Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: second stage
Subject: Mathematics II
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Lecture: Lec6

Infinite Series المتسلسلات غير المنتهية

A sequence of the form :

$$a_1 + a_2 + a_3 + a_4 + \cdots + a_n + \cdots$$

is called an infinite Series . The term the nth term of the Series .

The Sequence $\{S_n\}$ defined by :

$$S_1 = a_1$$

$$S_2 = a_1 + a_2 = S_1 + a_2$$

$$S_3 = a_1 + a_2 + a_3 = S_2 + a_2$$

$$\vdots$$

$$S_n = a_1 + a_2 + a_3 + \cdots + a_n = \sum_{k=1}^{r} a_k$$

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is the Sequence of partial sums of the Series and the number S_n being the nth partial Sum.

If the sequence of partial sum S_n converges to a limiting value (L), then the series is **convergent** and its Sum is (L). Hence,

$$a_1+a_2+a_3+a_4+\cdots+a_n+\cdots=\sum_{n=1}^{\infty} a_n = L$$

If S_n of Series does not converge, then the Series **divergent** and there is no sum.

* Any Series can be expressed by:

$$\sum_{n=1}^{\infty} a_n \left(\sum_{k=1}^{\infty} a_k\right) \text{ or Simply } \sum a_n \left(\sum a_k\right)$$

The following examples shows how the "partial fraction technique" can be used to compute the sums for some series. When this sum can be done, then the series is convergent.

Ex) Determine whether the following series is convergent or no? ① $\sum \frac{2}{n(n+1)}$

Sol.) $\sum \frac{2}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} \Rightarrow A(n+1) + Bn = 2 \rightarrow$

if $n=-1 \rightarrow B=-2$
$n=0 \rightarrow A=2 \quad \rightarrow$

$$\sum \frac{2}{n(n+1)} = \sum \left(\frac{2}{n} - \frac{2}{n+1}\right) \rightarrow$$

$$S_n = \underbrace{(2-\cancel{1})}_{n=1} + \underbrace{(\cancel{1}-\cancel{\tfrac{2}{3}})}_{n=2} + \underbrace{(\cancel{\tfrac{2}{3}}-\cancel{\tfrac{2}{4}})}_{n=3} + \cdots + \underbrace{(\cancel{\tfrac{2}{n-1}}-\cancel{\tfrac{2}{n}})}_{n=n-1} + \underbrace{(\cancel{\tfrac{2}{n}}-\tfrac{2}{n+1})}_{n=n}$$

$\therefore S_n = 2 - \frac{2}{n+1}$

$\therefore \lim_{n\to\infty} S_n = \lim_{n\to\infty} \left(2 - \frac{2}{n+1}\right) = 2 \rightarrow \therefore$ the series is convergent

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Ex) Discuss the convergence of this series : $\sum \frac{4}{(4n-3)(4n+1)}$?

Sol.) $\frac{4}{(4n-3)(4n+1)} = \frac{A}{4n-3} + \frac{B}{4n+1} \rightarrow A(4n+1) + B(4n-3) = 4$

if $n = 3/4 \rightarrow A = 1$ and if $n = -1/4 \rightarrow B = -1 \rightarrow$

$\therefore \sum \frac{4}{(4n-3)(4n+1)} = \sum \left(\frac{1}{4n-3} - \frac{1}{4n+1} \right)$

$$S_n = \underbrace{\left(1 - \cancel{\tfrac{1}{5}}\right)}_{n=1} + \underbrace{\left(\cancel{\tfrac{1}{5}} - \cancel{\tfrac{1}{9}}\right)}_{n=2} + \underbrace{\left(\cancel{\tfrac{1}{9}} - \cancel{\tfrac{1}{13}}\right)}_{n=3} \cancel{+} \cdots \cancel{+} \left(\cancel{\tfrac{1}{4n-3}} - \tfrac{1}{4n+1}\right)$$

$$= 1 - \frac{1}{4n+1} \longrightarrow \lim_{n \to \infty} S_n = \lim_{n \to \infty} \left(1 - \frac{1}{4n+1}\right) = 1 \rightarrow$$

$\therefore$ the series is convergent