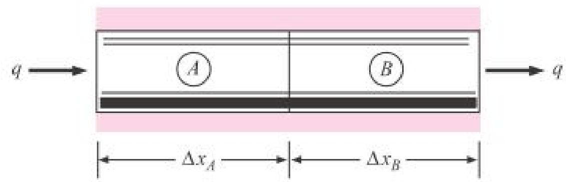
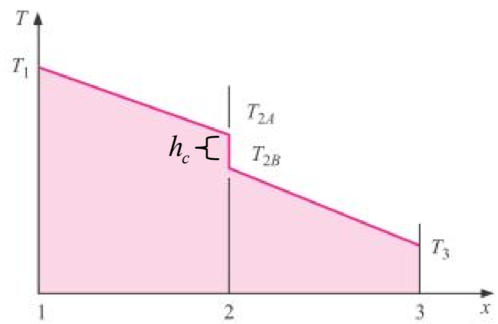
Thermal Contact Resistance





The temperature drop at plane 2, the contact plane between the two materials, is said to be the result of a *thermal contact resistance*. Performing an energy balance on the two materials, we obtain

$$q = k_A A \frac{T_1 - T_{2A}}{\Delta x_A} = \frac{T_{2A} - T_{2B}}{1/h_c A} = k_B A \frac{T_{2B} - T_3}{\Delta x_B}$$

or

$$q = \frac{T_1 - T_3}{\Delta x_A / k_A A + 1/h_c A + \Delta x_B / k_B A}$$

$$q_{interface} = h_c A (T_{2A} - T_{2B})$$

Where *1/hcA* is called the thermal contact resistance (*Rc*).
hc is called the contact coefficient.

Let the contact area by *Ac* and the void area by *Av*, then

$$q = \frac{T_{2A} - T_{2B}}{L_g/2k_A A_c + L_g/2k_B A_c} + k_f A_v \frac{T_{2A} - T_{2B}}{L_g} = \frac{T_{2A} - T_{2B}}{1/h_c A}$$

Where *Lg* is the thickness of the void space and *kf* is the thermal conductivity of the fluid which fills the void space. The total cross-sectional area of the bars is *A*. Solving for *hc*, the contact coefficient, we obtain

$$h_c = \frac{1}{L_g}\left(\frac{A_c}{A}\frac{2k_A k_B}{k_A + k_B} + \frac{A_v}{A}k_f\right)$$

EXAMPLE 2-12 Influence of Contact Conductance on Heat Transfer

Two 3.0-cm-diameter 304 stainless-steel bars, 10 cm long, have ground surfaces and are exposed to air with a surface roughness of about 1 μm. If the surfaces are pressed together with a pressure of 50 atm and the two-bar combination is exposed to an overall temperature difference of 100°C, calculate the axial heat flow and temperature drop across the contact surface.

$1/h_c = 5.28\times10^{-4}$ m^2.C°/W

Solution

The overall heat flow is subject to three thermal resistances, one conduction resistance for each bar, and the contact resistance. For the bars

$$R_{th} = \frac{\Delta x}{kA} = \frac{(0.1)(4)}{(16.3)\pi(3\times10^{-2})^2} = 8.679\,°\text{C/W}$$

$$R_c = \frac{1}{h_c A} = \frac{(5.28\times10^{-4})(4)}{\pi(3\times10^{-2})^2} = 0.747\,°\text{C/W}$$

The total thermal resistance is therefore

$$\sum R_{th} = (2)(8.679) + 0.747 = 18.105$$

and the overall heat flow is

$$q = \frac{\Delta T}{\sum R_{th}} = \frac{100}{18.105} = 5.52\text{ W} \quad [18.83\text{ Btu/h}]$$

The temperature drop across the contact is found by taking the ratio of the contact resistance to the total thermal resistance:

$$\Delta T_c = \frac{R_c}{\sum R_{th}}\Delta T = \frac{(0.747)(100)}{18.105} = 4.13\,°\text{C} \quad [39.43\,°\text{F}]$$

In this problem the contact resistance represents about 4 percent of the total resistance.

Unsteady State Conduction

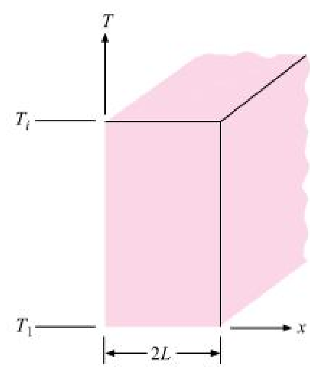
Unsteady-state heat-transfer processes (heating or cooling) takes place in the interim period before equilibrium is established. To analyze a transient heat-transfer problem, we can proceed by solving the general heat-conduction equation, which is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Consider the infinite plate of thickness $2L$ shown in Figure below. Initially the plate is at a uniform temperature Ti, and at time zero the surfaces are suddenly lowered to $T = T_1$. For one-dimension unsteady-state without heat generation, the differential equation is

$$\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha}\frac{\partial T}{\partial \tau}$$

Figure 4-1 | Infinite plate subjected to sudden cooling of surfaces.



Put $\theta = T - T_1$ (T = any temp.), then

$$\frac{\partial^2 \theta}{\partial x^2} = \frac{1}{\alpha}\frac{\partial \theta}{\partial \tau}$$

Types of boundary conditions

1- Constant wall temperature (variable heat flux).
2- Constant heat flux (variable wall temperature).
3- Convection boundary conditions.

The initial and boundary conditions for infinite plate (constant wall temperature) will be

I.C	$t = 0$	$0 \le x \le 2L$	$\theta = \theta_i = T_i - T_1$	
B.C.1	$x = 0$	$T = T_1$	$\theta = 0$	$t \rangle 0$
B.C.2	$x = 2L$	$T = T_1$	$\theta = 0$	$t \rangle 0$

The final solution is therefore

$$\frac{\theta}{\theta_i} = \frac{T - T_1}{T_i - T_1} = \frac{4}{\pi}\sum_{n=1}^{\infty}\frac{1}{n}e^{-[n\pi/2L]^2\alpha\tau}\sin\frac{n\pi x}{2L} \qquad n = 1, 3, 5 \ldots$$

$$\alpha = \frac{k}{\rho\, cp}$$