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PREREQUISITES FOR CALCULUS (المتطلبات الأساسية للتفاضل والتكامل)

Sets and Intervals (المجموعات والفترات)

DEFINITIONS:

Set: is a collection of things under certain conditions.

Example 1:

$$A = \{1, 3, 5, 8, 10\};$$

A is a set, 1, 3, 5, 8, 10 are elements.

Real Numbers (R): is a set of all rational and irrational numbers. $R = \{-\infty, +\infty\}$,

$$-\infty \longleftarrow \text{---} 0 \text{---} \longrightarrow +\infty$$

Integer Numbers (I): a set of all irrational numbers.

$I = \{-\infty, \text{---}, -3, -2, -1, 0, 1, 2, 3, \text{---}, +\infty\}$ negative and positive numbers only.

Natural Numbers (N): consist of zero and positive integer numbers only.

$$N = \{0, 1, 2, 3, \text{---}, +\infty\}$$

Intervals: is a set of all real numbers between two points on the real number line. (it is a subset of real numbers)

1. Open interval: is a set of all real numbers between A&B excluded (A&B are not elements in the set). $\{x: A < x < B\}$ or (A, B) .

$$-\infty \longleftarrow \text{---} A \text{---} (\text{---} X \text{---}) B \text{---} \longrightarrow +\infty$$

2. Closed interval: is a set of all real numbers between A&B included (A&B are elements in the set). $\{x: A \leq x \leq B\}$ or $[A, B]$.

$$-\infty \longleftarrow \text{---} A [\text{---} X \text{---}] B \text{---} \longrightarrow +\infty$$

3. Half-Open interval (Half-Close): is a set of all real numbers between A & B with one of the end-points as an element in the set.

a) $(A, B] = \{x: A < x \leq B\}$ $-\infty \longleftarrow \text{---} A (\text{---} X \text{---}] B \text{---} \longrightarrow +\infty$

b) $[A, B) = \{x: A \leq x < B\}$ $-\infty \longleftarrow \text{---} A [\text{---} X \text{---}) B \text{---} \longrightarrow +\infty$



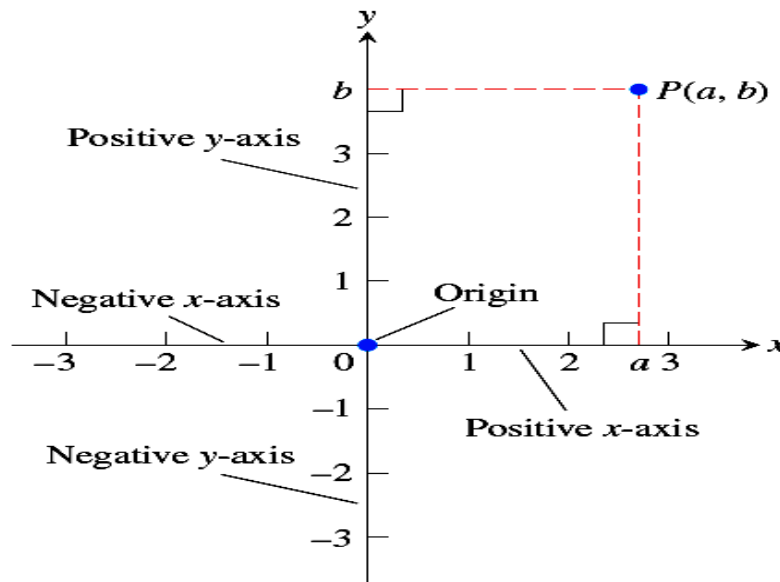
TABLE 1.1 Types of intervals

	Notation	Set description	Type	Picture
Finite:	(a, b)	$\{x a < x < b\}$	Open	
	$[a, b]$	$\{x a \leq x \leq b\}$	Closed	
	$[a, b)$	$\{x a \leq x < b\}$	Half-open	
	$(a, b]$	$\{x a < x \leq b\}$	Half-open	
Infinite:	(a, ∞)	$\{x x > a\}$	Open	
	$[a, \infty)$	$\{x x \geq a\}$	Closed	
	$(-\infty, b)$	$\{x x < b\}$	Open	
	$(-\infty, b]$	$\{x x \leq b\}$	Closed	
	$(-\infty, \infty)$	\mathbb{R} (set of all real numbers)	Both open and closed	



Coordinate in the Plane (الاحداثيات في الفراغ او المستوى)

Each point in the plane can be represented with a pair of real numbers (a,b) , the number a is the horizontal distance from the origin to point P , while b is the vertical distance from the origin to point P . The origin divides the x -axis into positive x axis to the right and the negative x -axis to the left, also, the origin divides the y -axis into positive y -axis upward and the negative y -axis downward. The axes divide the plane into four regions called quadrants.





Distance between Points and (Mid-Point Formula):

Distance between points in the plane is calculated with a formula that comes from Pythagorean Theorem:

❖ **Distance Formula for Points in the Plane**

The distance between $P(x_1, y_1)$ and $Q(x_2, y_2)$

$$d = \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

and the mid-point formula:

$$x_0 = \frac{x_1 + x_2}{2}, y_0 = \frac{y_1 + y_2}{2}$$

Example 2: find the distance between $P(-1,2)$ and $Q(3,4)$ and find the mid-point:

Sol.:

$$\begin{aligned} d &= \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(3 - (-1))^2 + (4 - 2)^2} = \sqrt{20} = 2\sqrt{5} \end{aligned}$$

$$x_0 = \frac{x_1 + x_2}{2}, x_0 = \frac{-1 + 3}{2} = 1 \text{ and } y_0 = \frac{y_1 + y_2}{2}, y_0 = \frac{2 + 4}{2} = 3.$$

Example 3: find the distance between $R(2,-3)$ and $S(6,1)$ and find the mid-point:

Sol.:

$$\begin{aligned} d &= \sqrt{(\Delta x)^2 + (\Delta y)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(6 - 2)^2 + (1 - (-3))^2} = \sqrt{16 + 16} = \sqrt{32} = 2\sqrt{8} \end{aligned}$$

$$x_0 = \frac{x_1 + x_2}{2}, x_0 = \frac{2 + 6}{2} = 4 \text{ and } y_0 = \frac{y_1 + y_2}{2}, y_0 = \frac{-3 + 1}{2} = -1.$$



Slope and Equation of Line

❖ **Slope (الميل):** The constant

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

is the slope of non-vertical line $P_1 P_2$.

Note1: Horizontal line have ($m=0$) ($\Delta y=0$), and the vertical line has no slope or the slope of vertical line is undefined ($\Delta x=0$).

Note2: Parallel lines have the same slope
In the the lines are parallel then ($m_1 = m_2$).

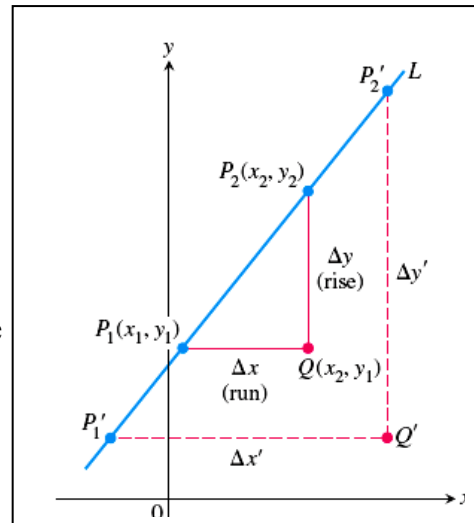
Note3: If two non-vertical lines L_1 and L_2 are perpendicular, their slopes m_1 and m_2 satisfy
 $m_1 * m_2 = -1$,
so each slope is the negative reciprocal of the other.

$$m_1 = \frac{1}{m_2} \text{ and } m_2 = \frac{1}{m_1}$$

Example 4: Find the slope of the straight line through the two points $P(3,2)$ and $Q(4,4)$:

Sol.:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 2}{4 - 3} = 2.$$





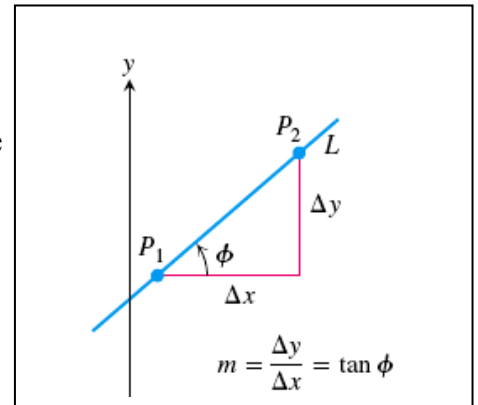
❖ Point-Slope Equation:

We can write an equation for a non-vertical straight line L if we know its slope m and the coordinate of one point $P_1(x_1, y_1)$ on it. If $P(x, y)$ is any other point on L , then we can use two points P_1 and P to compute the slope,

$$m = \frac{y - y_1}{x - x_1}$$

so that $y - y_1 = m(x - x_1)$

or $y = y_1 + m(x - x_1)$



The equation $y = y_1 + m(x - x_1)$

is the **point-slope equation** of the line that passes through the point $P_1(x_1, y_1)$ and has slope m .

Example 5: write an equation for the line pass through the point $(2,3)$ with slope $(-3/2)$.

Sol.: we substitute $x_1 = 2$, $y_1 = 3$, and $m = -3/2$ into the point-slope equation and obtain

$$y = y_1 + m(x - x_1)$$

$$y = 3 + \frac{-3}{2}(x - 2)$$

$$y = -\frac{3}{2}x + 6.$$



Example 6: A line pass through two points: write an equation for the line through (-2,-1) and (3,4)

Sol.: The line's slope is

$$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-1 - 4}{-2 - 3} = \frac{-5}{-5} = 1.$$

We can use this slope with either of the two given points in the point-slope equation;

With $(x_1, y_1) = (-2, -1)$

$$y = -1 + 1 \cdot (x - (-2))$$

$$y = -1 + x + 2$$

$$y = x + 1$$

With $(x_2, y_2) = (3, 4)$

$$y = 4 + 1 \cdot (x - 3)$$

$$y = 4 + x - 3$$

$$y = x + 1$$

Note: The equation:

$$y = mx + b$$

is called the **slope-intercept equation** of the line with slope m and y -intercept b

Note: The general form of straight line equation is

$$Ax + By + C = 0$$

Example 7: finding the slope and y -Intercept for the line $8x + 4y = 20$.

Sol.: solve the equation for y to put it in slope-intercept form :

$$8x + 4y = 20$$

$$4y = -8x + 20$$

$$y = -8/4 x + 4.$$

$$y = -2 x + 4.$$

The slope $m = -2$ the y -intercept is $b = 4$.



H.W:

1. finding the slope and y-Intercept for the line $4x + 2y = 4$.
2. write an equation for the line pass through $(-1,-1)$ and $(1,2)$.
3. write an equation for the line pass through the point $(1,-1)$ with slope (4) .
4. Find the **slope** of the straight line through the two points $P(3,-2)$ and $Q(3,6)$.
5. write an equation for the horizontal line pass through the point $(2,-2)$