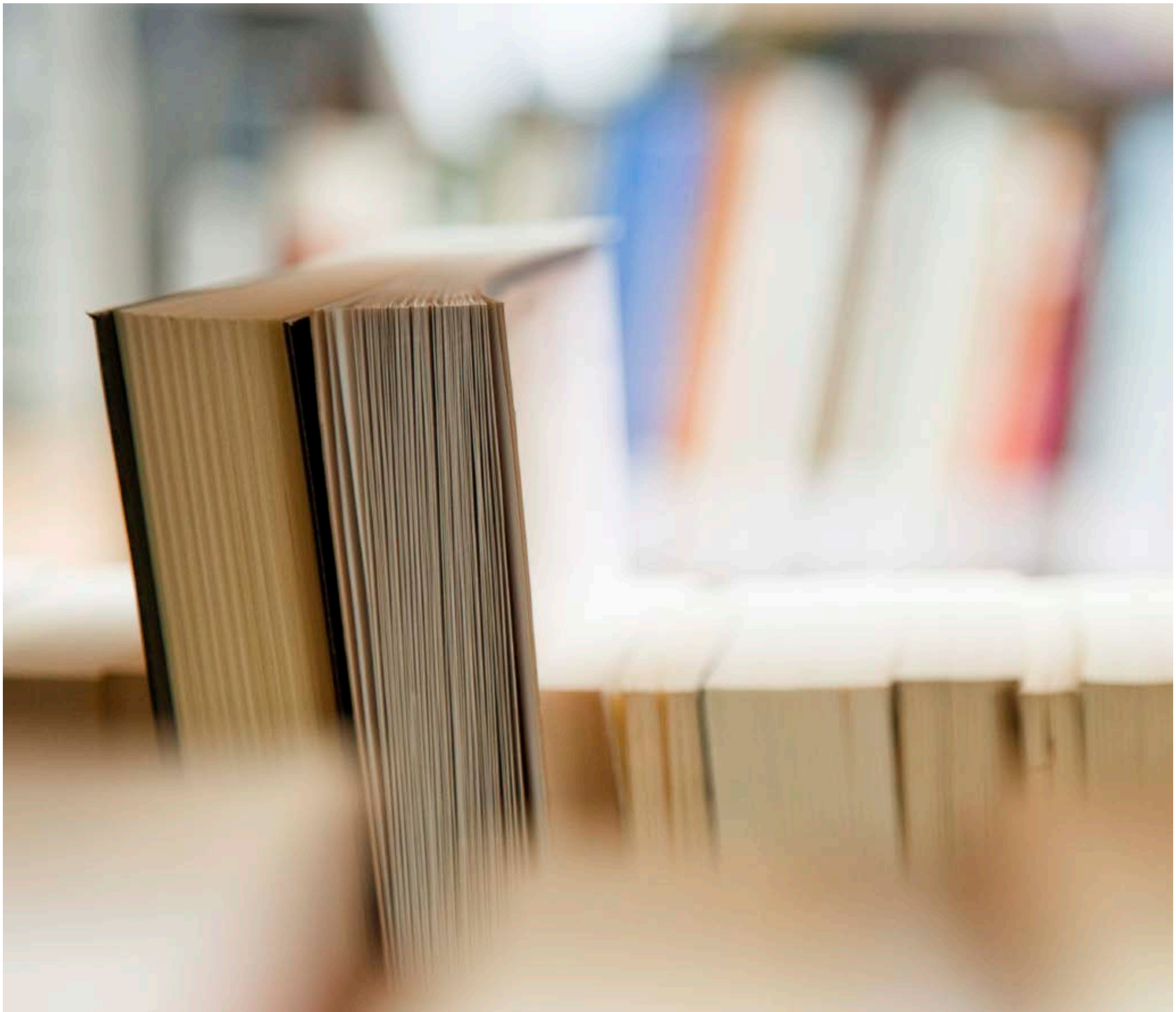


Information Theory and Coding

Forth Stage

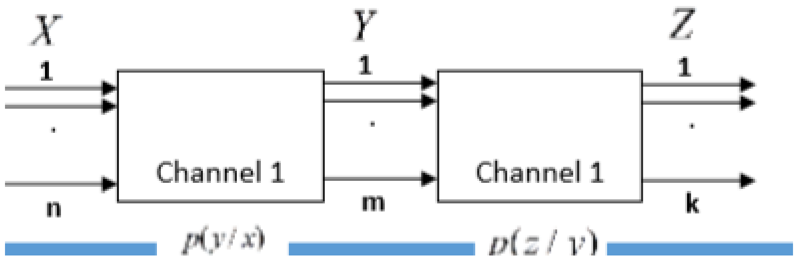
MSc. Zahraa Niema Kamal Lecture Nine First Course

1. Cascading of Channels

If two channels are cascaded, then the overall transition matrix is the product of the two transition matrices.

p(z / x)=p(y / x) . p(z / y)

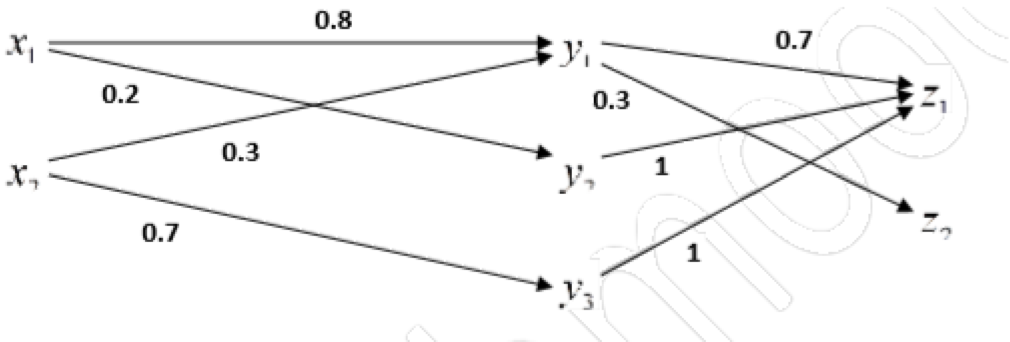
$(n \times k) \quad (n \times m) \quad (m \times k)$



For the series information channel, the overall channel capacity is not exceed any of each channel individually.

$I(X,Z) \le I(X,Y) \quad \& \quad I(X,Z) \le I(Y,Z)$

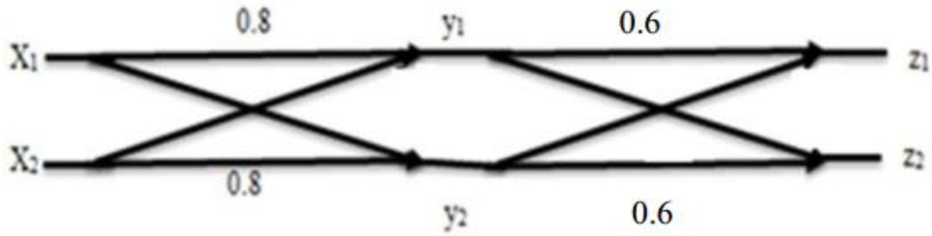
Example: Find the transition matrix p(z / x) for the cascaded channel shown:



$$p(Y|X) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix}, \qquad p(Z|Y) = \begin{bmatrix} 0.7 & 0.3 \\ 1 & 0 \\ 1 & 0 \end{bmatrix}$$

$$p(Z|X) = \begin{bmatrix} 0.8 & 0.2 & 0 \\ 0.3 & 0 & 0.7 \end{bmatrix} \cdot \begin{bmatrix} 0.7 & 0.3 \\ 1 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0.76 & 0.24 \\ 0.91 & 0.09 \end{bmatrix}$$

Example: Two BSC is cascaded as shown:



i) Find the resultant channel matrix; then plot the final model

ii) Find the channel capacity

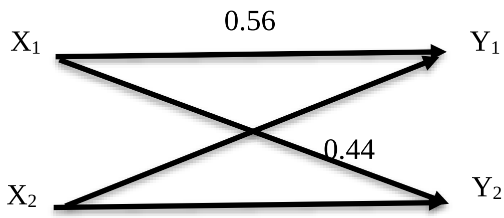
Solution

$P(Y/X)=\begin{bmatrix}0.8 & 0.2\\0.2 & 0.8\end{bmatrix}$

$P(Z/Y)=\begin{bmatrix}0.6 & 0.4\\0.4 & 0.6\end{bmatrix}$

$P(Z/X)=\begin{bmatrix}0.8 & 0.2\\0.2 & 0.8\end{bmatrix}\times\begin{bmatrix}0.6 & 0.4\\0.4 & 0.6\end{bmatrix}$

$P(Z/X)=\begin{bmatrix}0.56 & 0.44\\0.44 & 0.56\end{bmatrix}$



$C=\log_2 2+K$

$$k=\sum_{j=1}^{m}P(zj/xi)log_2P(zj/xi)=\frac{0.56ln0.56+0.44ln0.44}{ln2}=-0.989$$

C= 1- 0.989 = 0.011 bits / symbol