Gauss Law :-

- The total electric flux through any closed surface is proportional to the total electric charge inside the surface.

Point Charge Inside a Spherical Surface:

$$E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$$

$\vec{E} \,//\, d\vec{A}$ at each point

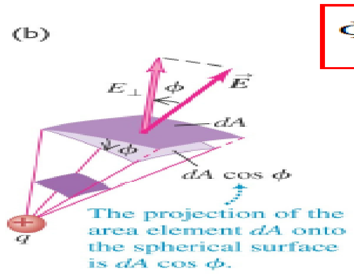
$$\Phi_E = E \cdot A = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}\left(4\pi R^2\right) = \frac{q}{\varepsilon_0}$$

- The flux is independent of the radius R of the sphere.

Point Charge Inside a Non spherical Surface:

- Divide irregular surface into d A elements, compute electric flux for each (E d A cos φ) and sum results by integrating.

- Each d A projects onto a spherical surface element → total electric flux through irregular surface = flux through sphere.



$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q}{\varepsilon_0}$$

Integral through a closed surface

Valid for + / - q

If enclosed q = 0 → $\Phi_E = 0$

Point charge outside a closed surface that encloses no charge. If an electric field line enters the surface at one point it must leave at another.

- Electric field lines can begin or end inside a region of space only when there is a charge in that region.

General form of Gauss's law:

$$\Phi_E = \oint \vec{E} \cdot d\vec{A} = \oint E \cos\varphi \, dA = \oint E_{\perp} dA = \frac{Q_{encl}}{\varepsilon_0}$$

Applications of Gauss's Law:-

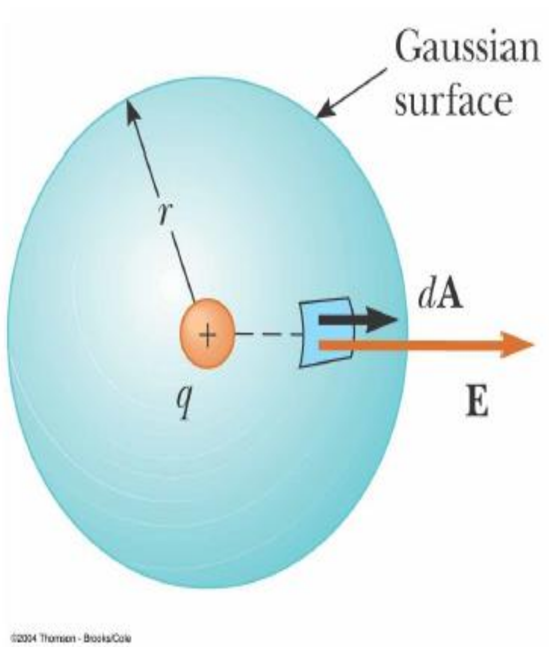
1- The Electric Field Due to a Point Charge:-

$$\phi_c = \oint \vec{E} . d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

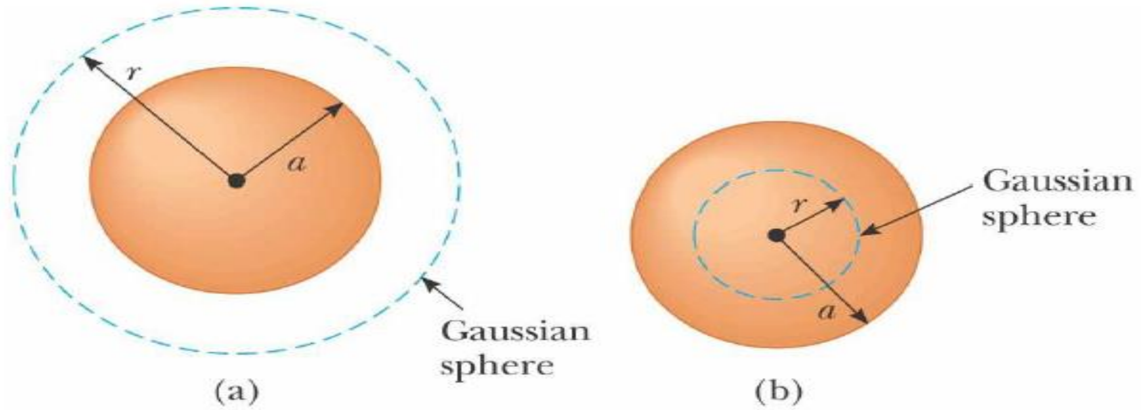
$$E(4\pi r^2) = \frac{q_{in}}{\varepsilon_0}$$

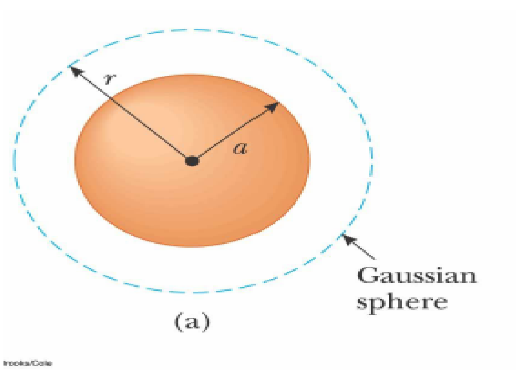
$$E = \frac{1}{4\pi\varepsilon_0} \frac{q}{r^2}$$

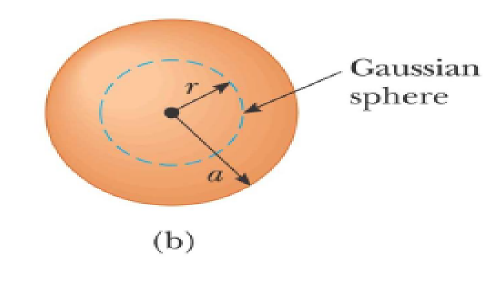


2 - A Spherically of radius a Symmetric Charge Distribution

2a) r > a (Gauss outside) 2b) r < a (Gauss inside)







$$\rho = \frac{Q}{V}$$

$$a)\ \ E = k_e \frac{Q}{r^2}, \qquad r \succ a$$

$$b)\, q_{in} = \rho V' = \ \rho(\frac{4}{3}\pi r^3), \qquad r \prec a$$

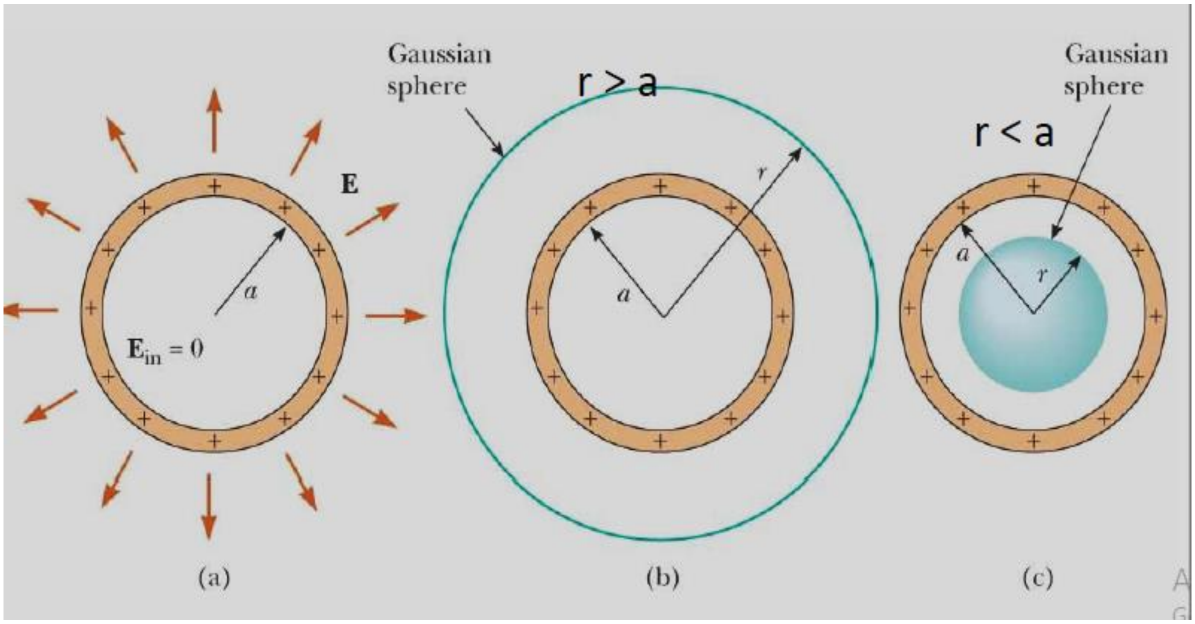
$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$$E = \frac{q_{in}}{4\pi\varepsilon_0 r^2} = \frac{\rho \frac{4}{3}\pi r^3}{4\pi\varepsilon_0 r^2} = \frac{\rho}{3\varepsilon_0} r$$

$$\rho = \frac{Q}{\frac{4}{3}\pi r^3}$$

$$E = \frac{Q r}{4\pi\varepsilon_0 a^3} = k_e \frac{Q}{a^3} r \quad \Rightarrow E \to 0\ as\ r \to 0$$

3- The Electric Field Due to a Thin Spherical Shell :-



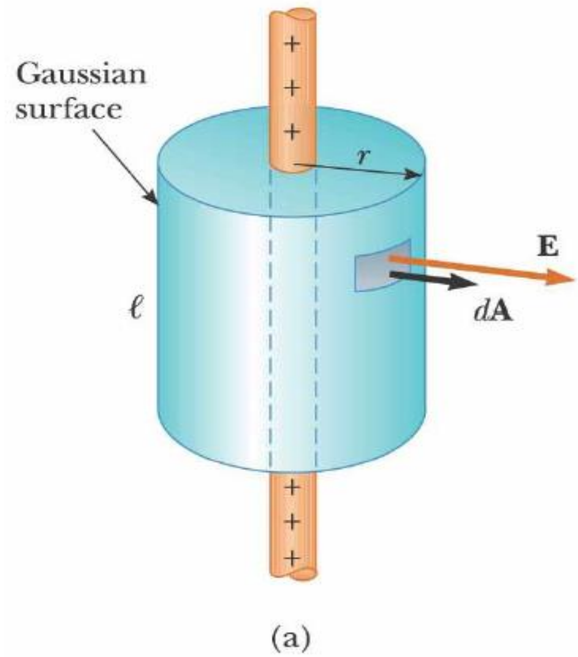
The electric field outside it is similar For the sphere outside the solid sphere,

• Gaussian surface closed

$$E = k_e \frac{Q}{r^2}, \qquad r \succ a$$

$$r \prec a, \quad q_{in} = 0 \qquad \Rightarrow E_{in} = 0$$

4- A Cylindrically Symmetric Charge Distribution :-



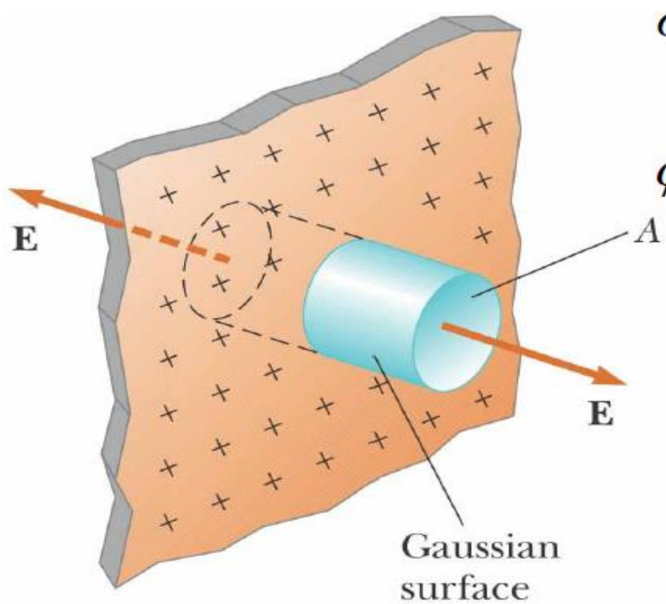
$$\lambda = Const. \;, \quad Q = \lambda \ell, \quad dA \,|\,|\, E$$

$$E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$E\,(2\pi r \ell) = \frac{\lambda \ell}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi \varepsilon_0 r} = 2k_e \frac{\lambda}{r}$$

5- A Plane of Charge :-



$$\sigma = \frac{q_{in}}{A}$$

$$\phi_c = E \oint dA = \frac{q_{in}}{\varepsilon_0}$$

$$2EA = \frac{\sigma A}{\varepsilon_0}$$

$$E = \frac{\sigma}{2\varepsilon_0}$$

Conductors in Electrostatic Equilibrium:-

Good electrical conductors contain free charges when they are in the net of their motion Inside the material is almost zero as a result of its occurrence under the influence of an external electric field.

A substance is in electrostatic equilibrium and has the following properties:

1-The field strength inside the conductor is zero.

2-Charges settle on the outer surface of an insulated conductor.

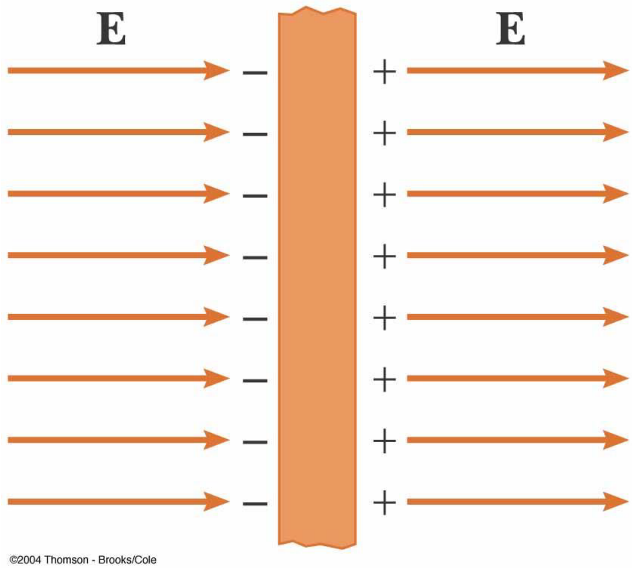
3- The surface charge density increases - for the uneven surface - on the pointed parts, which increases field strength near it,

4-At points outside the surface, the electric field is perpendicular to the surface at all points The intensity is “$E = \sigma/\varepsilon$” where the surface charge density varies from point to point other.

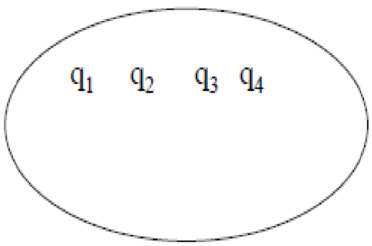
When placing a connector within a domain electric current, and new charges are generated conductive surface (as shown) That is, the surface density of the charge increases That generates a field that has the opposite direction

for the external field, so that it is The resultant field inside the conductor

equal to zero



Example :- Charges of (5, 9,27,80) micro coulombs were placed inside a submarine, calculate the net electric flow through the submarine and compare the number of electric field lines going outside it to the one inside it?



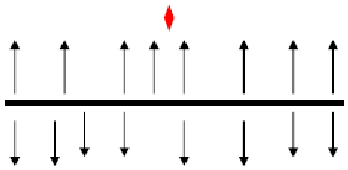
$$\phi = \frac{q_{in}}{\varepsilon_0}$$

$$= \frac{(5-9-27-80)10^{-6}}{8.9\ 10^{-12}}$$

$$= -6.9 x\, 10^{6}\ N.m^{2}/C$$

Since the net flow is a negative value, the number of intake lines entering the surface of the submarine greater than the number that emerges from the surface .

Example :- A large flat plate, the surface density of charge is 9 μC/m^2. find intensity The electric field above the surface and the magnitude at its center?



$$E = \frac{\sigma}{2\varepsilon_0}$$

$$E = \frac{9x10^{-6}}{8.9\,x10^{-12}} = 508\, kN/C$$

Homework :-

A solid sphere of radius 40.0 cm has a total positive charge of 26.0 μ C uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm, (b) 10.0 cm, (c) 40.0 cm, and (d) 60.0 cm from the center of the sphere ?