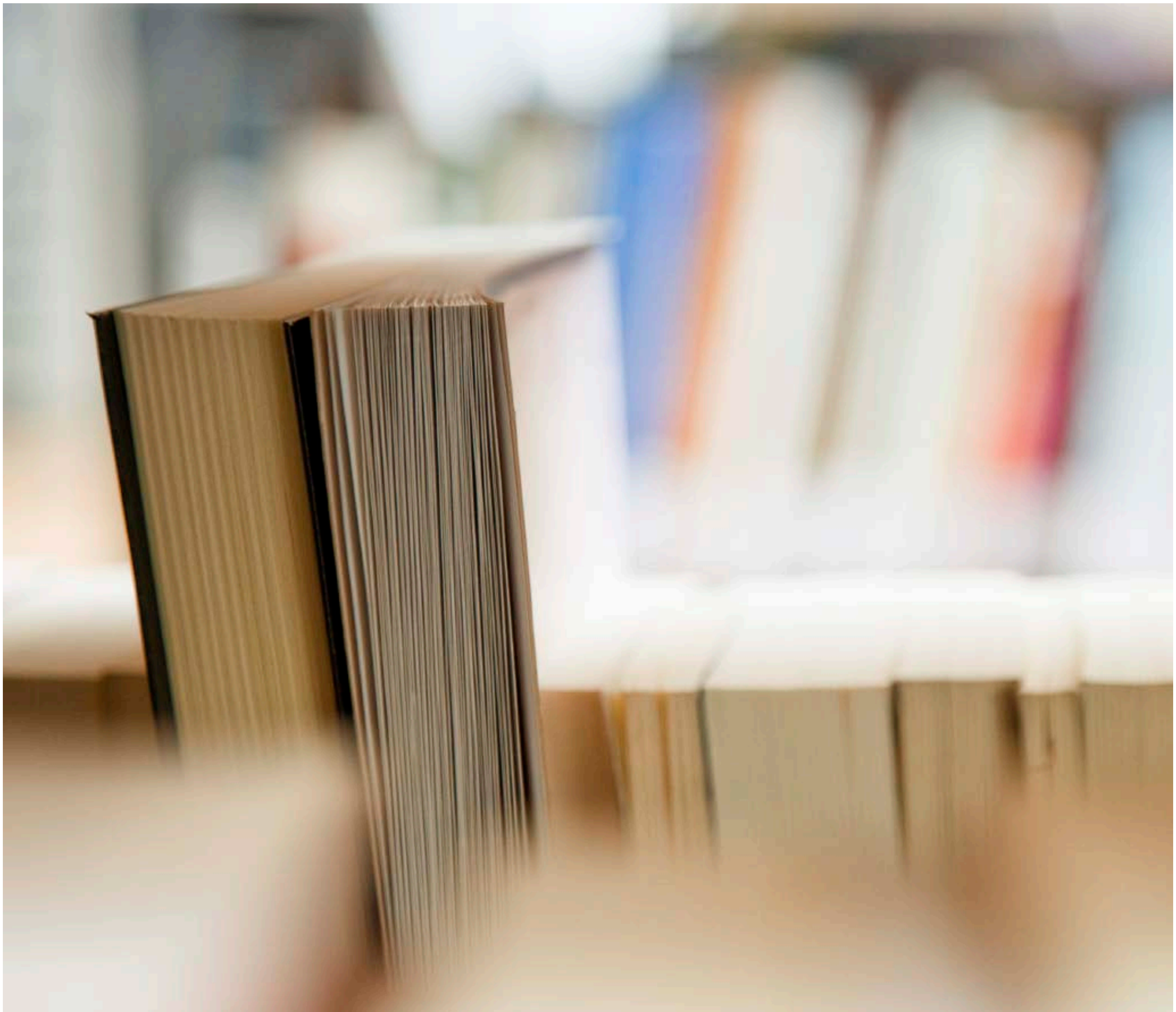

Information Theory and Coding

Forth Stage

MSc. Zahraa Niema Kamal Lecture Eight First Course

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1. Channel Capacity (Discrete channel):

This is defined as the maximum of I(X,Y):

$$C = channel\ capacity = \max[I(X,Y)] \qquad bits/symbol$$

Physically it is the maximum amount of information each symbol can carry to the receiver. Sometimes this capacity is also expressed in bits/sec if related to the rate of producing symbols r:

$$R(X,Y) = r \times I(X,Y) \qquad bits/\text{sec} \quad or\ R(X,Y) = I(X,Y)/\overline{\tau}$$

Channel capacity of Symmetric channels:

The channel capacity is defined as max $[I(X,Y)]$:

$$I(X,Y) = H(Y) - H(Y \mid X)$$

$$I(X,Y) = H(Y) + \sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i, y_j) \log_2 P(y_j|x_i)$$

But we have

$\boldsymbol{P(x_i, y_j) = P(x_i)P(y_j|x_i)}$ $\quad put\ in\ above\ equation\ yieldes$:

$$I(X,Y) = H(Y) + \sum_{j=1}^{m} \sum_{i=1}^{n} P(x_i)P(y_j|x_i) \log_2 P(y_j|x_i)$$

If the channel is symmetric the quantity:

$$\sum_{j=1}^{m} P(y_j|x_i) \log_2 P(y_j|x_i) = K$$

Where K is constant and independent of the row number i so that the equation becomes:

$$I(X,Y) = H(Y) + K \sum_{i=1}^{n} P(x_i)$$

Hence $I(X,Y) = H(Y) + K$ for symmetric channels

Max of $I(X,Y) = \max[H(Y) + K] = \max[H(Y)] + K$

When Y has equiprobable symbols then $\max[H(Y)] = log2m$

Then

$$I(X,Y) = \log_2 m + K$$

Or

$$C = \log_2 m + K$$

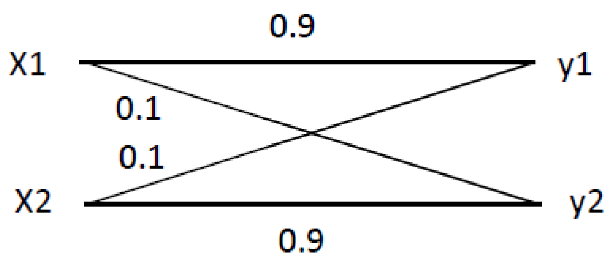
$$efficiency(\eta) = \frac{I(x,y)}{c} * 100\%$$

$$redundancy(R) = 100\% - efficiency$$

Example: Draw the channel model from the transmission matrix

$$P(y/x) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$$

Solution:

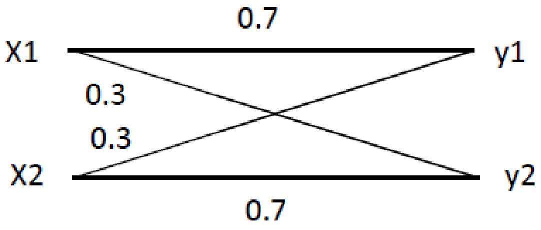


Example: A binary symmetric channel with error probability of 0.3 and marginal entropy H(y)= 0.97095 *bits/symbol.*

i- Draw the channel model.
ii- Calculate the channel capacity.
iii- Find the efficiency and redundancy.

Solution:

i-The channel model is as following:



ii- Then the channel capacity is calculated as follow:

$C = log_2 m + K$ and $n = m$

$$K = 0.7log_2 0.7 + 0.3log_2 0.3 = -0.88129$$

$$C = 1 - 0.88129 = 0.1187 \ \frac{bits}{symbol}$$

iii- The channel efficiency $\eta = \frac{I(X,Y)}{C}$

$I(X,Y) = H(Y) + K$

We have the marginal entropy $H(Y) = 0.97095\ bits/symbol$

$\therefore$ $I(X,Y) = 0.97095 + (-0.88129) = 0.0896\ bits/symbol$

Then $\eta = \frac{0.0896}{0.1187} = 75.6\%$

To find the channel redundancy:

$R = 1 - \eta = 1 - 0.756 = 0.244 \quad or\ 24.4\%$

Example: Given a binary symmetric channel with transmission matrix shown below; Determine the channel capacity, channel efficiency and redundancy. If p(x1=0.4).

$$\boldsymbol{P(y/x) = \begin{Bmatrix} 0.8 & 0.2 \\ 0.2 & 0.8 \end{Bmatrix}}$$

Solution:

P(x1)=0.4 , P(x2)= 1-0.4= 0.6

$$k = \sum_{j=1}^{m} P(yj/xi) log_2 P(yj/xi) = \frac{0.8ln0.8 + 0.2ln0.2}{ln2} = -0.721$$

$$C = log_2 m + k = log_2 2 - 0.721 = 1 - 0.721 = 0.279\ bit/symbol$$

$$P(x,y) = P(y/x) * P(x) = \begin{pmatrix} 0.8 * 0.4 & 0.2 * 0.4 \\ 0.2 * 0.6 & 0.8 * 0.6 \end{pmatrix} = \begin{pmatrix} 0.32 & 0.08 \\ 0.12 & 0.48 \end{pmatrix}$$

P(y1)=0.32+0.12=0.44, P(y2)=0.08+0.48=0.56

$$H(y) = -\sum_{j=1}^{m} P(yj) log_2 P(yj) = -\left[\frac{0.44ln0.44 + 0.56ln0.56}{ln2}\right] = 0.989\ bit/symbol$$

$$I(x,y) = H(y) + k = 0.989 - 0.721 = 0.268$$

$$efficiency = \frac{I(x,y)}{c} * 100\% = \frac{0.268}{0.279} * 100\% = 96.057\%$$

$$redundancy = 100\% - efficiency = 100\% - 96.057\% = 3.943\%$$