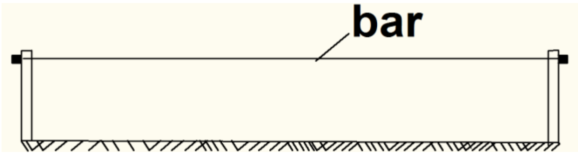
Thermal stresses

Consider an elastic bar as shown in Figure under the effect of temperature, increasing or decreasing the temperature of a material produces elongation or contraction. By the principles of superposition, the resulting axial strain ε is



$$\varepsilon = \varepsilon_t + \varepsilon_s$$

Where

$$\varepsilon_t = \alpha \,.\, \Delta t$$

is the thermal strain (strain due to changes of temperature) where α is the coefficient of thermal expansion (material property).
and

$$\varepsilon_s = \frac{\sigma}{E}$$

is the strain due to induced stress (σ)

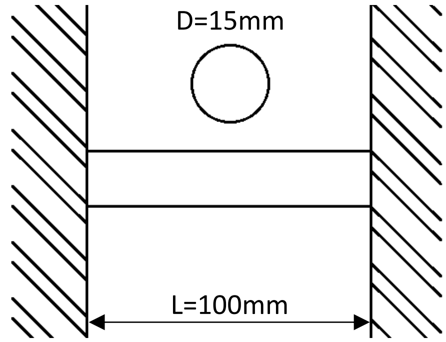
If the rise of temperature provided to a body free to expand in this case stains have no stresses associated with them

$$\varepsilon = \frac{\Delta L}{L}$$

$$\frac{\Delta L}{L} = \alpha \,.\, \Delta t$$

$$\boxed{\Delta L = \alpha \,.\, L.\, \Delta t}$$

Ex; A solid brass bar of length 100mm and diameter 15mm is used to fix two rigid surfaces. Find the stress induced into it if the temperature of the bar raised by 20 C°. Given α=19×10^{-6} , E =103 GPa



Sol:-

$$\Delta L_t + \Delta L_s = 0$$

$$\therefore \Delta L_t = -\Delta L_s$$

$$\alpha * L * \Delta t = \frac{-P * L}{A * E}$$

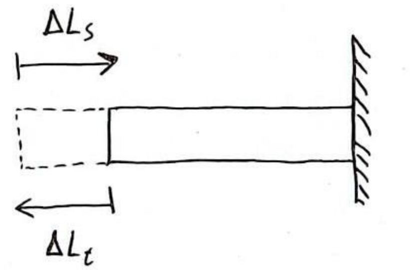
$$19 * 10^{-6} * 100 * 20 = \frac{-P * 100}{\frac{\pi}{4} * (15)^2 * 103 * 10^3}$$

$$0{,}038 = \frac{-100P}{18192375}$$

$$\therefore -100P = 691310{,}2$$

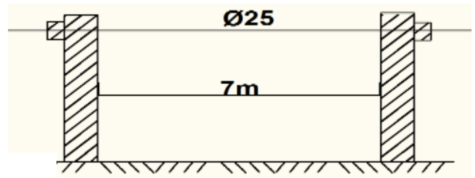
$$P = 6913{,}1 N = 6{,}913 KN$$

$$\therefore \sigma = \frac{P}{A} = \frac{6913{,}1}{\frac{\pi}{4} * (15)^2} = 39{,}14 MPa$$



Ex; Two parallel walls 7m apart, are held by steel bar of 25mm diameter, the bar passes through a metal plate and nut at each end, the nuts are screwed up to the plates which the bar is at 150°c. Find the pull exerted by the bar after cooling to 16°c. If

- **a-** the ends do not yield.
- **b-** the total yield at the two ends is 6.25mm.
  Given Es = 220 GPa and $\alpha = 11\text{x}10^{-6}/°c$



Sol:-

a) No yield

$$\Delta L_t + \Delta L_s = 0$$

$$\therefore \Delta L_t = -\Delta L_s$$

$$\therefore \alpha * L * \Delta t = \frac{-P*L}{A*E}$$

$$11*10^{-6} * 7*10^3 *(16-150) = \frac{-P*7*10^3}{\frac{\pi}{4}*(25)^2 * 220*10^3}$$

$$-10.318 = -6.485*10^{-5} P$$

$$\boxed{\therefore P = 159099\ N = 159.099\ KN}$$

b) Total yield = 6,25 mm (contraction)

$$\Delta L_t + \Delta L_s = -6,25$$

$$(\alpha * L * \Delta t) + \left(\frac{P*L}{A*E}\right) = -6,25$$

$$\left(11*10^{-6}*7*10^3*(16-150)\right) + \frac{P*7*10^3}{\frac{\pi}{4}*(25)^2*220*10^3} = -6,25$$

$$-10,318 + \frac{P*7*10^3}{107937500} = -6,25$$

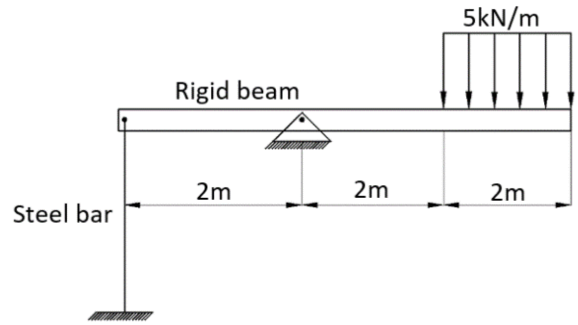
$$\therefore \frac{P*7*10^3}{107937500} = -6,25 + 10,318$$

$$\therefore P*7*10^3 = 439.089*10^6$$

$$\therefore P = 62727\ N$$

$$\boxed{\therefore P = 62,727\ KN}$$

Ex; Find the necessary change in temperature in the steel bar so that the rigid beam remains horizontal. The specification of steel bar is (A=1000mm^2, L= 2m, α=12×10^{-6}, E =200 GPa).



Sol.-

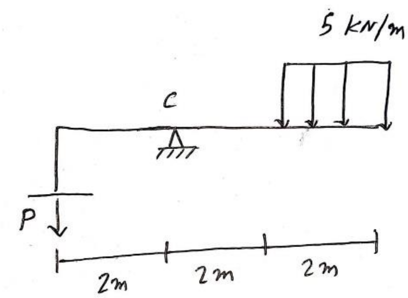
To Keep the member horizontal. The change in the steel cable Length must remain "Zero"

$$\therefore \Delta L_{t_{steel}} + \Delta L_{s_{steel}} = 0$$

$$\therefore \Delta L_t = -\Delta L_s$$

$$\therefore \alpha * L * \Delta t = \frac{-P*L}{A*E} \quad ---- (1)$$

To Find (P) in steel Cable



$$\Sigma M_c = 0$$

$$0 = 5*2*(2+\frac{2}{2}) - P*2$$

$$2P = 30 \quad \boxed{\therefore P = 15 \text{ KN}}$$

sub in (1)

$$\therefore 12*10^{-6}*2*10^3*\Delta t = \frac{-15*10^3*2*10^3}{1000*200*10^3}$$

$$\boxed{\therefore \Delta t = -6.25 \text{ decrease}}$$