Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: 3rd
Subject: Digital Signal Processing
Lecturer: Dr. Rami Qays Malik
Lecture: 4- Convolution

« Convolution »

the relationship between the input x(n) to a Linear Shift invariant system (LSI) h(n) to give output Response y(n)

$$y(n) = x(n) * h(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k)$$

Notes:-

* The above equation is Linear convolution of x(n) & h(n), this Linear convolution gives y(n). (total Response).
* this equation give the response of Linear shift (or time) invariant system (LSI or LTI) to an input x(n) & h(n).
* The behavior of the LSI system is completely characterized by unit sample response h(n).

ex // convolve the following two sequences to get y(n)?

$$x(n) = \{1, 1, \underset{\uparrow}{0}, 1, 1\}, \quad h(n) = \{1, -2, -3, \underset{\uparrow}{4}\}$$

sol // these two sequences can also be written as:-

$x(-2) = 1$	$h(-3) = 1$
$x(-1) = 1$	$h(-2) = -2$
$x(0) = 0 \leftarrow$	$h(-1) = -3$
$x(1) = 1$	$h(0) = 4 \leftarrow$
$x(2) = 1$	

* Lowest index of x(n) $\Rightarrow n_{xL} = -2$
  Highes index of x(n) $\Rightarrow n_{xH} = 2$

and Lowest index of h(n) $\Rightarrow n_{hL} = -3$
  Highest index of h(n) $\Rightarrow n_{hH} = 0$

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{n_{xL}=-2}^{n_{xH}=2} x(k)\, h(n-k)$$

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$$y(n) = x(-2)h(n+2) + x(-1)h(n+1) + x(0)h(n) + x(1)h(n-1) + x(2)h(n-2)$$

* Range of "n"

$$(nx_L + nh_L) \leq n \leq (nx_H + nh_H)$$

$$(-2-3) \leq n \leq (2+0)$$

$$-5 \leq n \leq 2$$

$n = -5$

$$y(-5) = x(-2)h(-5+2) + x(-1)h(-5+1) + x(0)h(-5) + x(1)h(-5-1) + x(2)h(-5-2) = 1$$

$n = -4$

$$y(-4) = x(-2)h(-2) + x(-1)h(-3) + x(0)h(-4) + x(1)h(-5) + x(2)h(-6) = -2+1 = -1$$

$n = -3 \Rightarrow y(-3) = -5$

$n = -2 \Rightarrow y(-2) = 2$

$n = -1 \Rightarrow y(-1) = 3$

$n = 0 \Rightarrow y(0) = -5$

$n = 1 \Rightarrow y(1) = 1$

$n = 2 \Rightarrow y(2) = 4$

$$\therefore y(n) = \{-5, 2, 3, \underset{\uparrow}{-5}, 1, 4\}$$

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Ex// Convolve the following two sequence x(n) & h(n) to get y(n)

$$x(n) = \begin{cases} 1 & \text{for } 3 \geq n \geq 0 \\ 0 & \text{elsewhere} \end{cases} \quad ; \quad h(n) = \begin{cases} 2 & , 1 \geq n \geq 0 \\ 0 & \text{elsewhere} \end{cases}$$

Sol// $x(n) = \{1, 1, 1, 1\}$; $h(n) = \{2, 2\}$

$x(0) = 1 \leftarrow n_{xL}$ $\qquad h(0) = 2 \leftarrow n_{hL}$

$x(1) = 1$ $\qquad h(1) = 2 \leftarrow n_{hH}$

$x(2) = 1$

$x(3) = 1 \leftarrow n_{xH}$

$$y(n) = \sum_{K=-\infty}^{\infty} x(K)\, h(n-K) = \sum_{k=n_{xL}=0}^{n_{xH}=3} x(K)\, h(n-K)$$

$y(n) = x(0)\,h(n) + x(1)\,h(n-1) + x(2)\,h(n-2) + x(3)\,h(n-3)$

Range of "n" $(n_{xH} + n_{hH}) n \geq (n_{xL} + n_{hL})$

$(3+1) \geq n \geq (0+0)$

$4 \geq n \geq 0$

$n = N_1 + N_2 - 1 = 4 + 2 - 1 = 5$

$n = 0$

$y(0) = x(0)\,h(0) + x(1)\,h(-1) + x(2)\,h(-2) + x(3)\,h(-3) = 2$

$n = 1$

$y(1) = x(0)\,h(1) + x(1)\,h(0) + x(2)\,h(-1) + x(3)\,h(-2) = 4$

$n = 2 \Rightarrow y(2) = 4$

$n = 3 \Rightarrow y(3) = 4$

⋮

$n = 4 \Rightarrow$

$y(4) = x(0)\,h(4) + x(1)\,h(3) + x(2)\,h(2) + x(3)\,h(1) = 2$

$\therefore y(n) = \{2, 4, 4, 4, 2\}$.

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Ex// Determine the convolution of the following two sequence by using Direct Method ?

$x(n) = \{1, 2, 3, 1\}$, $h(n) = \{2, 0, 2\}$.

Sol:

$$y(n) = \sum_{k=-\infty}^{\infty} x(k)\, h(n-k) = \sum_{k=0+0=0}^{n=3+2=5} x(k)\, h(n-k)$$

$$y(n) = x(0)h(n) + x(1)h(n-1) + x(2)h(n-2) + x(3)h(n-3) + x(4)h(n-4) + x(5)h(n-5)$$

for $n = 0$ $\quad 1*2 + 1*0 \quad 1*0$

$y(0) = x(0)h(0) + x(1)h(0-1) + x(2)h(0-2) + \dots$

$y(0) = 2$

for $n = 1$ $\quad 1*0 + 2*2 \quad 3*0$

$y(1) = x(0)h(1) + x(1)h(1-1) + x(2)h(1-2) + \dots$

$y(1) = 4$

for $n = 2$ $\quad 1*2 + 2*0 \quad 3*2 \quad 1*0$

$y(2) = x(0)h(2) + x(1)h(2-1) + x(2)h(2-2) + x(3)h(2-3) + \dots$

$y(2) = 8$

for $n = 3$ $\quad 1*0 + 2*2 + 3*0 \quad 1*2$

$y(3) = x(0)h(3) + x(1)h(3-1) + x(2)h(3-2) + x(3)h(3-3) + x(4)h(3-4) + \dots$

$y(3) = 6$

for $n = 4$ $\quad 1*0 \quad 2*0 \quad 3*2 \quad 1*0$

$y(4) = x(0)h(4) + x(1)h(4-1) + x(2)h(4-2) + x(3)h(4-3) + \dots$

$y(4) = 6$

for $n = 5$

$y(5) = 2$

$$\therefore y(n) = \{2, 4, 8, 6, 6, 2\}$$

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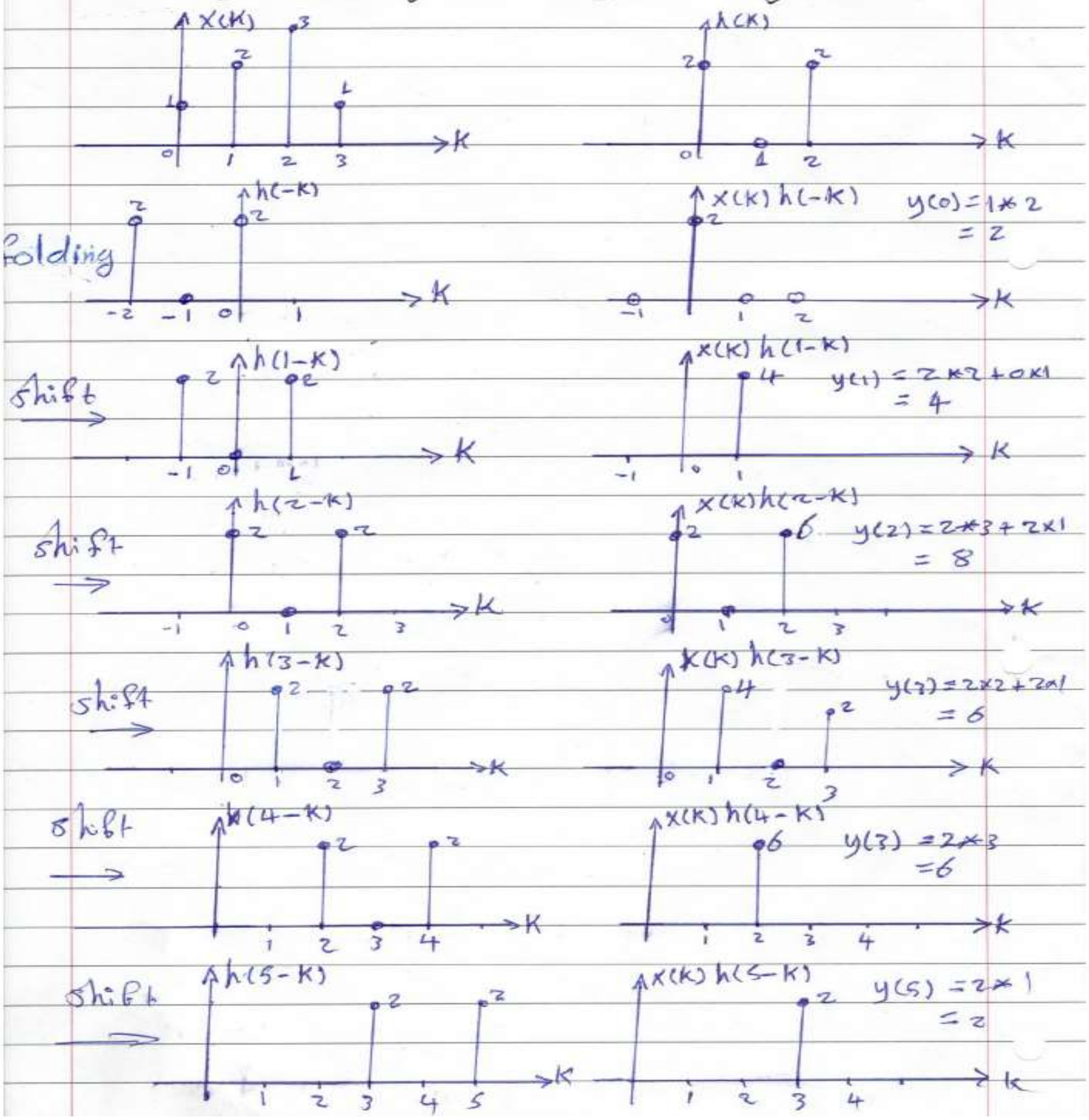
Ex:: Determine the convolution of the following two sequence by using Graphical method.

$x(n) = \{1, 2, 3, 1\}$, $h(n) = \{2, 0, 2\}$.



$\therefore y(n) = \{2, 4, 8, 6, 6, 2\}$