Normal Stresses

Stress is defined as the strength of a material per unit area of unit strength. It is the force on a member divided by area, which carries the force, formerly express in psi, now in N/mm^2 or MPa.

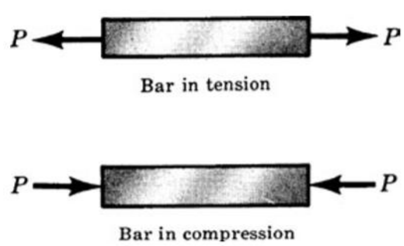


Fig. 1-1 Axially loaded bars.

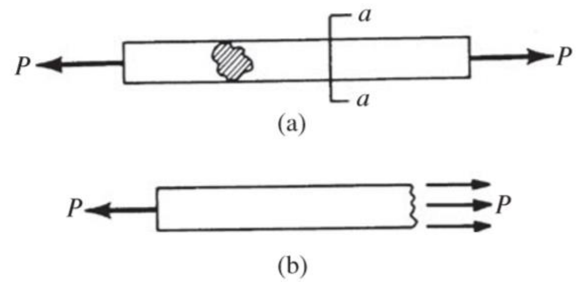


Fig. 1-2 Internal force.

$$\sigma = \frac{P}{A}$$

where **P** is the applied normal load in Newton and **A** is the area in mm^2. The maximum stress in tension or compression occurs over a section normal to the load.

Normal stress is either tensile stress or compressive stress. Members subject to pure tension (or tensile force) is under tensile stress, while compression members (members subject to compressive force) are under compressive stress.

Compressive force will tend to shorten the member. Tension force on the other hand will tend to lengthen the member.

Normal Strain

Let us suppose that the bar of Fig. 1-1 has tensile forces gradually applied to the ends. The elongation per unit length, which is termed normal strain and denoted by Δ, may be found by dividing the total elongation Δ by the length L, i.e.,

$$\varepsilon = \frac{\Delta}{L}$$

The strain is usually expressed in units of meters per meter and consequently is **dimensionless**.

Stress-Strain Curve

As the axial load in Fig. 1 is gradually increased, the total elongation over the bar length is measured at each increment of load and this is continued until fracture of the specimen takes place. Knowing the original cross-sectional area of the test specimen, the normal stress, denoted by s, may be obtained for any value of the axial load by the use of the relation

$$\sigma = \frac{P}{A}$$

experimental data may be plotted with these quantities considered as ordinate and abscissa, respectively. This is the stress-strain curve or diagram of the material for this type of loading. Stress-strain diagrams assume widely differing forms for various materials.

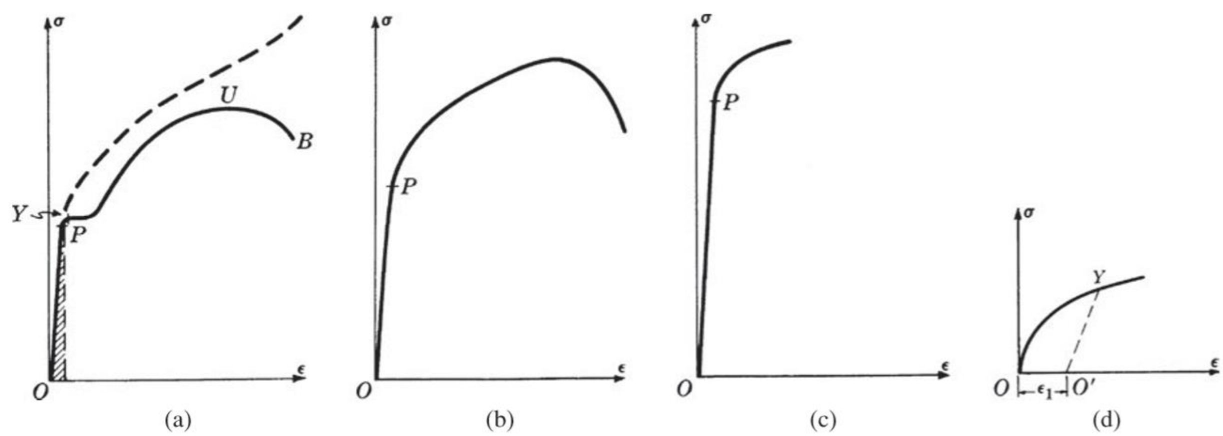
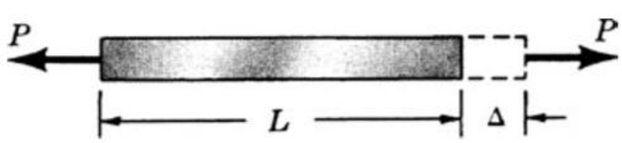


Fig. 1-3 Stress-strain diagrams.

Ex; In Fig, determine an expression for the total elongation of an initially straight bar of length **L**, cross-sectional area **A**, and modulus of elasticity **E** if a tensile load **P** acts on the ends of the bar.



$$E = \frac{\sigma}{\varepsilon} = \frac{P/_A}{\Delta/_L} = \frac{P * L}{A * \Delta} \qquad \Delta = \frac{P * L}{A * E}$$

Ex; a surveyor steel tape of 30m length has a cross-section of 0.5mm thickness and 15mm width. determine the elongation when the entire tape is stretched and held taut by a force of 1kN, the modulus of elasticity = 207 GPa.

$$E = \frac{\sigma}{\varepsilon} = \frac{P/A}{\Delta L/L} = \frac{P * L}{A * \Delta L}$$

$$\therefore \Delta L = \frac{PL}{AE}$$

$$L = 30\,m = 30 * 10^3\,mm$$

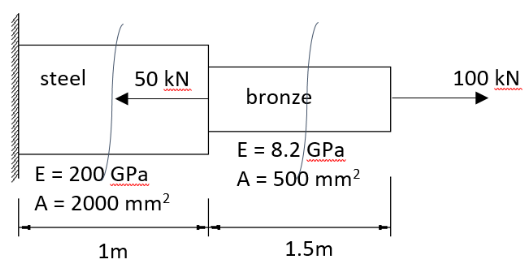
$$P = 1\,KN = 1000\,N$$

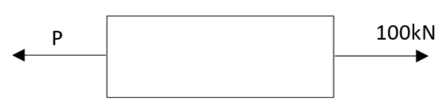
$$A = 0,5 * 15 = 7,5\,mm^2$$

$$E = 207\,GPa = 207 * 10^3\,MPa$$

$$\therefore \Delta L = \frac{PL}{AE} = \frac{1000 * 30 * 10^3}{7,5 * 207 * 10^3} = \boxed{19,32\,mm}$$

Ex; determine the elongation in the system shown





Sol:

1) For section (a-a)

$$\Sigma Fx = 0$$

$$0 = 100 - P \qquad \therefore P = 100\,KN \leftarrow$$

$$\therefore \Delta L = \frac{PL}{AE} = \frac{100 * 10^3 * 1,5 * 10^3}{500 * 8,2 * 10^3}$$

$$= \boxed{36.58\,mm}\ \text{elongation}$$

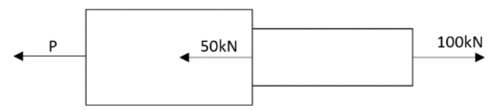
2) For section (b-b)

$\Sigma F_x = 0$

$0 = 100 - 50 - P$

$\therefore P = 100 - 50$

$\therefore P = 50\ KN \leftarrow$



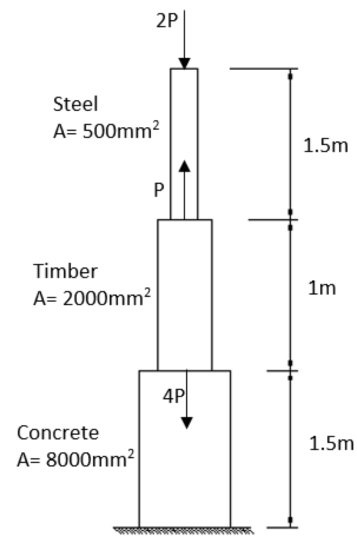
$\therefore \Delta L = \frac{PL}{AE} = \frac{50 * 10^3 * 1 * 10^3}{2000 * 200 * 10^3}$

$= \boxed{0,125\ mm}$ elongation

$\therefore$ Total elongation $= \Delta L_{bronze} + \Delta L_{steel}$

$= 36.58 + 0,125 = \boxed{36.705\ mm}$

Ex; find the maximum allowable value of (P) for the column shown in figure. The maximum allowable stress in steel = 120MPa, in timber is 12 MPa and in concrete is 16 MPa, Es= 200 GPa, Et =8.2 GPa, Ec = 17 GPa



SoL:-

1) section (a-a)

$\Sigma F_y = 0$

$0 = -2P + Y$

$\boxed{\therefore Y = 2P}$ compression

$\sigma_{steel} = \frac{Force}{A}$

$120 = \frac{2P}{500} \Rightarrow 2P = 120 * 500$

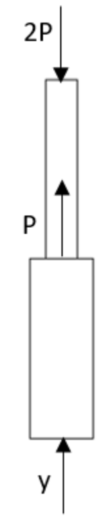
$\therefore P_s = 30000\ N = 30\ KN$

2) section (b-b)

$$\sum F_y = 0$$

$$0 = -2P + P + Y$$

$$\boxed{\therefore Y = P}$$ compression



$$\sigma_{timper} = \frac{Force}{A}$$

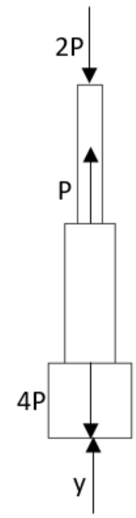
$$12 = \frac{P}{2000} \Longrightarrow P = 24000\ N = 24\ KN$$

3) section (c-c)

$$\sum F_y = 0$$

$$0 = -2P + P - 4P + Y$$

$$\boxed{\therefore Y = 5 \cdot P}$$ compression



$$\sigma_{concrete} = \frac{Force}{A}$$

$$16 = \frac{5P}{8000} \Longrightarrow P = 25600\ N = 25,6\ KN$$

$$\boxed{\therefore P = 24\ KN}$$ نختار الاصغر

To Find ΔL :-

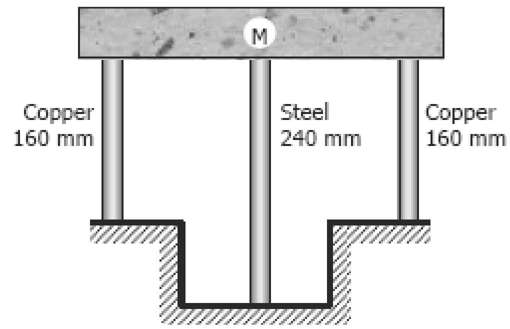
$$\Delta L_s = \frac{Force * L_s}{A_s * E_s} = \frac{2 * 24 * 10^3 * 1,5 * 10^3}{500 * 200 * 10^3} = 0,72\ mm$$

$$\Delta L_t = \frac{Force * L_t}{A_t * E_t} = \frac{24 * 10^3 * 1 * 10^3}{2000 * 8,2 * 10^3} = 1,463\ mm$$

$$\Delta L_c = \frac{Force * L_c}{A_c * E_c} = \frac{5 * 24 * 10^3 * 1,5 * 10^3}{8000 * 17 * 10^3} = 1,32\ mm$$

$$\therefore Total\ \Delta L = 0,72 + 1,463 + 1,32 = 3,5\ mm$$

Ex; A rigid block of mass (M) is supported by three symmetrically spaced rods as shown in fig. Each copper rod has an area of 900 mm^2; E = 120 GPa; and the allowable stress is 70 MPa. The steel rod has an area of 1200 mm^2; E = 200 GPa; and the allowable stress is 140 MPa. Determine the largest mass (M) which can be supported.

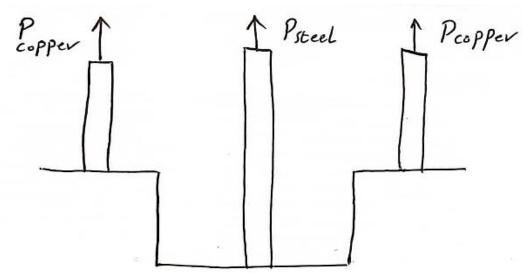


Sol;

$$2P_{copper} + P_{steel} = (M) \quad ---① $$

$$\Delta L_{steel} = \Delta L_{copper}$$

$$\frac{P_{steel} * L_{steel}}{A_{steel} * E_{steel}} = \frac{P_{copper} * L_{copper}}{A_{copper} * E_{copper}}$$



$$\frac{P_{steel} * 240}{1200 * 200 * 10^3} = \frac{P_{copper} * 160}{900 * 120 * 10^3}$$

$$1 * \cancel{10^{-6}} P_{steel} = 1,481 * \cancel{10^{-6}} P_{copper}$$

$$\therefore P_{steel} = 1,481 \, P_{copper}$$

sub in ①

$$2P_{copper} + 1,481 \, P_{copper} = (M) \quad ---②$$

$$2 * \left(\frac{P_{steel}}{1,481}\right) + P_{steel} = (M) \quad ---③$$

$$\sigma_{copper} = \frac{P_{copper}}{A_{copper}}$$

$$\therefore P_{copper} = \sigma_{copper} * A = 70 * 900 = 63000 \, N = 63 \, KN$$

$$\sigma_{steel} = \frac{P_{steel}}{A_{steel}}$$

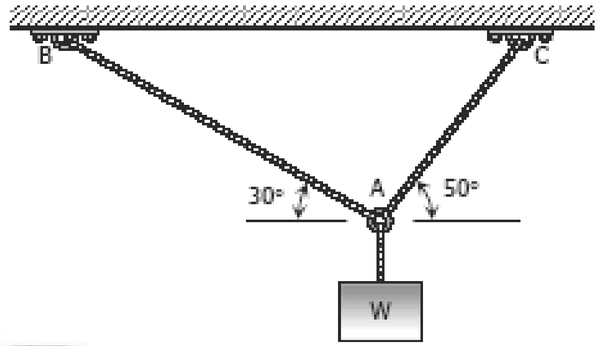
$$\therefore P_{steel} = \sigma_{steel} * A_{steel} = 1200 * 140 = 168000 \, N = 168 \, KN$$

$$\therefore M_1 = 2 * 63 + 1,481 * 63 = 219,303 \, KN$$

$$M_2 = 2 * \left(\frac{168}{1,481}\right) + 168 = 394,8 \, KN$$

$$\boxed{\therefore M = 219,303 \, KN}$$

Ex; Determine the largest weight **W** that can be supported by two wires shown in Fig. The stress in either wire is not to exceed 30 MPa. The cross-sectional areas of wires AB and AC are 40 mm^2 and 50 mm^2, respectively.



Sol:-

$\Sigma F_x = 0$

$0 = -AB * \cos 30 + Ac * \cos 50$

$0 = -AB * 0,866 + Ac * 0,642$

$AB * 0,866 = Ac * 0,642$

$\boxed{\therefore AB = Ac * 0,741}$ or $\boxed{Ac = AB * 1,349}$

$\Sigma F_y = 0$

$0 = -W + AB * \sin 30 + AC * \sin 50$

$0 = -W + 0,5 * AB + 0,766 * AC$

$0 = -W + 0,5 * (0,741\ Ac) + 0,766\ Ac$

$\boxed{\therefore W = 1,136\ Ac}$ —①

$\boxed{W = 1,533\ AB}$ —②

For AB:-

$\sigma = \frac{P}{A}$

$\therefore 30 = \frac{AB}{40} \Rightarrow AB = 30 * 40 = 1200 N = 1.2 KN$

$\boxed{\therefore W = 1,533 * 1.2 = 1,839 KN}$

For AC:-

$\sigma = \frac{P}{A}$

$\therefore 30 = \frac{AC}{50} \Rightarrow Ac = 30 * 50 = 1500 N = 1,5 KN$

$\boxed{\therefore W = 1.136 * 1,5 = 1.704 KN}$

$\boxed{\therefore W = 1,704 KN}$

Ex; The assembly in Fig. P-242 consists of a light rigid bar AB, pinned at O, that is attached to the steel and aluminum rods. In the position shown, bar AB is horizontal and there is a gap, $\Delta = 5$ mm, between the lower end of the steel rod and its pin support at C. Compute the stress in the aluminum rod when the lower end of the steel rod is attached to its support.