Introduction

Heat transfer is the science that seeks to predict the energy transfer that may take place between material bodies as a result of a temperature difference. Thermodynamics teaches that this energy transfer is defined as heat. The science of heat transfer seeks not merely to explain how heat energy may be transferred, but also to predict the rate at which the exchange will take place under certain specified conditions. The fact that a heat-transfer *rate* is the desired objective of an analysis points out the difference between heat transfer and thermodynamics. Thermodynamics deals with systems in equilibrium; it may be used to predict the amount of energy required to change a system from one equilibrium state to another; it may not be used to predict how fast a change will take place since the system is not in equilibrium during the process. Heat transfer supplements the first and second principles of thermodynamics by providing additional experimental rules that may be used to establish energy-transfer rates. As in the science of thermodynamics, the experimental rules used as a basis of the subject of heat transfer are rather simple and easily expanded to encompass a variety of practical situations.

Modes of Heat Transfer

1- **Conduction**

$\frac{Q}{A} = q = -k\frac{dT}{dx}$ (Fourier's law of heat conduction)

2- **Convection**

a) Forced Convection
b) Natural Convection

$q = hA(T_W - T_b)$ (Newton's law of cooling)

q = heat transfer (W)
h = heat transfer coefficient (W/m^2K)
A = surface area (m^2)
T_W = wall temperature
T_b = balk temperature of fluid

3- **Radiation**

$q = \sigma A(T_1^4 - T_2^4)$ (Stefan-Boltzmann law)

σ = Stefan-Boltzmann constant with the value of 5.669×10^{-8} W/m^2K^4

For two **Black infinite** Bodies

We therefore introduce two new factors in the last Equation to take into account both situations, so that

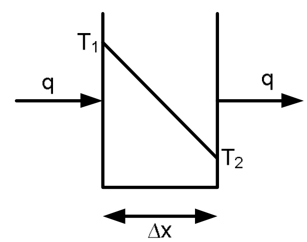
$$q = F_{\varepsilon} F_G \sigma A (T_1^4 - T_2^4)$$

F_{ϵ} is the emissivity function
F_G is the geometric function

Conduction Heat Transfer

When a temperature gradient exists in a body, experience has shown that there is an energy transfer from the high-temperature region to the low-temperature region.

$$\frac{q}{A} \alpha \frac{\Delta T}{\Delta x}$$



For small difference in temperature in the thin wall

$$\frac{q}{A} \alpha \frac{dT}{dx}$$

The constant of proportionality is inserted is the thermal conductivity of solid material (*k*)

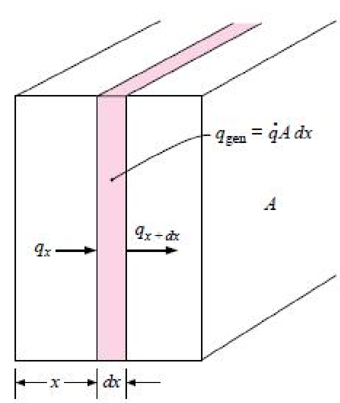
$$\frac{q}{A} = -k\frac{dT}{dx} \quad or \quad q = -kA\frac{dT}{dx}$$

This is called Fourier's law of heat conduction. *q* is the heat-transfer rate and *dT/dx* is the temperature gradient in the direction of the heat flow. The minus sign is inserted to make clear that the heat must flow in a direction of temperature decrease.

The general equation for heat transfer by conduction can be derived by making energy balance on a solid system. For the element of thickness *dx*, the following energy balance may be made:

Energy in = Energy out + Energy Accumulated

Energy conducted in left face + heat generated within element = change in internal energy + energy conducted out right face



$$\text{Energy in left face} = q_x = -kA\,\frac{\partial T}{\partial x}$$

$$\text{Energy generated within element} = \dot{q}A\,dx$$

$$\text{Change in internal energy} = \rho c A\,\frac{\partial T}{\partial \tau}\,dx$$ (Energy accumulated within element)

$$\text{Energy out right face} = q_{x+dx} = -kA\,\frac{\partial T}{\partial x}\bigg]_{x+dx}$$

$$= -A\left[k\,\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right)dx\right]$$

where

$\dot{q}$ = energy generated per unit volume, W/m^3
c = specific heat of material, J/kg · °C
ρ = density, kg/m^3

Combining the relations above gives

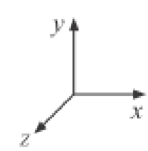
$$-kA\,\frac{\partial T}{\partial x} + \dot{q}A\,dx = \rho c A\,\frac{\partial T}{\partial \tau}\,dx - A\left[k\,\frac{\partial T}{\partial x} + \frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right)dx\right]$$

or

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \dot{q} = \rho c\,\frac{\partial T}{\partial \tau}$$

This is the one-dimensional heat-conduction equation. In the same way can treat the multiple dimensions to obtain **the general three-dimensional heat-conduction** equation, which is:

$$\frac{\partial}{\partial x}\left(k\,\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\,\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k\,\frac{\partial T}{\partial z}\right) + \dot{q} = \rho c\,\frac{\partial T}{\partial \tau}$$

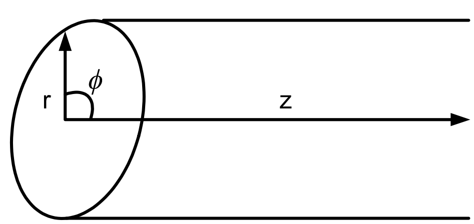


For constant thermal conductivity

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\,\frac{\partial T}{\partial \tau}$$

Where the quantity $\alpha = k / \rho\, c_p$ is called the thermal diffusivity of the material (m^2/s).

B) Cylindrical Coordinate

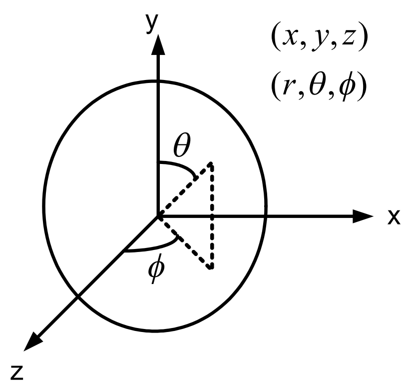


$x = r\,Cos\phi$
$y = r\,Sin\phi$
$z = z$

The general equation is written:

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

C) Spherical Coordinate



$$x = r\,Cos\theta + Sin\phi$$
$$y = r\,Sin\theta + Cos\phi$$
$$z = r\,Cos\theta$$

The general equation is written:

$$\frac{1}{r}\frac{\partial^2}{\partial r^2}(rT) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta \frac{\partial T}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2 T}{\partial \phi^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Some simplifications of the general equation are:

1-Steady-state one-dimensional heat flow (no heat generation):

$$\frac{d^2T}{d^2x} = 0 \Rightarrow \frac{dT}{dc} = C_1 \Rightarrow T = C_1 x + C_2$$

2-Steady-state one-dimensional heat flow in cylindrical coordinates (no heat generation):

$$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} = 0$$

3-Steady-state one-dimensional heat flow with heat sources:

$$\frac{d^2T}{dx^2} + \frac{\dot{q}}{k} = 0$$

4-Two-dimensional steady-state conduction without heat sources:

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \quad \Rightarrow T = f(x, y)$$