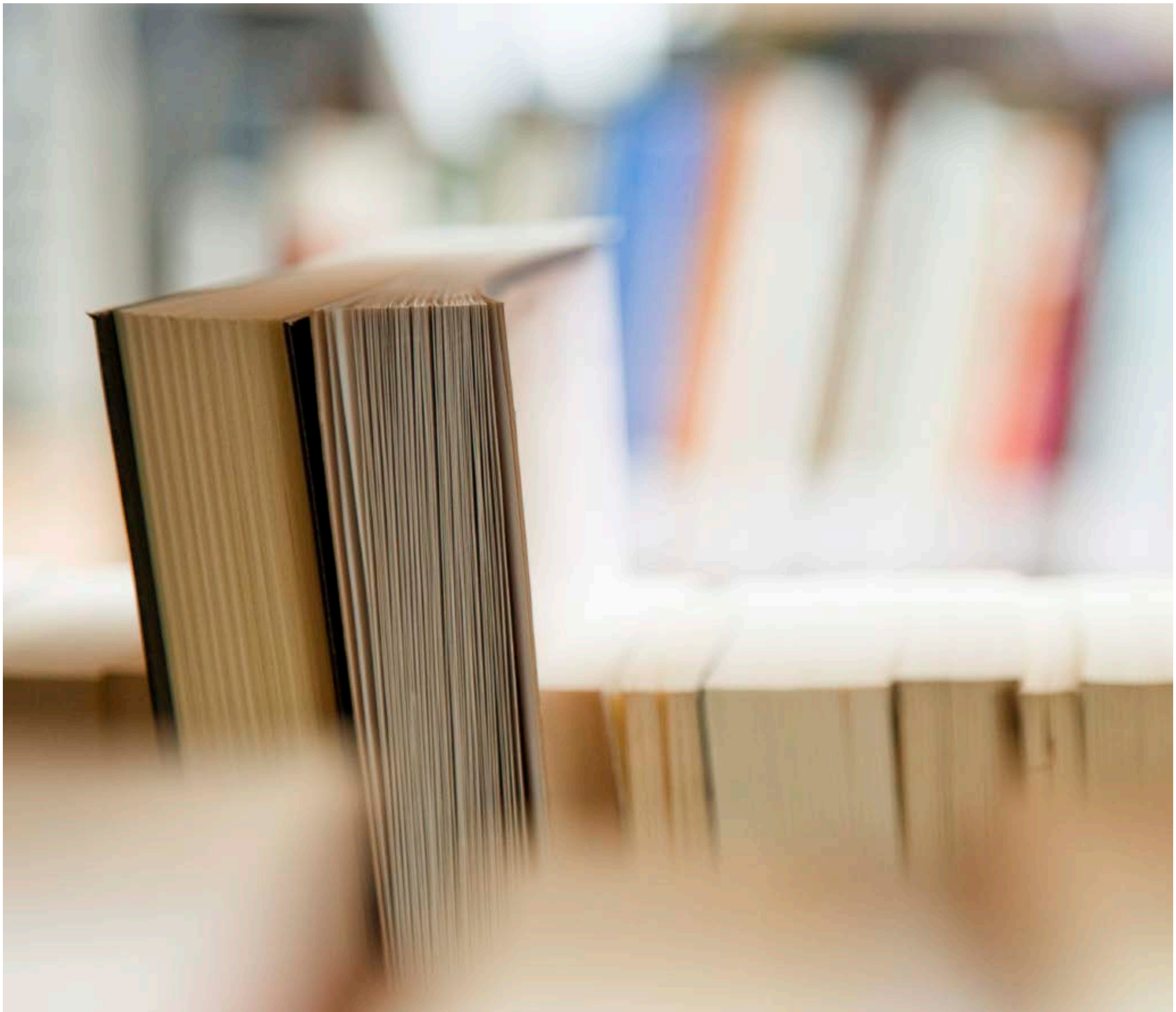


Information Theory and Coding

Forth Stage

MSc. Zahraa Niema Kamal **Lecture Three** **First Course**

Conditional Probability:

It is happened when there are dependent events. We have to use the symbol "|" to mean "given":

- P(B|A) means "Event B **given** Event A has occurred".
- P(B|A) is also called the "Conditional Probability" of B given A has occurred .
- And we write it as:

$$P(A \mid B) = \frac{number\ of\ elements\ of\ A\ and\ B}{number\ of\ elements\ of\ B}$$

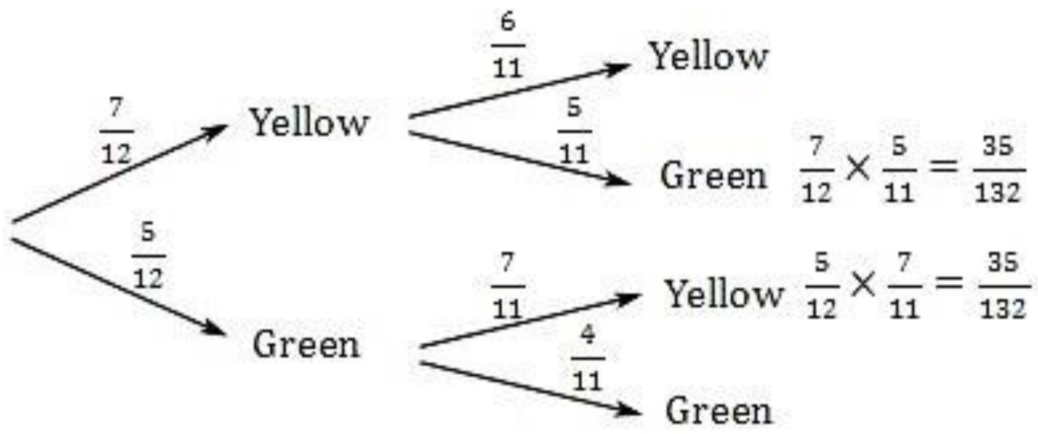
Or

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

Where $P(B) > 0$

Example: A box contains 5 green pencils and 7 yellow pencils. Two pencils are chosen at random from the box without replacement. What is the probability they are different colors?

Solution: Using a tree diagram:



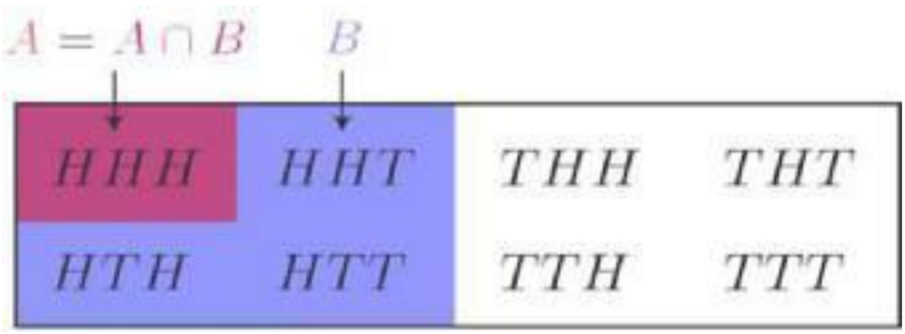
Example: Find the conditional prob. For Toss a fair coin 3 times :1- What is the probability of 3 heads? 2- What is the probability of head?

Solution:

S= {HHH,HHT,HTH,HTT,THH,THT,TTH,TTT}

A={HHH}

B= {HHH,HHT,HTH,HTT}



The conditional probability of A given B is written P(A|B) and is defined

$$P(A \mid B) = \frac{P(A \cap B)}{P(B)}$$

We have P(A)=1/8

P(B)=4/8

P(A ∩ B)= 1/8

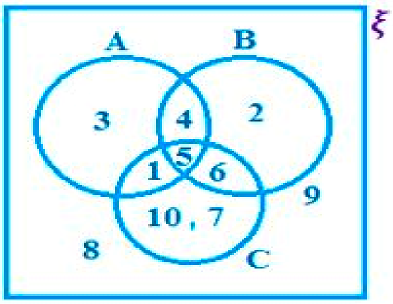
$P(A|B) = \frac{1/8}{4/8} = \frac{1}{4}$

Venn's Diagram

A Venn diagram is a diagram that shows all possible logical relations between a finite collections of different sets. These diagrams depict elements as points in the plane, and sets as regions inside closed curves. A Venn diagram consists of multiple overlapping closed curves, usually circles, each representing a set. The points inside a curve labelled S represent elements of the set S, while points outside the boundary represent elements not in the set S. Fig. 5 shows the set $A = \{1,2,3\}, B = \{4,5\}\ and\ U = \{1,2,3,4,5,6\}$.

Example:

From the adjoining Venn diagram, find the following sets.



(i) A

(ii) B

(iii) ξ

(iv) A'

(v) B'

(vi) C'

(vii) C - A

(viii) B - C

(ix) A - B

(x) A ∪ B

(xi) B ∪ C

(xii) A ∩ C

(xiii) B ∩ C

(xiv) (B ∪ C)'

(xv) (A ∩ B)'

(xvi) (A ∪ B) ∩ C

Answers for examples on Venn diagram are given below:

(i) **A**

= {1, 3, 4, 5}

(ii) **B**

= {4, 5, 6, 2}

(iii) **ξ**

= {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

(iv) **A'**

= {2, 6, 7, 8, 9, 10} **all elements of universal set leaving the elements of set A.**

(v) **B'**

= {1, 3, 7, 8, 9, 10} **all elements of universal set leaving the elements of set B.**

(vi) **C'** = To find

C = {1, 5, 6, 7, 10}

Therefore, C' = {2, 3, 4, 8, 9} **all elements of universal set leaving the elements of set C.**

(vii) **C - A**

Here C = {1, 5, 6, 7, 10}

A = {1, 3, 4, 5}

then C – A = {6, 7, 10} **excluding all elements of A from C.**

(viii) **B - C**

Here B = {4, 5, 6, 2}

C = {1, 5, 6, 7, 10}

B - C = {4, 2} **excluding all elements of C from B.**

(ix) **B - A**

Here B = {4, 5, 2}

A = {1, 3, 4, 5}

B - A = {6, 2} excluding all elements of A from C.

(x) **A ∪ B**

Here A = {1, 3, 4, 5}

B = (4, 5, 6, 2}

A ∪ B = {1, 2, 3, 4, 5, 6}

(xi) **B ∪ C**

Here B = {4, 5, 6, 2}

C = {1, 5, 6, 7, 10}

B ∪ C = {1, 2, 4, 5, 6, 7, 10}

(xii) **(B ∪ C)'**

Since, B ∪ C = {1, 2, 4, 5, 6, 7, 10}

Therefore, (B ∪ C)' = {3, 8, 9}

(xiii) **(A ∩ B)'**

A = {1, 3, 4, 5}

B = {4, 5, 6, 2}

(A ∩ B) = {4, 5}

(A ∩ B)' = {1, 2, 3, 6, 7, 8, 9, 10}

(xiv) **(A ∪ B) ∩ C**

A = {1, 2, 3, 4}

B = {4, 5, 6, 2}

C = {1, 5, 6, 7, 10}

A ∪ B= {1, 2, 3, 4, 5, 6}

(A ∪ B) ∩ C = {1, 5, 6}

(xv) **A ∩ (B ∩ C)**

A = {1, 3, 4, 5}

B = {4, 5, 6, 2}

C = {1, 5, 6, 7, 10}

B ∩ C = {5, 6}

A ∩ (B ∩ C) = {5}