Al-Mustaqbal University College

Department of Medical Instrumentation Techniques Engineering

Subject : **Mathematics (I)**

Class : First

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Lecture One : Limit and Continuity

Limit and Continuity:

When f (x) close to the number **L** as x close to the number **a**, we write

$f(x) \to L \ \ as \ \ x \to a \qquad means: \ \lim_{x \to a} f(x) = L$

Example 1: Let $f(x) = 2x + 5$ $evaluate\ f(x)\ at\ x = 1$

Sol:

$$\lim_{x \to a} f(x) = \lim_{x \to 1} (2x + 5) = 2 * 1 + 5 = 7$$

The Limit Laws:

If L, M, C, and k are real numbers and $\lim_{x \to c} f(x) L$ *and* $\lim_{x \to c} g(x) = M$

1. Sum Rule: $\lim_{x \to c} (f(x) + g(x)) = L + M$

2. Difference Rule: $\lim_{x \to c} (f(x) - g(x)) = L - M$

3. Constant Multiple Rule: $\lim_{x \to c} k.f(x) = k.L$

4. Product Rule: $\lim_{x \to c} (f(x).g(x)) = L.M$

5. Quotient Rule: $\lim_{x \to c} \frac{f(x)}{g(x)} = \frac{L}{M}$, $M \neq 0$

6. Power Rule: $\lim_{x\to c}[f(x)]^n = L^n$

7. Root Rule: $\lim_{x\to c}\sqrt[n]{f(x)} = \sqrt[n]{L}$, n is a positive integer.

Indeterminate quantities

$\frac{0}{0}$,	$\frac{\infty}{\infty}$,	$0*\infty$,	$\infty - \infty$,	∞^0 ,	0^0

Example 2: Evaluate the following limits

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = \frac{0}{0} = \infty, \textit{then the solution} \;, x \neq 1$$

Sol:

$$\lim_{x\to 1}\frac{x^3-1}{x-1} = \lim_{x\to 1}\frac{(x-1)(x^2+x+1)}{x-1} = \lim_{x\to 1} x^2+x+1 = 1^2+1+1 = 3$$

Example 3: $\lim_{h\to 0}\frac{1}{h}(\frac{1}{x+h}-\frac{1}{x}), h \neq 0$

Sol:

$$\lim_{h\to 0}[\frac{1}{h}(\frac{1}{x+h}-\frac{1}{x})] = \lim_{h\to 0}[\frac{1}{h}(\frac{x-x-h}{x(x+h)})] = \lim_{h\to 0}[\frac{1}{h}(\frac{-h}{x(x+h)})]$$

$$\lim_{h\to 0}\frac{-1}{x(x+h)} = -\frac{1}{x(x+0)} = -\frac{1}{x^2}$$

Example 4 : $\lim_{x\to-2}\sqrt{4x^2-3} = \sqrt{\lim_{x\to-2}(4x^2-3)} = \sqrt{4*(-2)^2-3} =$

$\sqrt{16-3} = \sqrt{13}$ → Root Rule

Example5: Evaluate the following limits if they exist.

1) $\lim_{x\to-1}\frac{\sqrt{2+x}-1}{x+1} \quad ,x \neq -1,\ x \neq -2$

Sol:

$$\lim_{x\to-1}\frac{\sqrt{2+x}-1}{x+1} * \frac{\sqrt{2+x}+1}{\sqrt{2+x}+1} = \lim_{x\to-1}\frac{2+x-1}{x+1(\sqrt{2+x}+1)}$$

$$\lim_{x\to-1}\frac{x+1}{x+1(\sqrt{2+x}+1)} = \lim_{x\to-1}\frac{1}{(\sqrt{2+x}+1)} = \frac{1}{(\sqrt{2-1}+1)} = \frac{1}{2}$$

Limits of trigonometric functions.

$\lim_{x\to 0} \sin x = 0$	$\lim_{x\to 0} \cos x = 1$	$\lim_{x\to 0} \tan x = 0$
$\lim_{x\to 0} \frac{\sin x}{x} = 1$	$\lim_{x\to 0} \frac{\sin ax}{ax} = 1$	$\lim_{x\to 0} \frac{\tan x}{x} = 1$
$\lim_{x\to 0} \frac{\tan ax}{ax} = 1$	$\lim_{x\to 0} \frac{1-\cos x}{x} = 0$	

Examples 6

1. $$\lim_{x\to 0} \frac{\sin 2x}{5x} = \frac{1}{5} \lim_{x\to 0} \frac{\sin 2x}{x} = \frac{1}{5} \lim_{x\to 0} \frac{\sin 2x}{x} * \frac{2}{2}$$

$$= \frac{2}{5} \lim_{x\to 0} \frac{\sin 2x}{2x} = \frac{2}{5} * 1 = \frac{2}{5}$$

2. $$\lim_{x\to 0} \frac{\sin 2x}{\sin 3x} = \lim_{x\to 0} \frac{\sin 2x \, \frac{2x}{2x}}{\sin 3x \, \frac{3x}{3x}} = \lim_{x\to 0} \frac{2x \, \frac{\sin 2x}{2x}}{3x \, \frac{\sin 3x}{3x}}$$

$$= \frac{2}{3} \lim_{x\to 0} \frac{\frac{\sin 2x}{2x}}{\frac{\sin 3x}{3x}} = \frac{2}{3} \frac{\lim_{x\to 0} \frac{\sin 2x}{2x}}{\lim_{x\to 0} \frac{\sin 3x}{3x}} = \frac{2}{3} * \frac{1}{1} = \frac{2}{3}$$

3. $$\lim_{\theta\to 0} \frac{\theta}{\sin \theta} = \lim_{\theta\to 0} \frac{1}{\frac{\sin \theta}{\theta}} = \frac{1}{\lim_{\theta\to 0} \frac{\sin \theta}{\theta}} = \frac{1}{1} = 1$$

4. $$\lim_{\theta\to 0} \frac{\sin^2 \theta}{\theta^2} = \left(\lim_{\theta\to 0} \frac{\sin \theta}{\theta} \right)^2 = (1)^2 = 1$$

$$\text{or} \quad \lim_{\theta\to 0} \frac{\sin \theta}{\theta} * \lim_{\theta\to 0} \frac{\sin \theta}{\theta} = 1 * 1 = 1$$

Limits of infinity:

1- If k is constant, then $\lim_{x\to\infty} k = k$

2- $\lim_{x\to\infty} \frac{1}{x} = 0, \ \lim_{x\to-\infty} \frac{1}{x} = \frac{1}{\infty} = 0$

3- $\lim_{x\to 0} \frac{1}{x} = \infty$

Examples 7:

1. $\lim_{x\to\infty} \frac{x}{2x+3} = \lim_{x\to\infty} \frac{1}{2+\frac{3}{x}} = \frac{1}{2+0} = \frac{1}{2}$

2. $\lim_{x\to\infty} \frac{2x^2+3x+5}{5x^2-4x+1} = \lim_{x\to\infty} \frac{2+\frac{3}{x}+\frac{5}{x^2}}{5-\frac{4}{x}+\frac{1}{x^2}} = \frac{2+0+0}{5-0+0} = \frac{2}{5}$

3. $\lim_{x\to\infty} \frac{2x^2+1}{3x^2-2x^2+5x-2} = \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{1}{x^3}}{3-\frac{2}{x}+\frac{5}{x^2}-\frac{2}{x^3}} = 0$

4. $\lim_{x\to\infty} \frac{2x^2+1}{3x^2-2x^2+5x-2} = \lim_{x\to\infty} \frac{\frac{2}{x}+\frac{1}{x^3}}{3-\frac{2}{x}+\frac{5}{x^2}-\frac{2}{x^3}} = 0$

H.W. :

1. If $f(x) = \frac{x^2-3x+2}{x-2}$, $x \neq 2$; $find \lim\limits_{x\to 2} f(x)$.

2. $\lim\limits_{x\to -2} \sqrt{4x^2-3} = \sqrt{\lim\limits_{x\to -2}(4x^2-3)}$

3. $\lim\limits_{t\to 0} \frac{\tan t \ \sec 2t}{3t}$

4. $\lim\limits_{x\to \infty} \frac{11x+2}{2x^3-1}$

5. $\lim\limits_{x\to\infty} \frac{2x^2+1}{3x^2-2x^2+5x-2}$