Information Theory and Coding

Forth Stage

MSc. Zahraa Niema Kamal Lecture Seven First Course

1. Channel:

In telecommunications and computer networking, a communication channel or **channel**, refers either to a physical transmission medium such as a wire, or to a logical connection over a multiplexed medium such as a radio channel. A channel is used to convey an information signal, for example a digital bit stream, from one or several senders (or transmitters) to one or several receivers. A channel has a certain capacity for transmitting information, often measured by its bandwidth in Hz or its data rate in bits per second.

2. Symmetric channel:

The symmetric channel has the following condition:

a- Equal number of symbol in X&Y, i.e. P(Y|X) is a square matrix.

b- Any row in P(Y|X) matrix comes from some permutation of other rows.

For example, the following conditional probability of various channel types as shown:

a- $P(Y|X) = \begin{bmatrix} 0.9 & 0.1 \\ 0.1 & 0.9 \end{bmatrix}$ is a BSC, because it is square matrix and 1st row is the permutation of 2nd row.

b- $P(Y|X) = \begin{bmatrix} 0.9 & 0.05 & 0.05 \\ 0.05 & 0.9 & 0.05 \\ 0.05 & 0.05 & 0.9 \end{bmatrix}$ is TSC, because it is square matrix and each row is a permutation of others.

c- $P(Y|X) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \end{bmatrix}$ is a non-symmetric since since it is not square although each row is permutation of others.

d- $P(Y|X) = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.7 & 0.2 \\ 0.1 & 0.1 & 0.8 \end{bmatrix}$ is a non-symmetric although it is square since 2nd row is not permutation of other rows.

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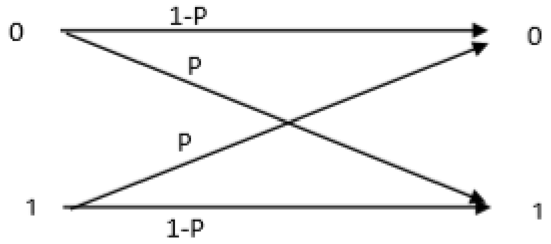
2.1- Binary symmetric channel (BSC): It is a common communications channel model used in coding theory and information theory. In this model, a transmitter wishes to send a bit (a zero or a one), and the receiver receives a bit. It is assumed that the bit is usually transmitted correctly, but that it will be "flipped" with a small probability (the "crossover probability").

$$P(Y/X) = \begin{matrix} & y1 & y2 \\ x1 & 1-p & p \\ x2 & p & 1-p \end{matrix}$$



A binary symmetric channel with crossover probability p denoted by BSCp, is a channel with binary input and binary output and probability of error p; that is, if X is the transmitted random variable and Y the received variable, then the channel is characterized by the conditional probabilities:

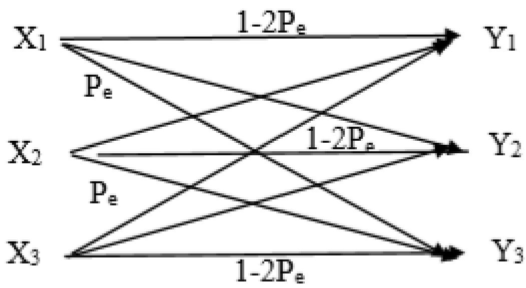
$$\Pr(Y=0|X=0) = 1-P$$
$$\Pr(Y=0|X=1) = P$$
$$\Pr(Y=1|X=0) = P$$
$$\Pr(Y=1|X=1) = 1-P$$

2.2- Ternary symmetric channel (TSC):

The transitional probability of TSC is:

$$P(Y|X) = \begin{matrix} \\ x_1 \\ x_2 \\ x_3 \end{matrix} \begin{bmatrix} y_1 & y_2 & y_3 \\ 1-2P_e & P_e & P_e \\ P_e & 1-2P_e & P_e \\ P_e & P_e & 1-2P_e \end{bmatrix}$$

The TSC is symmetric but not very practical since practically $x1$ and $x3$ are not affected so much as $x2$. In fact the interference between $x1$ and $x3$ is much less than the interference between $x1$ and $x2$ or $x2$ and $x3$.



Shannon's theorem:

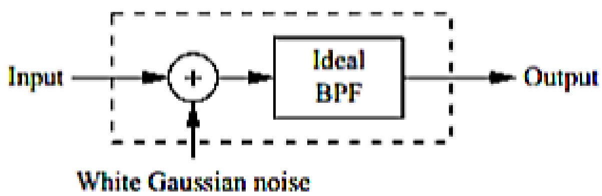
a- A given communication system has a maximum rate of information C known as the channel capacity.

b- If the information rate R is less than C, then one can approach arbitrarily small error probabilities by using intelligent coding techniques.

c- To get lower error probabilities, the encoder has to work on longer blocks of signal data. This entails longer delays and higher computational requirements.

Thus, if R ≤ C then transmission may be accomplished without error in the presence of noise. The negation of this theorem is also true: if R > C, then errors cannot be avoided regardless of the coding technique used.

Consider a bandlimited Gaussian channel operating in the presence of additive Gaussian noise:



The Shannon-Hartley theorem states that the channel capacity is given by:

$$C = B\log_2(1 + \frac{S}{N})$$

Where C is the capacity in bits per second, B is the bandwidth of the channel in Hertz, and S/N is the signal-to-noise ratio.

Example: If the channel capacity is 1500 bps, find its bandwidth if the SNR is 3

Solution:

$$C = Blog_2(1 + SNR) \rightarrow 1500 = Blog_2 4 \rightarrow 1500 = B * 2$$

$$\therefore B = 750\ Hz$$

Example: find the capacity of a channel if the bandwidth is 1MHz and the power of signal is 21 W and the power of noise is 3 W.

Solution:

$$C = B\ log_2\left(1 + \frac{S}{N}\right) \rightarrow C = 1M\ log_2(1 + \frac{21}{3}) \rightarrow C = 1Mlog_2 8 \rightarrow C = 3M\ bps$$