#### ALMUSTAQBAL UNIVERSITY COLLEGE

**Biomedical Engineering Department** 

**Stage**: Second year students

**Subject : Chemistry 1 - Lecture 10** 

Lecturer: Assistant professor Dr. SADIQ . J. BAQIR



# **Buffer solutions**

The buffer is a mixture of a weak acid and its conjugate base (Salt) or a weak base and its conjugate acid(Salt), that resists change in pH of a solution due to dilution or addition of a small amounts of strong acid or base. Buffers are used to Maintain the pH of solutions at relatively constant and predetermined level.

Usually, buffers have a useful  $\underline{pH}$  range =  $pK_a \pm 1$ ,

# Calculation of the pH of different types of Buffer solutions

# 1. acidic buffers

Consists of weak acid (HA) and its salt (A<sup>-</sup>) . Typical example is (acetic acid – acetate salt (CH<sub>3</sub>COOH – CH<sub>3</sub>COO<sup>-</sup>)

$$HA + H_2O \rightleftharpoons H_3O^+ + A^- \longrightarrow 1$$

$$Ka = \frac{[H_3O^+][A^-]}{[HA]}$$

$$A^- + H_2O \rightleftharpoons OH^- + HA \longrightarrow 2$$

$$K_b = \frac{[OH][HA]}{[A-]} = \frac{Kw}{Ka}$$

equilibrium 1) will decrease  $C_{HA}$  by amount  $[H_3O^+]$  and equilibrium 2) will increase it by amount  $[OH^-]$ .

$$[HA]_{equil.} = C_{HA} - [H_3O^+] + [OH^-]$$

Similarly equilibrium ① will increase  $[A^-]$  by amount  $[H_3O^+]$  while equilibrium ② will decrease  $[A^-]$  by amount  $[OH^-]$  then

$$[A^{-}]_{equil.} = C_{A-} + [H_{3}O^{+}] - [OH^{-}]$$

As we have acid then

$$[HA] = C_{HA} - [H_3O^+]$$

$$[A^{-}] = C_{A^{-}} + [H_{3}O^{+}]$$

And because we have weak acid then

$$[HA] \cong C_{HA}$$

$$[A^-] \cong C_{A-}$$

$$Ka = \frac{[H_3O^+][A^-]}{[HA]}$$

$$[H_3O^+] = K_a \frac{[HA]}{[A^-]}$$

$$[H_3O^+] = K_a \frac{c_{HA}}{c_{A^-}}$$

- log [H<sub>3</sub>O] = -log K<sub>a</sub> - log 
$$\frac{C_{HA}}{C_{A-}}$$

$$pH = pKa + log \; \frac{c_A}{c_{HA}}$$

\*\* 
$$pH = pKa + log \frac{c_{salt}}{c_{acid}}$$
 ( Henderson equation)

**Example:** what is the pH of a solution that is (0.40 M) in formic acid  $(\text{Ka}=1.80 \times 10^{-4})$  and (1.0 M) in sodium formate?

#### **Solution:**

The pH of the solution will be effected by Ka of formic acid (HCOOH) and K<sub>b</sub> of formate ion (HCOO<sup>-</sup>)

$$HCOONa \rightarrow HCOO^- + Na^+$$

$$HCOOH + H_2O \rightleftharpoons HCOO^- + H_3O^+$$
  $Ka = 1.77 \times 10^{-4}$ 

$$HCOO^{-} + H_{2}O \rightleftharpoons HCOOH + OH^{-}$$
  $K_{b} = \frac{Kw}{Ka} = 5.65 \times 10^{-11}$ 

Since Ka of formic acid >>  $K_b$  for formate the solution will be acidic and Ka will determine the  $H_3O^+$  conc.

$$pH = pKa + log \frac{c_{salt}}{c_{acid}}$$

$$pKa = -\log Ka = -\log (1.77 \times 10^{-4}) = 3.75$$

$$pH = 3.75 + log \frac{1.0}{0.4} = 4.14$$

$$[H_3O^+] = 10^{-pH} = 10^{-4.14} = 7.2 \text{ x } 10^{-5}$$

Check if  $\frac{[\text{H3O}^+]}{[\text{HCOOH}]} \times 100 < 10 \%$  Then approximation is valid  $\frac{7.2 \times 10^{-5}}{0.4} \times 100 = 0.018 \%$ 

The approximation is valid:

### B) basic Buffers

It is composed of a solution of a weak base (B) and it's conjugate acid (Salt)  $BH^+$  e.g :  $NH_3$ - $NH_4Cl$ .

1) 
$$B + H_2O \rightleftharpoons BH^+ + OH^ K_b = \frac{[OH^-][BH^+]}{[B]}$$

$$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^-$$

2) 
$$BH^{+} + H_{2}O \rightleftharpoons H_{3}O^{+} + B$$
  $K_{a} = \frac{Kw}{Kb} = \frac{[H_{3}O^{+}][B]}{[BH^{+}]}$ 

$$NH_4^+ + H_2O \rightleftharpoons H_3O^+ + NH_3$$

[B] will decrease in equilibrium ①by amount [OH-] & increase in equilibrium ②by [H<sub>3</sub>O+]

Then 
$$[B] = C_B - [OH^-] + [H_3O^+]$$

Similarly [BH<sup>+</sup>] will increase in equilibrium (1) By [OH<sup>-</sup>] and decrease in equilibrium (2) by [H<sub>3</sub>O<sup>+</sup>].

Then 
$$[BH^+] = C_{BH^+} + [OH^-] - [H_3O^+]$$

$$[B] \equiv C_B$$
 and  $[BH^+] \equiv C_{BH^+}$  (by approximation)

$$K_b = \frac{[\text{OH}^-][\text{BH}^+]}{[\text{B}]}$$

$$[OH^{-}] = K_b \frac{C_B}{C_{BH^{+}}}$$

$$pOH = pK_b + log \frac{[BH^+]}{[B]}$$

$$pOH = pK_b + log \frac{C_{salt}}{C_{base}}$$
 (Henderson equation)

In General:

$$pH = pK_a + log \frac{C_{salt}}{C_{acid}}$$
 (for acidic buffer)

$$pOH = pK_b + log \frac{C_{salt}}{C_{base}}$$
 (for basic buffer)

$$pH = 14 - pOH$$

**Example:** calculate the pH of a solution that is 0.2 M in NH<sub>3</sub> and 0.3 M in NH<sub>4</sub>Cl (  $K_b = 1.75 \times 10^{-5}$ ).

#### **Solution:**

$$NH_4Cl \rightarrow NH_4^+ + Cl^-$$

$$NH_4^+ + H_2O \rightleftharpoons NH_3 + H_3O^+$$
  $Ka = \frac{K_w}{K_b} = \frac{10^{-14}}{1.75 \times 10^{-5}} = 5.7 \times 10^{-10}$ 

$$NH_3 + H_2O \Rightarrow NH_4^+ + OH^ K_b = 1.75 \times 10^{-5}$$

because  $K_b >> K_a$  the solution is assumed to be basic

$$pOH = pK_b + log \frac{C_{salt}}{C_{base}}$$
 (for basic buffer)

$$pK_b = -\log K_b = -\log (1.75 \times 10^{-5}) = 4.75$$

$$pOH = 4.75 + log \frac{0.3}{0.2} = 4.93$$

To check the validity of approximation we calculate [OH-]

$$[OH^{-}] = 10^{-pOH} = 10^{-4.93} = 1.17 \times 10^{-5}$$

then Check if  $\frac{[OH^-]}{[Base]} \times 100 < 10 \%$  Then approximation is valid

Then 
$$\frac{1.17 \times 10^{-5}}{0.2} \times 100 = 5.85 \times 10^{-3}$$
 (approximation is valid)

$$pH = 14 - 4.93 = 9.07$$

# **Properties of buffer solution:**

# 1 Effect of dilution:

The pH of the buffer solution remains independent of dilution until the concentration of species it's contain are decreased to the point where the approximation mentioned above become invalid.

**Example:** calculate the change in pH of a buffer containing (0.4M) formic acid HCOOH (Ka= $1.77x10^{-4}$ ) and (1M) sodium formate HCOONa after dilution by a factor of 50 times .

### **Solution:**

$$HCOONa \rightarrow HCOO^{-} + Na^{+}$$

$$HCOOH + H_2O \rightleftharpoons HCOO^- + H_3O^+$$
  $Ka = 1.77 \times 10^{-4}$ 

$$HCOO^{-} + H_{2}O \rightleftharpoons HCOOH + OH^{-}$$
  $K_{b} = \frac{Kw}{Ka} = 5.65 \times 10^{-11}$ 

Since Ka of formic acid  $\gg$  K<sub>b</sub> for formate the solution will be acidic.

$$pH = pKa + log \frac{[HCOONa]}{[HCOOH]}$$

$$pKa = -\log(1.77x10^{-4}) = 3.75$$

#### a. Before Dilution

$$pH = 3.75 + \log \frac{1}{0.4} = 4.15$$

$$[H_3O^+] = 10^{-pH} = 10^{-4.15} = 7.2 \text{ x } 10^{-5}$$

Check if  $\frac{[\text{H}30^+]}{[\text{H}COOH]} \times 100 < 10 \%$  Then approximation is valid  $\frac{7.1 \times 10^{-5}}{0.4} \times 100 = 0.0177 \%$ 

The approximation is valid.

b. After dilution with 50 times.

$$M_1V_1 = M_2V_2$$

$$0.4 \text{ x} 1 = M_2 \text{ x} 50$$

[HCOOH] = 
$$\frac{0.4}{50}$$
 = 8 x 10<sup>-3</sup> M

For [HCOO-]

$$1 \times 1 = M_2 \times 50$$

$$[HCOO^{-}] = \frac{1}{50} = 2 \times 10^{-2} M$$

$$pH = 3.75 + \log_{\frac{8 \times 10^{-2}}{8 \times 10^{-3}}} = 4.15$$

$$[H_3O^+] = 10^{-pH} = 10^{-4.15} = 7.2 \text{ x } 10^{-5}$$

Check if 
$$\frac{[\text{H30}^+]}{[\text{HCOOH}]} \times 100 < 10 \%$$
 Then approximation is valid  $\frac{7.2 \times 10^{-5}}{0.4} \times 100 = 0.018 \%$ 

The approximation is valid.

 $\therefore$  NO change in pH occur after 50 times dilution(  $\Delta PH = 0$ )

# (2) Effect of adding strong acid or base:

Buffer solution resist the pH change after addition of small amount of strong acid or base.

Example: Calculate the pH change that take place when 100 mL portion of: **a)** 0.05 M NaOH **b)** 0.05 M HCl

is added seperately to 400 mL of buffer solution of (0.2M)  $NH_3$  and (0.3M)  $NH_4Cl$  ( $K_b=1.75x10^{-5}$ ).

## **Solution:**

## a. The original buffer before addition

$$NH_4Cl \rightarrow NH_4^+ + Cl^-$$

$$NH_4^+ + H_2O \Rightarrow NH_3 + H_3O^+$$
  $Ka = \frac{K_W}{K_b} = \frac{10^{-14}}{1.75 \times 10^{-5}} = 5.7 \times 10^{-10}$ 

$$NH_3 + H_2O \rightleftharpoons NH_4^+ + OH^ K_b = 1.75 x 10^{-5}$$

because  $K_b >> K_a$  the solution is assumed to be basic

$$pOH = pK_b + log \frac{c_{salt}}{c_{base}}$$
 (for basic buffer)

$$pK_b = -\log K_b = -\log (1.75 \times 10^{-5}) = 4.75$$

$$pOH = pK_b + log \frac{c_{NH_4Cl}}{c_{NH_3}} = 4.76 + log \frac{0.3}{0.2} = 4.93$$

$$[OH^{-}] = 10^{-pOH} = 10^{-4.93} = 1.17 \times 10^{-5}$$

To check the validity of approximation we calculate [OH-] then

Check if 
$$\frac{[OH-]}{[Base]} \times 100 < 10 \%$$
 Then approximation is valid

Then 
$$\frac{1.17 \times 10^{-5}}{0.2} \times 100 = 5.85 \times 10^{-3}$$
 % (approximation is valid)

$$pH = 14 - 4.93 = 9.07$$

#### b. after addition of strong base or acid

1) addition of NaOH converts part of NH<sub>4</sub><sup>+</sup> in the buffer to NH<sub>3</sub>

$$NH_4^+ + OH^- \rightleftharpoons NH_3 + H_2O$$
 (OH from NaOH)

The analytical concentration of NH<sub>3</sub> and NH<sub>4</sub>Cl become :

$$C_{NH_3} = \frac{\text{original No.of moles of NH}_3 + \text{moles of produced NH}_3}{\text{Total New volume (L)}}$$

Or 
$$C_{NH_3} = \frac{\text{original No.of mmoles of NH}_3 + \text{mmoles of produced NH}_3}{\text{Total New volume (mL)}}$$

No. of moles of produced  $NH_3 = No.$  of moles of reacted NaOH

$$C_{NH_3} = \frac{M_{NH_3}V_{NH_3} + M_{NaOH}V_{NaOH}}{V_{NH_3} + V_{NaOH}}$$

$$C_{NH_3} = \frac{400 \times 0.2 + 100 \times 0.05}{[400 + 100]} = \frac{85}{500} = 0.17 \text{M}$$

$$C_{NH4+} = \frac{\text{original No. of moles of NH}_4^+ - \text{moles of reacted NH}_4^+}{\text{Total New volume(L)}}$$

$$Or \ C_{NH4+} \ = \ \frac{original \ No. \ of \ mmoles \ of \ NH_4^+ - \ mmoles \ of \ reacted \ NH_4^+}{Total \ New \ volume(mL)}$$

No. of moles of consumed  $NH_4^+$  = No. of moles of reacted NaOH

$$C_{NH_4Cl} = \frac{M_{NH_4Cl}V_{NH_4Cl} - M_{NaOH}V_{NaOH}}{VNH_4Cl + V_{NaOH}}$$

$$C_{NH_4Cl} = \frac{400 \times 0.3 - 100 \times 0.05}{[400 + 100]} = \frac{115}{500} = 0.23$$
M

pOH = 4.76+ log 
$$\frac{0.23}{0.17}$$
 = 4.89 (Henderson equation)

$$[OH^{-}] = 10^{-pOH} = 10^{-4.93} = 1.17 \times 10^{-5}$$

To check the validity of approximation we calculate

if 
$$\frac{[OH^-]}{[Base]} \times 100 < 10 \%$$
 Then approximation is valid

Then 
$$\frac{1.17 \times 10^{-5}}{0.2} \times 100 = 5.85 \times 10^{-3}$$
 (approximation is valid)

$$pH = 14 - 4.89 = 9.11$$

$$\Delta pH = 9.11 - 9.07 = 0.04$$

2) addition of HCl converts part of NH<sub>3</sub> to NH<sub>4</sub>Cl

$$NH_3 + H_3O^+ \rightleftharpoons NH_4^+ + H_2O (H_3O^+ \text{ from HCl})$$

$$C_{NH3} \ = \ \frac{\text{original No.of moles of NH}_3 - \text{moles of reacted NH}_3}{\text{Total New volume (L)}}$$

$$Or \ C_{NH3} \ = \ \frac{original \ No. of \ mmoles \ of \ NH_3 - \ mmoles \ of \ reacted \ NH_3}{Total \ New \ volume \ (mL)}$$

No. of moles of consumed  $NH_3 = No.$  of moles of reacted HCl

$$C_{NH_3} = \frac{M_{NH_3}V_{NH_3} - M_{HCl}V_{HCl}}{V_{NH_3} + V_{HCl}}$$

$$C_{NH_3} = \frac{400 \times 0.2 - 100 \times 0.05}{[400 + 100]} = 0.150$$
M

$$C_{\mathrm{NH_4}^+}$$
 =  $\frac{\mathrm{original\ No\ of\ moles\ of\ NH_4}^+ + \mathrm{moles\ of\ produced\ NH_4}^+}{\mathrm{Total\ New\ volume\ (L)}}$ 

Or 
$$C_{\text{NH}_4}^+ = \frac{\text{original No of mmoles of NH}_4^+ + \text{mmoles of produced NH}_4^+}{\text{Total New volume (mL)}}$$

No. of moles of produced  $NH_4 = No.$  of moles of reacted HCl

$$C_{NH_4Cl} = \frac{\mathsf{M}_{NH_4Cl} \mathsf{V}_{NH_4Cl} + \mathsf{M}_{\mathsf{HCl}} \mathsf{V}_{\mathsf{HCl}}}{\mathsf{V}_{NH_4Cl} + \mathsf{V}_{\mathsf{HCl}}}$$

$$C_{NH_4Cl} = \frac{400 \times 0.3 + 100 \times 0.05}{[400 + 100]} = 0.25 M$$

pOH= pK<sub>b</sub> + 
$$log \frac{NH_4Cl}{C_{NH_3}}$$
 (Henderson equation)

pOH = 
$$4.76 + \log \frac{0.25}{0.15} = 4.98$$

$$[OH^{-}] = 10^{-pOH} = 10^{-4.98} = 1.05 \times 10^{-5}$$

To check the validity of approximation we calculate [OH-] then

Check if 
$$\frac{[OH^-]}{[Base]} \times 100 < 10 \%$$
 Then approximation is valid

Then 
$$\frac{1.05 \times 10^{-5}}{0.2} \times 100 = 5.51 \times 10^{-3}$$
 (approximation is valid)

$$pH = 14 - 4.98 = 9.02$$

$$\Delta pH = 9.02 - 9.07 = -0.05$$

| Addition           | ΔрН    |
|--------------------|--------|
| 100 mL 0.05 M NaOH | 0.04   |
| 100 mL 0.05 M HCl  | - 0.05 |

# **Exercise:**

Calculate the pH change that take place when 100 mL portion of:

**a)** 0.05 M NaOH **b)** 0.05 M HCl is added seperately to 400 mL of buffer solution of (0.1M) CH<sub>3</sub>COOH and (0.2M) CH<sub>3</sub>COONa ( $K_a = 1.74 \times 10^{-5}$ ).

#### Hint:

$$CH_3COOH + NaOH \rightleftharpoons CH_3COONa + H_2O$$

$$CH_3COO^- + HCl \rightleftharpoons CH_3COOH$$

# **Preparation of buffer:**

To prepare a buffer, it is to choose the acid with the pK<sub>a</sub> close to the desired <u>pH</u>. Usually, buffers have a useful <u>pH</u> range = pK<sub>a</sub>  $\pm$  1, but the closer it is to the weak acid's pKa, is the better.

### **Example:**

Describe how you might prepare approximately (500 mL) of pH 4.5 buffer solution from 1 M acetic acid (CH<sub>3</sub>COOH) and sodium acetate (CH<sub>3</sub>COONa) (82.03 g /mole) , (Ka= $1.74 \times 10^{-5}$ ) .

## **Solution:**

For acidic buffer(pH= 4.5)

$$CH_3COOH + H_2O \rightleftharpoons CH_3COO^- + H_3O^+$$

$$pH = pKa + log \frac{c_{salt}}{c_{acid}}$$
 ( Henderson equation for acidic buffers)

$$pKa = -log(1.74 \times 10^{-5}) = 4.76$$

$$4.5 = 4.76 + \log \frac{[CH_3COO^-]}{[CH_3COOH]}$$

$$4.5 = 4.76 + \log \frac{[CH_3COO^-]}{[1]}$$

$$4.5 - 4.76 = \log [CH_3COO^{-}] - \log 1$$

$$\log [CH_3COO^{-}] = -0.26$$

$$[CH_3COO^-] = 10^{-0.26} = 0.549 M$$

Mass of  $CH_3COONa$  needed =  $Molarity(M) \times V(liter) \times Mwt$ 

Mass of CH<sub>3</sub>COONa = 0.549(mol/L) x 
$$\frac{500}{1000}$$
 L x 82.03(g/mol) = 22.52 g

The required buffer is to be made by dissolution of 22.52 g of CH<sub>3</sub>COONa and completing the volume to 500 mL with 1M CH<sub>3</sub>COOH

# **Buffer capacity**

It is defined as the number of moles of an acid or base added to 1 liter of a buffer solution to cause its pH to change by 1 unit.

Buffer capacity depends on the amount of acid and base used to prepare the buffer. For example, if you have a 1-L buffer solution made of (1 M CH<sub>3</sub>COOH and 1 M CH<sub>3</sub>COONa) and a 1-L buffer solution that is (0.1 M CH<sub>3</sub>COOH and 0.1 M CH<sub>3</sub>COONa), although they will both have the same pH, the first buffer solution will have a greater buffer capacity because it has a higher amount of CH<sub>3</sub>COOH and CH<sub>3</sub>COO<sup>-</sup>.

To calculate buffer capacity, we use the following formula:

$$\beta = n / \Delta pH$$

 $\beta$  is buffer capacity (it is unitless)

**n** is the number of moles of an acid or base (that were added to the buffer) per liter of the buffer

$$\mathbf{n} = \frac{\textit{No.of moles of acid or base added}}{\textit{vol of buffer(L)}}$$

 $\Delta pH$  is the difference between the initial pH of the buffer and the pH of the buffer after the acid or base is added

The higher the capacity ( $\beta$ ), the more acid and base can be added to the buffer before its pH changes significantly.

#### **Example:**

A volume of 150 mL of 0.2 M HCl was added to 600 mL of buffer with a pH of 7.39. which gives the buffer solution a new pH of 7.03. What is the capacity of this buffer solution?

#### **Solution:**

No. of moles = Molarity (M)  $\times$  V(L)

Number of moles of HCl =  $0.2 \text{ M} \times 0.150 \text{ L} = 0.03 \text{ mol}$ 

$$n = \frac{\textit{No.of moles of acid or base added}}{\textit{vol of buffer(L)}} = \frac{0.03 \, \textit{mole}}{0.6 \, \textit{L}} = 0.05 \, \, \textit{mol/L}$$

n = 0.05 mol/L

$$\Delta pH = |7.03 - 7.39| = 0.36$$

$$\beta = \frac{n}{\Delta pH} = \frac{0.05}{0.36} = 0.14$$

Thus, the buffer capacity of our buffer solution is 0.14.