \frac{y}{x}$$

$$tan\theta = \frac{5}{12}$$

$$tan\theta = 12$$

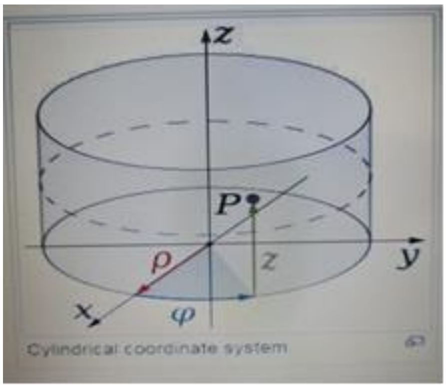
θ=??

$$\theta = tan^{-1}\left(\frac{5}{12}\right)$$

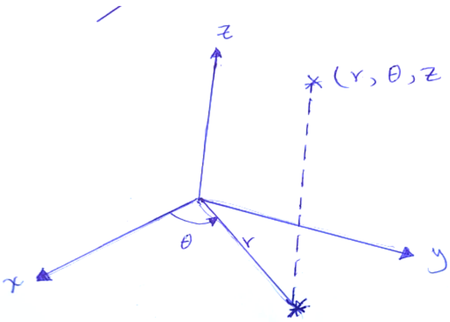
θ=22,6°

❖ Cylindrical coordinate systems

- There are two common methods for extending the polar coordinate system to three dimensions.
- In the cylindrical coordinate system, a z coordinate with the same meaning as in Cartesian coordinates is added to the r and θ polar coordinates giving a triple (r , θ , z).



❖ Convert from Cartesian coordinates to cylindrical coordinates



(x, y,z) → (r,θ,z)

$$\theta = tan^{-1}\left(\frac{y}{x}\right)$$

$$z = z$$

❖ Convert from cylindrical coordinates to Cartesian coordinates

(r,θ,z) → (x,y,z)

$$x = rcos\theta$$

$$y = rsin\theta$$

z=z

Example: Convert from cylindrical coordinates to Cartesian coordinates $(4, \frac{2\pi}{3}, -2)$

Solution : $x = r\cos\theta$

$x = 4\cos\frac{2\pi}{3}$

$x = -2$

$y = r\sin\theta$

$y = 4\sin\frac{2\pi}{3}$

$y = 2\sqrt{3}$

z=z

z=-2

p=(-2,2$\sqrt{3}$,-2)

Example: Convert from Cartesian coordinates to cylindrical coordinates $(1, -3, 5)$

Solution :

$r = \sqrt{x^2 + y^2}$

$r = \sqrt{1^2 + -3^2}$

$r = \sqrt{10}$

$\theta = tan^{-1}\left(\frac{y}{x}\right)$

$\theta = tan^{-1}\left(\frac{-3}{1}\right)$

$\theta = -1.249$

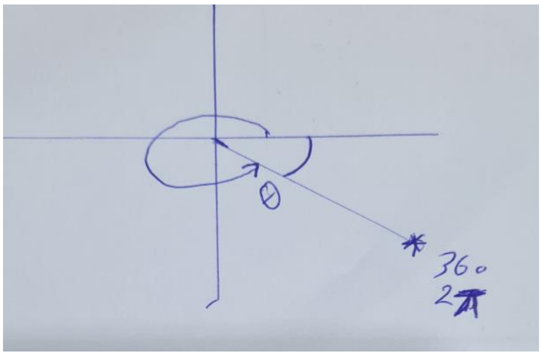
$\theta = -1.249 + 2\pi$

$\theta = 5.03$

z = z

z = 5

(r,θ,z) = ($\sqrt{10}$, 5.03 , 5)



Homework :

Convert from Cartesian coordinates to polar coordinates (3,-3,-7) .