Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class:3td
Subject: Medical Communication
Lecturer: Asst. Lect. Mays Khalid
Lecture: 3

Signal Spectrum using Fourier Transform

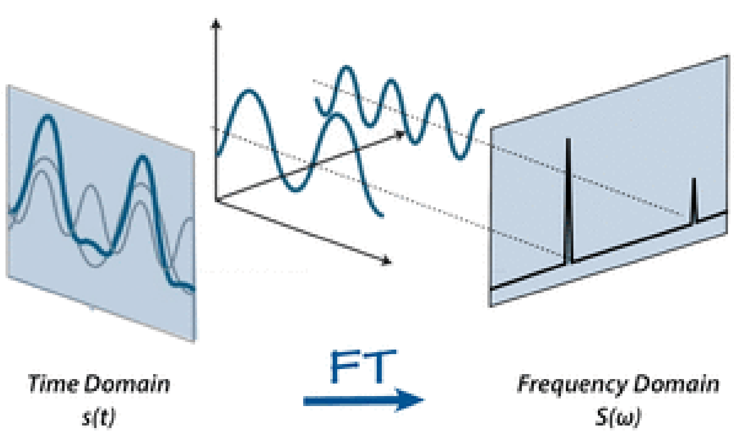
3.1 Fourier Transform

The Fourier transform is used to convert a continuous and non-periodic time-domain signal into the frequency domain. The resulting frequency domain representation from performing the Fourier transform is continuous and non-periodic. The Fourier transform of $x(t)$:

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

The inverse Fourier transform of $X(w)$:

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(w)\, e^{jwt}\, dw$$



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Example 1: Find the Fourier Transform of the rectangular pulse signal $\boldsymbol{x}(\boldsymbol{t})$ defined by.

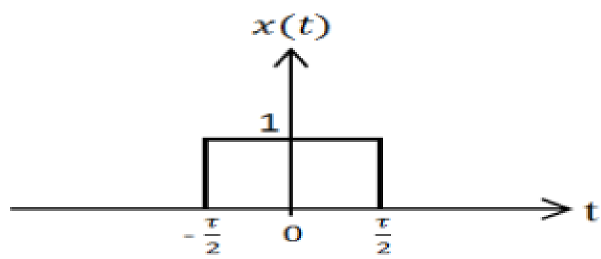
$$x(t) = \begin{cases} 1 & |t| < \frac{\tau}{2} \\ 0 & |t| > \frac{\tau}{2} \end{cases}$$

Solution:

$$X(w) = \int_{-\infty}^{\infty} x(t)\, e^{-jwt}\, dt$$

$$X(w) = \int_{-\tau/2}^{\tau/2} 1\, e^{-jwt}\, dt$$

$$X(w) = \frac{1}{-jw}\left[e^{\frac{-jw\tau}{2}} - e^{\frac{jw\tau}{2}}\right]$$



$$X(w) = \frac{1}{jw}\left[e^{\frac{jw\tau}{2}} - e^{\frac{-jw\tau}{2}}\right]$$

$$X(w) = \frac{2}{w}\left[\frac{e^{\frac{jw\tau}{2}} - e^{\frac{-jw\tau}{2}}}{2j}\right]$$

$$X(w) = \frac{2}{w}\sin\left(\frac{w\tau}{2}\right)$$

$$X(w) = \tau\, \mathrm{sinc}\left(\frac{w\tau}{2}\right)$$

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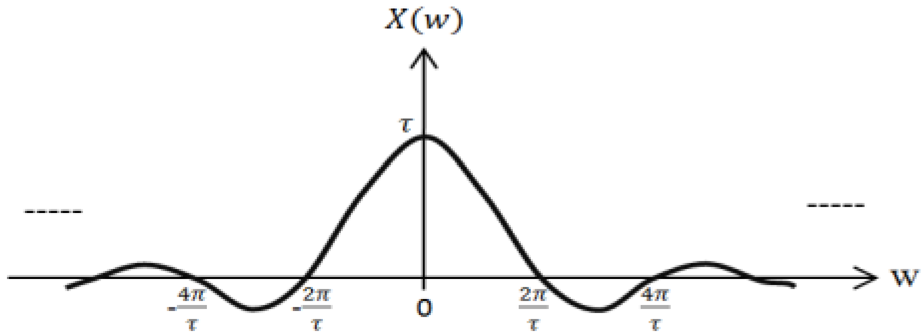


Example 2: Suppose that a signal gets turned on at t = 0 and then decays exponentially, so that

$$f(t) = \begin{cases} e^{-at} & t \geq 0 \\ 0 & t < 0 \end{cases}$$

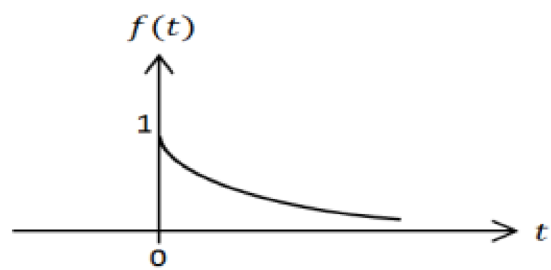
for $a > 0$. Find the Fourier Transform of this signal.

Solution:

$$F(w) = \int_{-\infty}^{\infty} f(t)\, e^{-jwt}\, dt$$

$$F(w) = \int_{0}^{\infty} e^{-at}\, e^{-jwt}\, dt$$

$$F(w) = \int_{0}^{\infty} e^{-t(a+jw)}\, dt$$



$$F(w) = \frac{1}{-(a+jw)}\left[\, e^{-\infty} - e^{0}\, \right]$$

$$F(w) = \frac{1}{(a+jw)}$$

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3.2 Properties of the Continuous-Time Fourier Transform

There are a number of properties of the Fourier transform that may be used to simplify the evaluation of the Fourier transform and its inverse. Some of these properties are described below:

Property	Signal	Fourier Transform
Linearity	$ax(t) + by(t)$	$aX(w) + bY(w)$
Time-Shift	$x(t - t_o)$	$e^{-jwt_o} X(w)$
Time-Reversal	$x(-t)$	$X(-w)$
Frequency Shifting	$e^{jw_o t} x(t)$	$X(w - w_o)$
Time Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{w}{a}\right)$
Convolution	$x(t) * y(t)$	$X(w)\,Y(w)$
Time differentiation	$\frac{dx(t)}{dt}$	$jwX(w)$
Parseval's relation	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} \lvert X(w)\rvert^2\, dw$

3.3 Frequency response

In Lecture1 we showed that the output $\boldsymbol{y(t)}$ of a continuous-time LTI system equals the convolution of the input $\boldsymbol{x(t)}$ with the impulse response $\boldsymbol{h(t)}$; that is

$$\boldsymbol{y(t) = x(t) * h(t)}$$

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Applying the convolution property, we obtain,

$$\boldsymbol{Y}(\boldsymbol{w}) = \boldsymbol{X}(\boldsymbol{w}).\boldsymbol{H}(\boldsymbol{w})$$

Where $\boldsymbol{Y}(\boldsymbol{w})$,$\boldsymbol{X}(\boldsymbol{w})$ and $\boldsymbol{H}(\boldsymbol{w})$ are the Fourier transforms of $y(t)$, $x(t)$ and $h(t)$, respectively. we have

$$\mathbf{H}(\mathbf{w}) = \frac{\mathbf{Y}(\mathbf{w})}{\mathbf{X}(\mathbf{w})}$$

Example 4: Consider the linear time-invariant system characterized by the second-order linear constant coefficient difference equation

$$\boldsymbol{y}'(\boldsymbol{t}) + 2\boldsymbol{y}(\boldsymbol{t}) = \boldsymbol{x}(\boldsymbol{t}) + \boldsymbol{x}'(\boldsymbol{t})$$

By using the Fourier Transform, find the impulse response, $h(t)$.

Solution:

$$jwY(w) + 2Y(w) = X(w) + jwX(w)$$

$$Y(w)[\,jw + 2\,] = X(w)[\,1 + jw\,]$$

$$\frac{Y(w)}{X(w)} = \frac{jw + 1}{jw + 2}$$

$$H(w) = \frac{jw + 2 - 1}{jw + 2}$$

$$H(w) = 1 - \frac{1}{jw + 2}$$

$$h(t) = \delta(t) - e^{-2t}u(t)$$

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1. Find the Fourier Transform of the unit impulse signal $x(t)$ defined by.

$$x(t) = \delta(t)$$

2. Consider a continuous-time LTI system described by

$$\frac{dy(t)}{dt} + 2y(t) = x(t)$$

Using the Fourier Transform, to find the output $y(t)$ if the input signal is:

$$x(t) = e^{-t}\, u(t)$$

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