# *Information Theory and Coding*

***Forth Stage***

**MSc. Zahraa Niema Kamal** **Lecture Two** **First Course**

## Random Variables:

A random variable, usually written X, is a variable whose possible values are numerical outcomes of a random phenomenon. There are two types of random variables, discrete and continuous. All random variables have a cumulative distribution function. It is a function giving the probability that the random variable X is less than or equal to x, for every value x.

### 1. Discrete Random Variable

A discrete random variable is one which may take on only a countable number of distinct values such as 0,1,2,3,4,........ If a random variable can take only a finite number of distinct values, then it must be discrete. Examples of discrete random variables include the number of children in a family, the number of defective light bulbs in a box of ten. The probability distribution of a discrete random variable is a list of probabilities associated with each of its possible values. It is also sometimes called the probability function or the probability mass function.

When the sample space Ω has a finite number of equally likely outcomes, so that the discrete uniform probability law applies. Then, the probability of any event x is given by:

$$p(A) = \frac{\text{Number of elements of x}}{\text{Number of elements of } \Omega}$$

This distribution may also be described by the ***probability histogram.*** Suppose a random variable $X$ may take $k$ different values, with the probability that $X = xi$ defined to be

$P(X = xi) = Pi$ . The probabilities $Pi$ must satisfy the following:

1- $0 < Pi < 1$ for each i

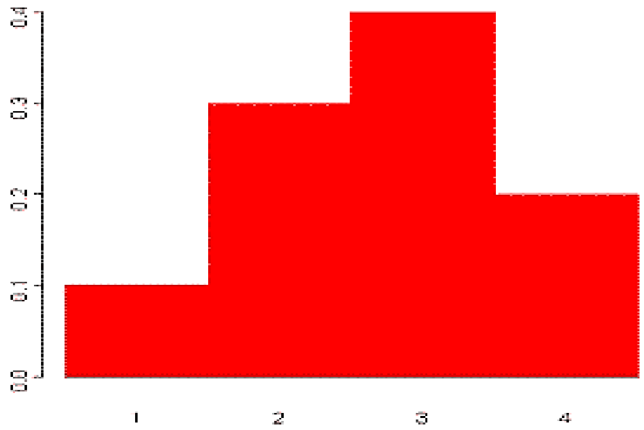
2- $P1 + P2 + \cdots + Pk = 1$ or,

$$\sum_{i=1}^{k} P_i = 1$$

**Example**: Suppose a variable X can take the values 1, 2, 3, or 4. The probabilities associated with each outcome are described by the following table:

| | | | | |
|---|---|---|---|---|
| Outcome: | 1 | 2 | 3 | 4 |
| Probability: | 0.1 | 0.3 | 0.4 | 0.2 |

plot the probability distribution and the cumulative distribution.




probability distribution

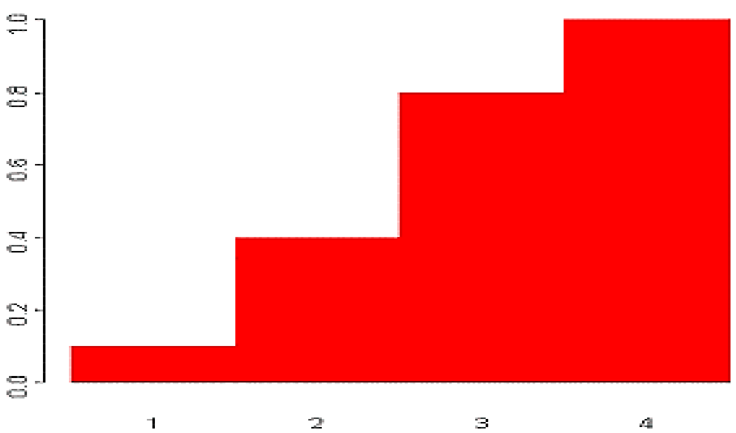
The cumulative distribution function for the above probability distribution is calculated as follows:

The probability that X is less than or equal to 1 is 0.1,

the probability that X is less than or equal to 2 is 0.1+0.3 = 0.4,

the probability that X is less than or equal to 3 is 0.1+0.3+0.4 = 0.8,

and, the probability that X is less than or equal to 4 is 0.1+0.3+0.4+0.2 = 1.




cumulative distribution

**H.W:** Having a text of (ABCAABDCAA). Calculate the probability of each letter, plot the probability distribution and the cumulative distribution.

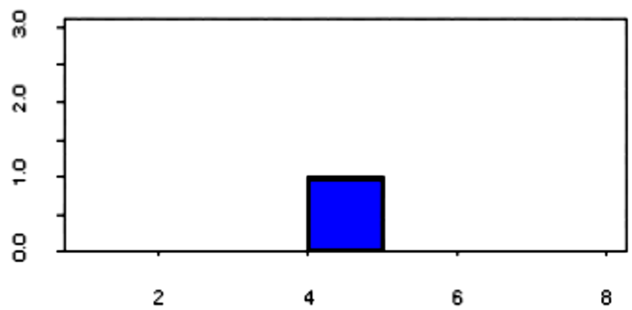
## 2. Continuous Random Variables

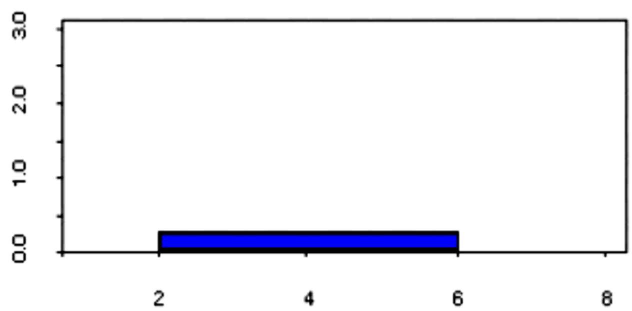
A ***continuous random variable*** is one which takes an infinite number of possible values. Continuous random variables are usually measurements. Examples include height, weight and the amount of sugar in an orange. A continuous random variable is not defined at specific values. Instead, it is defined over an *interval* of values, and is represented by the ***area under a curve***. The curve, which represents a function *p(x)*, must satisfy the following:

**1:** *The curve has no negative values ($p(x) \geq 0$ for all x)*

**2:** *The total area under the curve is equal to 1.*

A curve meeting these requirements is known as a ***density curve***. If any interval of numbers of equal width has an equal probability, then the curve describing the distribution is a rectangle, with constant height across the interval and 0 height elsewhere, these curves are known as uniform distributions.

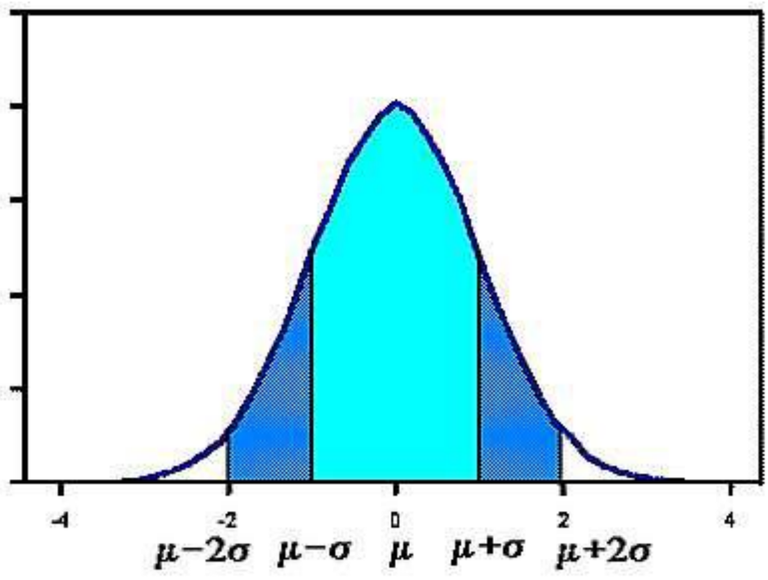



Uniform distribution

Another type of distribution is the normal distribution having a bell-shaped density curve described by its mean $\mu$ and standard deviation $\sigma$. The height of a normal density curve at a given point x is given by:

$$h = \frac{1}{\sigma\sqrt{2\pi}} e^{-0.5\left(\frac{x-\mu}{\sigma}\right)^2}$$




The Standard normal curve

## Joint Probability:

Joint probability is the probability of event $Y$ occurring at the same time event $X$ occurs. Its notation is $P(X \cap Y) or\ P(X, Y)$, which reads; the joint probability of $X$ and $Y$.

$$P(X, Y) = P(X) \times P(Y)$$

**Example:** For discrete random variable, if the probability of rolling a four on one die is $P(X)$ and if the probability of rolling a four on second die is $P(Y)$. Find $P(X,Y)$.

Solution: We have $P(X) = P(Y) = 1/6$

$$P(X,Y) = P(X) \times P(Y)$$
$$= 1/6 \times 1/6$$
$$=1/36$$
$$= 0.0277 = 2.8\%$$

**Example :** if you have the joint probability as shown in this matrix; find the probability of the each single value

$$P(x,y) = \begin{bmatrix} 0.1 & 0.25 \\ 0 & 0.2 \\ 0.25 & 0.2 \end{bmatrix}$$

Solution:

$p(x1) = \sum_{j=1}^{2} p\ (x1, yj) = 0.1 + 0.25 = 0.35$

$p(x2) = \sum_{j=1}^{2} p\ (x2, yj) = 0 + 0.2 = 0.2$

$p(x3) = \sum_{j=1}^{2} p\ (x3, yj) = 0.25 + 0.2 = 0.45$

$p(y1) = \sum_{i=1}^{3} p\ (xi, y1) = 0.1 + 0 + 0.25 = 0.35$

$p(y2) = \sum_{i=1}^{3} p\ (xi, y2) = 0.25 + 2 + 0.2 = 0.65$

**Then:**

$p(x) = [0.35 \quad 0.2 \quad 0.45]$ **and** $p(y)= [0.35 \quad 0.65]$