2. Work

Work is defined as a force acting through a displacement, where the displacement is in the direction of the force. If a system exists in which a force at the boundary of the system is moved through a distance, then work is done by or on the system. Gas contained in a cylinder by a piston can do work on that piston provided its initial pressure is greater than that of the surroundings.

1. Reversible Work

Consider an ideal frictionless fluid contained in a cylinder behind a piston. Assume that the pressure and temperature of the fluid are uniform and that there is no friction between the piston and cylinder walls. Assume the cross sectional area of the piston is A. Let the piston move a distance dl during the process.

The force exerted by the fluid on the piston = $(P \times A)$

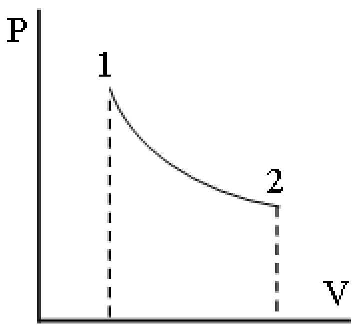
Therefore the work done on the piston by the fluid = $(P \times Adl)$

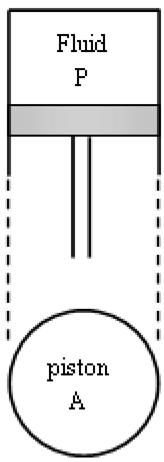
Let $Adl = dV$

Then $dW = P \times dV$

In a reversible process, when the piston moves between two states 1 and 2 i.e. the fluid undergoes a reversible process from 1 to 2:

$$W = \int_1^2 PdV = area\ under\ the\ curve$$

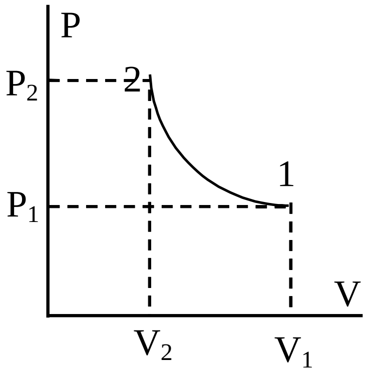




Example (2.1): One kg of a fluid is compressed reversibly according to a law $Pv = 0.25$ where P in bar and v in m^3/kg. The final volume is 1/4 of the initial volume. Calculate the work done on the fluid and sketch the process on a (P-V) diagram.

Solution:

Since we have 1 kg of the fluid, then $V = v$



We know that:

$$V_2 = \frac{1}{4}V_1 \rightarrow \frac{V_2}{V_1} = \frac{1}{4}$$

$$P \times V = 0.25 \rightarrow P = \frac{0.25}{V}$$

$$W = \int_1^2 PdV = \int_1^2 \frac{0.25}{V} dV \times 10^5 = 0.25 \times 10^5 \times \int_1^2 \frac{dV}{V}$$

$$= 0.25 \times 10^5 \times \ln\left(\frac{V_2}{V_1}\right) = 0.25 \times 10^5 \times \ln\left(\frac{1}{4}\right)$$

$\boldsymbol{W = -34657}$ **J** **Ans.**

Example (2.2): Unit mass of a fluid at a pressure of 3 bar and a specific volume of 0.18 m³/kg, is contained in a cylinder behind a piston. The fluid expands reversibly to a pressure of 0.6 bar according to a law $PV^2 = C$. Calculate the work done during the process.

Solution:

$$V_1 = v_1 \times m = 0.18 \times 1 = 0.18 \text{ m}^3$$

$$C = PV^2 = P_1V_1^2 = 3 \times 10^5 \times 0.18^2 = 9720$$

$$PV^2 = C \rightarrow P = \frac{C}{V^2} \rightarrow P = \frac{9720}{V^2}$$

$$\frac{P_1}{P_2} = \left(\frac{V_2}{V_1}\right)^n \rightarrow \frac{3}{0.6} = \left(\frac{V_2}{0.18}\right)^2 \rightarrow V_2 = 0.402 \text{ m}^3$$

$$W = \int_1^2 PdV = 9720 \int_1^2 \frac{dV}{V^2} = 9720 \times \frac{-1}{V} \Big|_1^2 = 9720 \times \left[-\frac{1}{0.402} + \frac{1}{0.18}\right]$$

$\boldsymbol{W = 29820}$ **J** **Ans.**

2. Irreversible Work

It has been stated above that work is given by $\int PdV$ for a reversible process only. It can be easily shown that $\int PdV$ is not equal to the work done if the process is irreversible. For example, consider a cylinder divided into a number of compartments by sliding partitions. Initially, compartment A is filled with a mass of fluid at pressure P_1.When the sliding partition 1 is removed quickly, the fluid fills the compartments A and B. When the system settles to a new equilibrium state the pressure and volume are fixed and the state can be marked on the (P-V) diagram as shown below. Sliding partition 2 is removed and the fluid expands to occupy compartments A, B and C. Again the equilibrium state can be marked on the diagram. The same procedure can be adopted with partitions 3 and 4 until finally the fluid is at P_2 and occupies a volume V_2 when filling compartments A, B, C, D and E. The area under the curve 1-2 is given by $\int_1^2 PdV$, but no work has been done. No piston has moved, no turbine wheel has been revolved; in other word, no external force has been moved through a distance. This is the extreme case of an irreversible process in which $\int_1^2 PdV$ has a value and yet the work done is zero.