Dr. ALI KAMIL

1st CLASS

LECTURE NO.1,2

Force Systems

2

CHAPTER OUTLINE

2/1 Introduction

In this and the following chapters, we study the effects of forces which act on engineering structures and mechanisms. The experience gained here will help you in the study of mechanics and in other subjects such as stress analysis, design of structures and machines, and fluid flow. This chapter lays the foundation for a basic understanding not only of statics but also of the entire subject of mechanics, and you should master this material thoroughly.

2/2 Force

Before dealing with a group or *system* of forces, it is necessary to examine the properties of a single force in some detail. A force has been defined in Chapter 1 as an action of one body on another. In dynamics we will see that a force is defined as an action which tends to cause acceleration of a body. A force is a *vector quantity*, because its effect depends on the direction as well as on the magnitude of the action. Thus, forces may be combined according to the parallelogram law of vector addition.

The action of the cable tension on the bracket in Fig. 2/1a is represented in the side view, Fig. 2/1b, by the force vector $\mathbf{P}$ of magnitude P. The effect of this action on the bracket depends on P, the angle θ, and the location of the point of application A. Changing any one of these three specifications will alter the effect on the bracket, such as the force

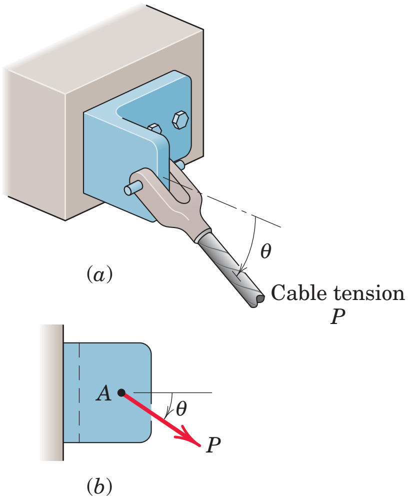


Figure 2/1

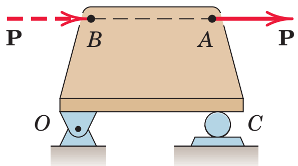


Figure 2/2

The forces associated with this lifting rig must be carefully identified, classified, and analyzed in order to provide a safe and effective working environment.

in one of the bolts which secure the bracket to the base, or the internal force and deformation in the material of the bracket at any point. Thus, the complete specification of the action of a force must include its *magnitude*, *direction*, and *point of application*, and therefore we must treat it as a fixed vector.

External and Internal Effects

We can separate the action of a force on a body into two effects, *external* and *internal*. For the bracket of Fig. 2/1 the effects of **P** external to the bracket are the reactive forces (not shown) exerted on the bracket by the foundation and bolts because of the action of **P**. Forces external to a body can be either *applied* forces or *reactive* forces. The effects of **P** internal to the bracket are the resulting internal forces and deformations distributed throughout the material of the bracket. The relation between internal forces and internal deformations depends on the material properties of the body and is studied in strength of materials, elasticity, and plasticity.

Principle of Transmissibility

When dealing with the mechanics of a rigid body, we ignore deformations in the body and concern ourselves with only the net external effects of external forces. In such cases, experience shows us that it is not necessary to restrict the action of an applied force to a given point. For example, the force **P** acting on the rigid plate in Fig. 2/2 may be applied at A or at B or at any other point on its line of action, and the net external effects of **P** on the bracket will not change. The external effects are the force exerted on the plate by the bearing support at O and the force exerted on the plate by the roller support at C.

This conclusion is summarized by the *principle of transmissibility*, which states that a force may be applied at any point on its given line of action without altering the resultant effects of the force *external* to the *rigid* body on which it acts. Thus, whenever we are interested in only the resultant external effects of a force, the force may be treated as a *sliding* vector, and we need specify only the *magnitude*, *direction*, and *line of action* of the force, and not its *point of application*. Because this book deals essentially with the mechanics of rigid bodies, we will treat almost all forces as sliding vectors for the rigid body on which they act.

Force Classification

Forces are classified as either *contact* or *body* forces. A contact force is produced by direct physical contact; an example is the force exerted on a body by a supporting surface. On the other hand, a body force is generated by virtue of the position of a body within a force field such as a gravitational, electric, or magnetic field. An example of a body force is your weight.

Forces may be further classified as either *concentrated* or *distributed*. Every contact force is actually applied over a finite area and is therefore really a distributed force. However, when the dimensions of the area are very small compared with the other dimensions of the body, we may consider the force to be concentrated at a point with negligible loss of accuracy. Force can be distributed over an *area*, as in the case of mechanical contact, over a *volume* when a body force such as weight is acting, or over a *line*, as in the case of the weight of a suspended cable.

The *weight* of a body is the force of gravitational attraction distributed over its volume and may be taken as a concentrated force acting through the center of gravity. The position of the center of gravity is frequently obvious if the body is symmetric. If the position is not obvious, then a separate calculation, explained in Chapter 5, will be necessary to locate the center of gravity.

We can measure a force either by comparison with other known forces, using a mechanical balance, or by the calibrated movement of an elastic element. All such comparisons or calibrations have as their basis a primary standard. The standard unit of force in SI units is the newton (N) and in the U.S. customary system is the pound (lb), as defined in Art. 1/5.

Action and Reaction

According to Newton's third law, the *action* of a force is always accompanied by an *equal* and *opposite reaction*. It is essential to distinguish between the action and the reaction in a pair of forces. To do so, we first *isolate* the body in question and then identify the force exerted *on* that body (not the force exerted *by* the body). It is very easy to mistakenly use the wrong force of the pair unless we distinguish carefully between action and reaction.

Concurrent Forces

Two or more forces are said to be *concurrent at a point* if their lines of action intersect at that point. The forces $\mathbf{F}_1$ and $\mathbf{F}_2$ shown in Fig. 2/3*a* have a common point of application and are concurrent at the point *A*. Thus, they can be added using the parallelogram law in their common plane to obtain their sum or *resultant* $\mathbf{R}$, as shown in Fig. 2/3*a*. The resultant lies in the same plane as $\mathbf{F}_1$ and $\mathbf{F}_2$.

Suppose the two concurrent forces lie in the same plane but are applied at two different points as in Fig. 2/3*b*. By the principle of transmissibility, we may move them along their lines of action and complete their vector sum $\mathbf{R}$ at the point of concurrency *A*, as shown in Fig. 2/3*b*. We can replace $\mathbf{F}_1$ and $\mathbf{F}_2$ with the resultant $\mathbf{R}$ without altering the external effects on the body upon which they act.

We can also use the triangle law to obtain $\mathbf{R}$, but we need to move the line of action of one of the forces, as shown in Fig. 2/3*c*. If we add the same two forces as shown in Fig. 2/3*d*, we correctly preserve the magnitude and direction of $\mathbf{R}$, but we lose the correct line of action, because $\mathbf{R}$ obtained in this way does not pass through *A*. Therefore this type of combination should be avoided.

We can express the sum of the two forces mathematically by the vector equation

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2$$

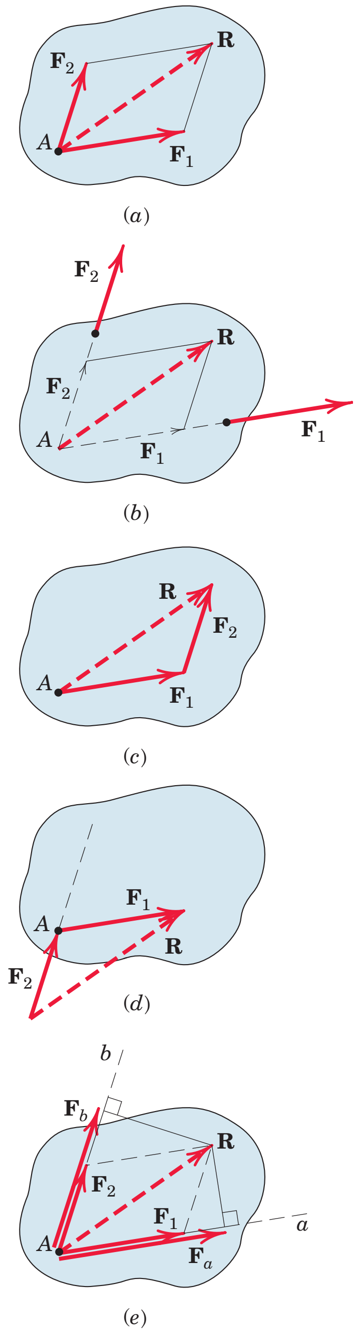


Figure 2/3

Vector Components

In addition to combining forces to obtain their resultant, we often need to replace a force by its *vector components* in directions which are convenient for a given application. The vector sum of the components must equal the original vector. Thus, the force **R** in Fig. 2/3*a* may be replaced by, or *resolved* into, two vector components $\mathbf{F}_1$ and $\mathbf{F}_2$ with the specified directions by completing the parallelogram as shown to obtain the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$.

The relationship between a force and its vector components along given axes must not be confused with the relationship between a force and its perpendicular* projections onto the same axes. Figure 2/3*e* shows the perpendicular projections $\mathbf{F}_a$ and $\mathbf{F}_b$ of the given force **R** onto axes *a* and *b*, which are parallel to the vector components $\mathbf{F}_1$ and $\mathbf{F}_2$ of Fig. 2/3*a*. Figure 2/3*e* shows that the components of a vector are not necessarily equal to the projections of the vector onto the same axes. Furthermore, the vector sum of the projections $\mathbf{F}_a$ and $\mathbf{F}_b$ is not the vector **R**, because the parallelogram law of vector addition must be used to form the sum. The components and projections of **R** are equal only when the axes *a* and *b* are perpendicular.

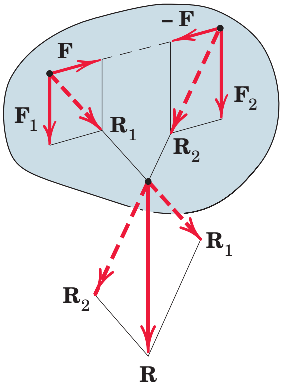


Figure 2/4

A Special Case of Vector Addition

To obtain the resultant when the two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are parallel as in Fig. 2/4, we use a special case of addition. The two vectors are combined by first adding two equal, opposite, and collinear forces **F** and $-\mathbf{F}$ of convenient magnitude, which taken together produce no external effect on the body. Adding $\mathbf{F}_1$ and **F** to produce $\mathbf{R}_1$, and combining with the sum $\mathbf{R}_2$ of $\mathbf{F}_2$ and $-\mathbf{F}$ yield the resultant **R**, which is correct in magnitude, direction, and line of action. This procedure is also useful for graphically combining two forces which have a remote and inconvenient point of concurrency because they are almost parallel.

It is usually helpful to master the analysis of force systems in two dimensions before undertaking three-dimensional analysis. Thus the remainder of Chapter 2 is subdivided into these two categories.

SECTION A TWO-DIMENSIONAL FORCE SYSTEMS

2/3 Rectangular Components

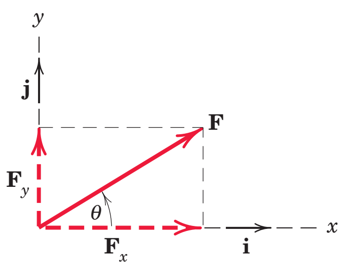


Figure 2/5

The most common two-dimensional resolution of a force vector is into rectangular components. It follows from the parallelogram rule that the vector **F** of Fig. 2/5 may be written as

$$\mathbf{F} = \mathbf{F}_x + \mathbf{F}_y \tag{2/1}$$

where $\mathbf{F}_x$ and $\mathbf{F}_y$ are *vector components* of **F** in the *x*- and *y*-directions. Each of the two vector components may be written as a scalar times the

*Perpendicular projections are also called *orthogonal* projections.

appropriate unit vector. In terms of the unit vectors **i** and **j** of Fig. 2/5, $\mathbf{F}_x = F_x\mathbf{i}$ and $\mathbf{F}_y = F_y\mathbf{j}$, and thus we may write

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j} \tag{2/2}$$

where the scalars F_x and F_y are the x and y *scalar components* of the vector **F**.

The scalar components can be positive or negative, depending on the quadrant into which **F** points. For the force vector of Fig. 2/5, the x and y scalar components are both positive and are related to the magnitude and direction of **F** by

$$\begin{aligned} F_x &= F\cos\theta & F &= \sqrt{F_x^{\,2} + F_y^{\,2}} \\ F_y &= F\sin\theta & \theta &= \tan^{-1}\frac{F_y}{F_x} \end{aligned} \tag{2/3}$$

Conventions for Describing Vector Components

We express the magnitude of a vector with lightface italic type in print; that is, $|\mathbf{F}|$ is indicated by F, a quantity which is always *nonnegative*. However, the scalar components, also denoted by lightface italic type, will include sign information. See Sample Problems 2/1 and 2/3 for numerical examples which involve both positive and negative scalar components.

When both a force and its vector components appear in a diagram, it is desirable to show the vector components of the force with dashed lines, as in Fig. 2/5, and show the force with a solid line, or vice versa. With either of these conventions it will always be clear that a force and its components are being represented, and not three separate forces, as would be implied by three solid-line vectors.

Actual problems do not come with reference axes, so their assignment is a matter of arbitrary convenience, and the choice is frequently up to the student. The logical choice is usually indicated by the way in which the geometry of the problem is specified. When the principal dimensions of a body are given in the horizontal and vertical directions, for example, you would typically assign reference axes in these directions.

Determining the Components of a Force

Dimensions are not always given in horizontal and vertical directions, angles need not be measured counterclockwise from the x-axis, and the origin of coordinates need not be on the line of action of a force. Therefore, it is essential that we be able to determine the correct components of a force no matter how the axes are oriented or how the angles are measured. Figure 2/6 suggests a few typical examples of vector resolution in two dimensions.

Memorization of Eqs. 2/3 is not a substitute for understanding the parallelogram law and for correctly projecting a vector onto a reference axis. A neatly drawn sketch always helps to clarify the geometry and avoid error.

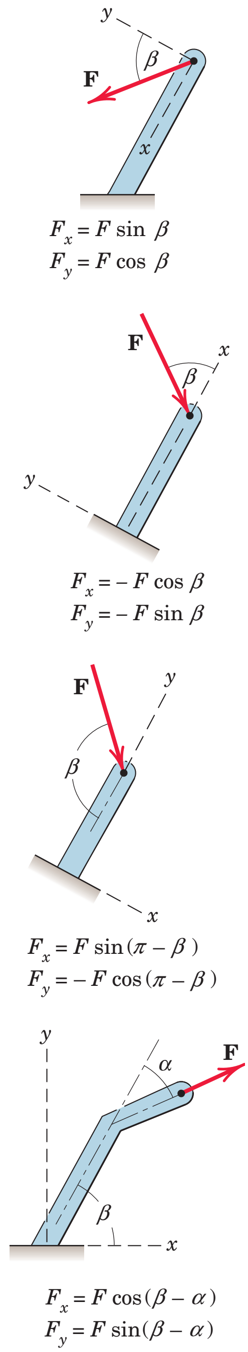


Figure 2/6

Rectangular components are convenient for finding the sum or resultant $\mathbf{R}$ of two forces which are concurrent. Consider two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ which are originally concurrent at a point O. Figure 2/7 shows the line of action of $\mathbf{F}_2$ shifted from O to the tip of $\mathbf{F}_1$ according to the triangle rule of Fig. 2/3. In adding the force vectors $\mathbf{F}_1$ and $\mathbf{F}_2$, we may write

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 = (F_{1_x}\mathbf{i} + F_{1_y}\mathbf{j}) + (F_{2_x}\mathbf{i} + F_{2_y}\mathbf{j})$$

or

$$R_x\mathbf{i} + R_y\mathbf{j} = (F_{1_x} + F_{2_x})\mathbf{i} + (F_{1_y} + F_{2_y})\,\mathbf{j}$$

from which we conclude that

$$\begin{aligned} R_x &= F_{1_x} + F_{2_x} = \Sigma F_x \\ R_y &= F_{1_y} + F_{2_y} = \Sigma F_y \end{aligned} \qquad \textbf{(2/4)}$$

The term ΣF_x means "the algebraic sum of the x scalar components". For the example shown in Fig. 2/7, note that the scalar component F_{2_y} would be negative.

The structural elements in the foreground transmit concentrated forces to the brackets at both ends.

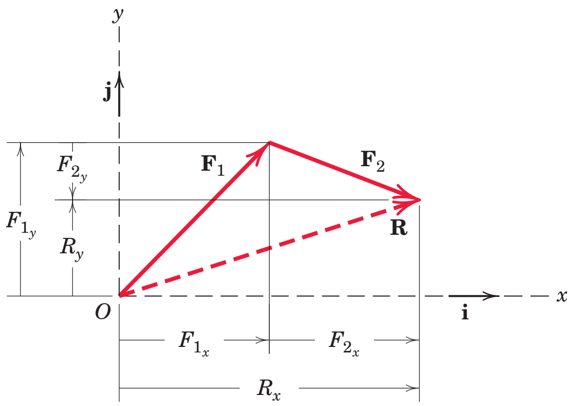
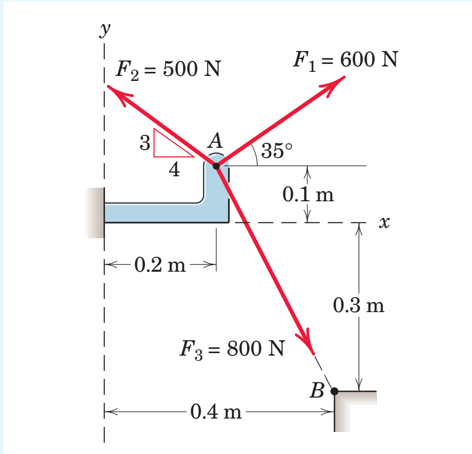


Figure 2/7

SAMPLE PROBLEM 2/1

The forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$, all of which act on point A of the bracket, are specified in three different ways. Determine the x and y scalar components of each of the three forces.



Solution. The scalar components of $\mathbf{F}_1$, from Fig. a, are

$$F_{1_x} = 600 \cos 35° = 491 \text{ N} \qquad \textit{Ans.}$$

$$F_{1_y} = 600 \sin 35° = 344 \text{ N} \qquad \textit{Ans.}$$

The scalar components of $\mathbf{F}_2$, from Fig. b, are

$$F_{2_x} = -500(\tfrac{4}{5}) = -400 \text{ N} \qquad \textit{Ans.}$$

$$F_{2_y} = 500(\tfrac{3}{5}) = 300 \text{ N} \qquad \textit{Ans.}$$

Note that the angle which orients $\mathbf{F}_2$ to the x-axis is never calculated. The cosine and sine of the angle are available by inspection of the 3-4-5 triangle. Also note that the x scalar component of $\mathbf{F}_2$ is negative by inspection.

The scalar components of $\mathbf{F}_3$ can be obtained by first computing the angle α of Fig. c.

$$\alpha = \tan^{-1}\left[\frac{0.2}{0.4}\right] = 26.6°$$

1 Then,

$$F_{3_x} = F_3 \sin \alpha = 800 \sin 26.6° = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -F_3 \cos \alpha = -800 \cos 26.6° = -716 \text{ N} \qquad \textit{Ans.}$$

Alternatively, the scalar components of $\mathbf{F}_3$ can be obtained by writing $\mathbf{F}_3$ as a magnitude times a unit vector $\mathbf{n}_{AB}$ in the direction of the line segment AB. Thus,

2

$$\mathbf{F}_3 = F_3\mathbf{n}_{AB} = F_3\frac{\overrightarrow{AB}}{\overline{AB}} = 800\left[\frac{0.2\mathbf{i} - 0.4\mathbf{j}}{\sqrt{(0.2)^2 + (-0.4)^2}}\right]$$

$$= 800\,[0.447\mathbf{i} - 0.894\mathbf{j}]$$

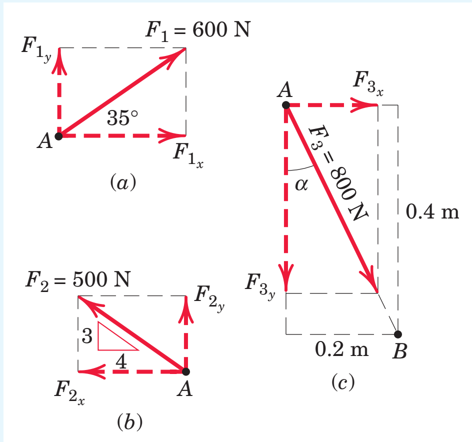
$$= 358\mathbf{i} - 716\mathbf{j} \text{ N}$$

The required scalar components are then

$$F_{3_x} = 358 \text{ N} \qquad \textit{Ans.}$$

$$F_{3_y} = -716 \text{ N} \qquad \textit{Ans.}$$

which agree with our previous results.



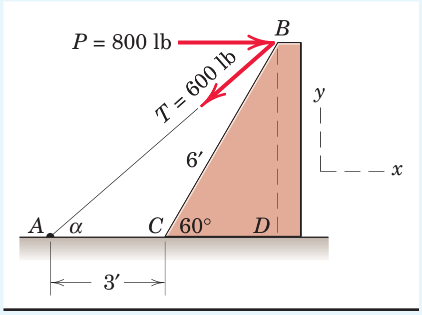
Helpful Hints

1 You should carefully examine the geometry of each component determination problem and not rely on the blind use of such formulas as $F_x = F \cos\theta$ and $F_y = F \sin\theta$.

2 A unit vector can be formed by dividing *any* vector, such as the geometric position vector $\overrightarrow{AB}$, by its length or magnitude. Here we use the overarrow to denote the vector which runs from A to B and the overbar to determine the distance between A and B.

SAMPLE PROBLEM 2/2

Combine the two forces **P** and **T**, which act on the fixed structure at B, into a single equivalent force **R**.

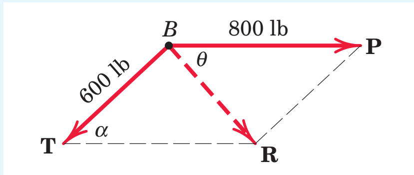


Graphical solution. The parallelogram for the vector addition of forces **T** and **P** is constructed as shown in Fig. a. The scale used here is 1 in. = 800 lb; a scale of 1 in. = 200 lb would be more suitable for regular-size paper and would give greater accuracy. Note that the angle α must be determined prior to construction of the parallelogram. From the given figure

$$\tan\alpha = \frac{\overline{BD}}{\overline{AD}} = \frac{6\sin 60°}{3 + 6\cos 60°} = 0.866 \qquad \alpha = 40.9°$$

Measurement of the length R and direction θ of the resultant force **R** yields the approximate results

$$R = 525 \text{ lb} \qquad \theta = 49° \qquad \textit{Ans.}$$



(a)

Helpful Hints

1 Note the repositioning of **P** to permit parallelogram addition at B.

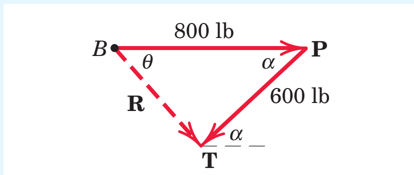
Geometric solution. The triangle for the vector addition of **T** and **P** is shown in Fig. b. The angle α is calculated as above. The law of cosines gives

$$R^2 = (600)^2 + (800)^2 - 2(600)(800)\cos 40.9° = 274{,}300$$

$$R = 524 \text{ lb} \qquad \textit{Ans.}$$

From the law of sines, we may determine the angle θ which orients **R**. Thus,

$$\frac{600}{\sin\theta} = \frac{524}{\sin 40.9°} \qquad \sin\theta = 0.750 \qquad \theta = 48.6° \qquad \textit{Ans.}$$



(b)

2 Note the repositioning of **F** so as to preserve the correct line of action of the resultant **R**.

Algebraic solution. By using the x-y coordinate system on the given figure, we may write

$$R_x = \Sigma F_x = 800 - 600\cos 40.9° = 346 \text{ lb}$$

$$R_y = \Sigma F_y = -600\sin 40.9° = -393 \text{ lb}$$

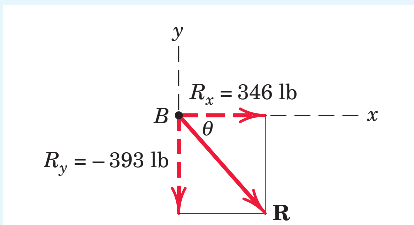
The magnitude and direction of the resultant force **R** as shown in Fig. c are then

$$R = \sqrt{R_x{}^2 + R_y{}^2} = \sqrt{(346)^2 + (-393)^2} = 524 \text{ lb} \qquad \textit{Ans.}$$

$$\theta = \tan^{-1}\frac{|R_y|}{|R_x|} = \tan^{-1}\frac{393}{346} = 48.6° \qquad \textit{Ans.}$$

The resultant **R** may also be written in vector notation as

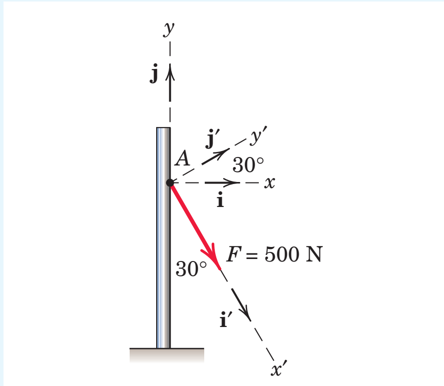
$$\mathbf{R} = R_x\mathbf{i} + R_y\mathbf{j} = 346\mathbf{i} - 393\mathbf{j} \text{ lb} \qquad \textit{Ans.}$$

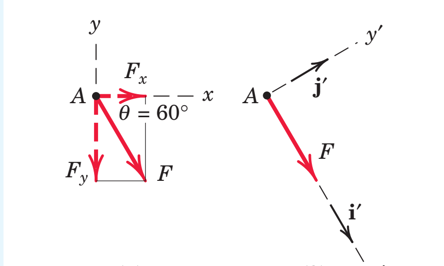


(c)

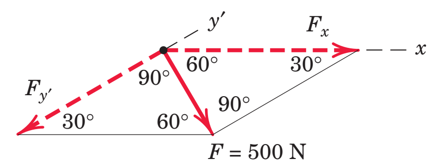
SAMPLE PROBLEM 2/3

The 500-N force **F** is applied to the vertical pole as shown. (1) Write **F** in terms of the unit vectors **i** and **j** and identify both its vector and scalar components. (2) Determine the scalar components of the force vector **F** along the x'- and y'-axes. (3) Determine the scalar components of **F** along the x- and y'-axes.





(*a*) (*b*)



(*c*)

Solution. ***Part (1).*** From Fig. *a* we may write **F** as

$$\mathbf{F} = (F\cos\theta)\mathbf{i} - (F\sin\theta)\mathbf{j}$$

$$= (500\cos 60°)\mathbf{i} - (500\sin 60°)\mathbf{j}$$

$$= (250\mathbf{i} - 433\mathbf{j})\text{ N} \qquad \textit{Ans.}$$

The scalar components are $F_x = 250$ N and $F_y = -433$ N. The vector components are $\mathbf{F}_x = 250\mathbf{i}$ N and $\mathbf{F}_y = -433\mathbf{j}$ N.

Part (2). From Fig. *b* we may write **F** as $\mathbf{F} = 500\mathbf{i}'$ N, so that the required scalar components are

$$F_{x'} = 500\text{ N} \qquad F_{y'} = 0 \qquad \textit{Ans.}$$

Part (3). The components of **F** in the x- and y'-directions are nonrectangular and are obtained by completing the parallelogram as shown in Fig. *c*. The magnitudes of the components may be calculated by the law of sines. Thus,

1 $$\frac{|F_x|}{\sin 90°} = \frac{500}{\sin 30°} \qquad |F_x| = 1000\text{ N}$$

$$\frac{|F_{y'}|}{\sin 60°} = \frac{500}{\sin 30°} \qquad |F_{y'}| = 866\text{ N}$$

The required scalar components are then

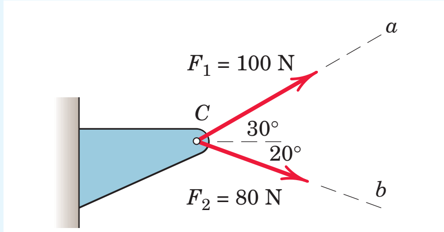
$$F_x = 1000\text{ N} \qquad F_{y'} = -866\text{ N} \qquad \textit{Ans.}$$

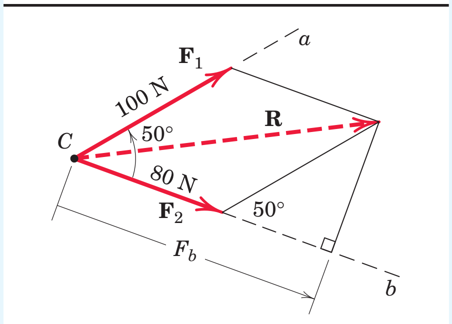
Helpful Hint

1 Obtain F_x and $F_{y'}$ graphically and compare your results with the calculated values.

SAMPLE PROBLEM 2/4

Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the bracket as shown. Determine the projection F_b of their resultant **R** onto the b-axis.





Solution. The parallelogram addition of $\mathbf{F}_1$ and $\mathbf{F}_2$ is shown in the figure. Using the law of cosines gives us

$$R^2 = (80)^2 + (100)^2 - 2(80)(100)\cos 130° \qquad R = 163.4\text{ N}$$

The figure also shows the orthogonal projection $\mathbf{F}_b$ of **R** onto the b-axis. Its length is

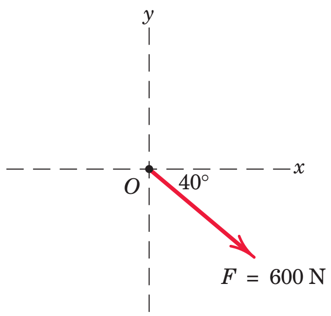
$$F_b = 80 + 100\cos 50° = 144.3\text{ N} \qquad \textit{Ans.}$$

Note that the components of a vector are in general not equal to the projections of the vector onto the same axes. If the a-axis had been perpendicular to the b-axis, then the projections and components of **R** would have been equal.

PROBLEMS

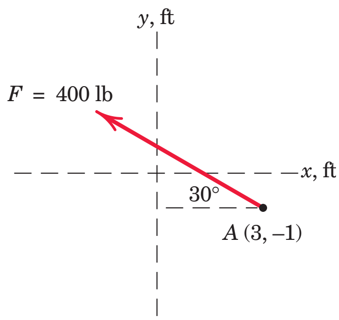
Introductory Problems

2/1 The force **F** has a magnitude of 600 N. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify the x and y scalar components of **F**.



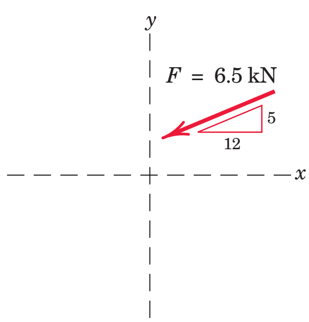
Problem 2/1

2/2 The magnitude of the force **F** is 400 lb. Express **F** as a vector in terms of the unit vectors **i** and **j**. Identify both the scalar and vector components of **F**.



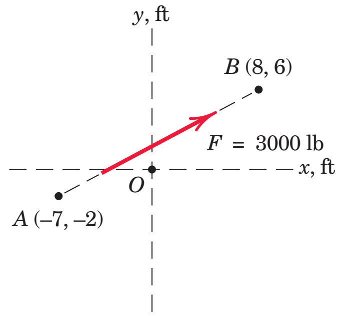
Problem 2/2

2/3 The slope of the 6.5-kN force **F** is specified as shown in the figure. Express **F** as a vector in terms of the unit vectors **i** and **j**.



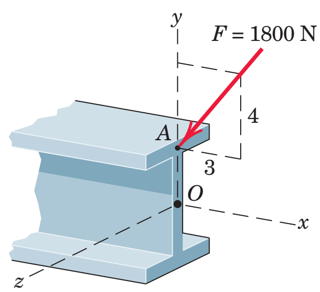
Problem 2/3

2/4 The line of action of the 3000-lb force runs through the points A and B as shown in the figure. Determine the x and y scalar components of **F**.



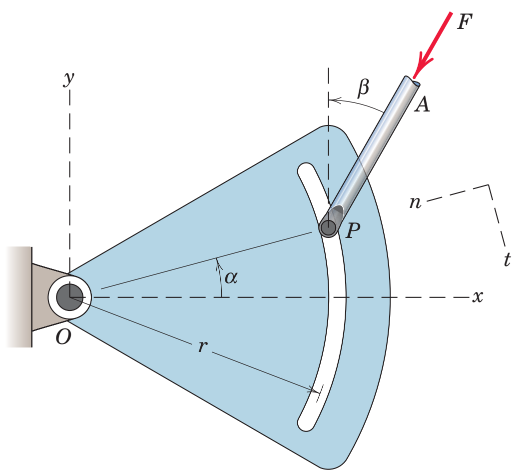
Problem 2/4

2/5 The 1800-N force **F** is applied to the end of the I-beam. Express **F** as a vector using the unit vectors **i** and **j**.



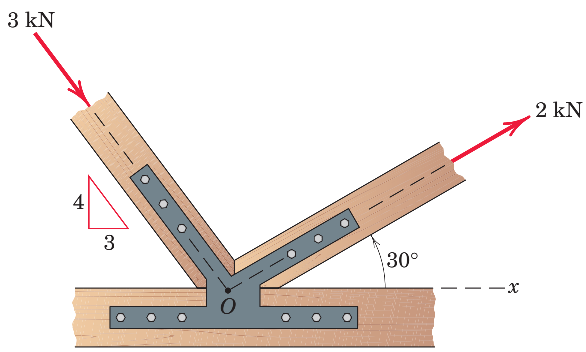
Problem 2/5

2/6 The control rod AP exerts a force **F** on the sector as shown. Determine both the x-y and the n-t components of the force.



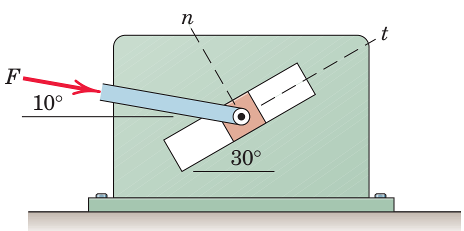
Problem 2/6

2/7 The two structural members, one of which is in tension and the other in compression, exert the indicated forces on joint O. Determine the magnitude of the resultant $\mathbf{R}$ of the two forces and the angle θ which $\mathbf{R}$ makes with the positive x-axis.



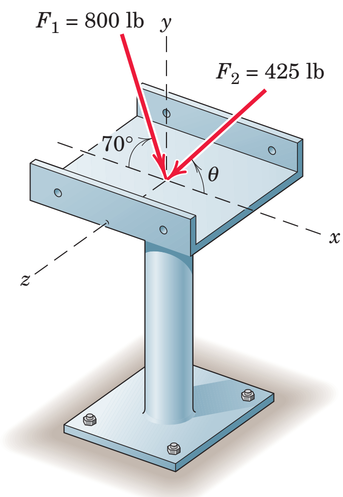
Problem 2/7

2/8 The t-component of the force $\mathbf{F}$ is known to be 75 N. Determine the n-component and the magnitude of $\mathbf{F}$.



Problem 2/8

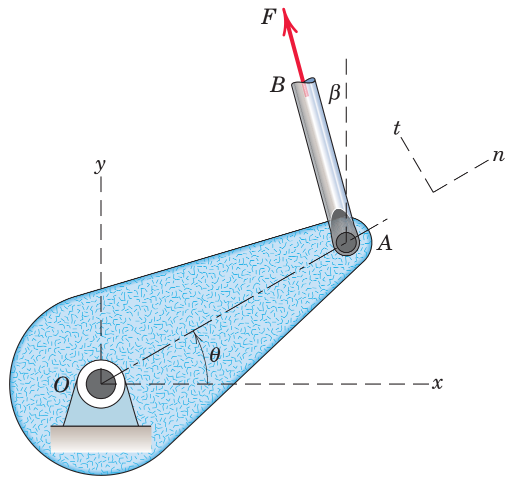
2/9 Two forces are applied to the construction bracket as shown. Determine the angle θ which makes the resultant of the two forces vertical. Determine the magnitude R of the resultant.



Problem 2/9

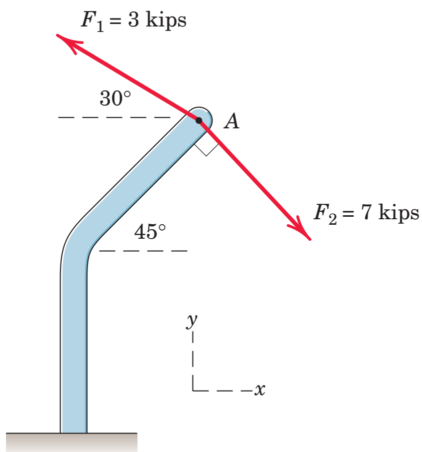
Representative Problems

2/10 Determine the n- and t-components of the force $\mathbf{F}$ which is exerted by the rod AB on the crank OA. Evaluate your general expression for $F = 100$ N and (*a*) $\theta = 30°$, $\beta = 10°$ and (*b*) $\theta = 15°$, $\beta = 25°$.



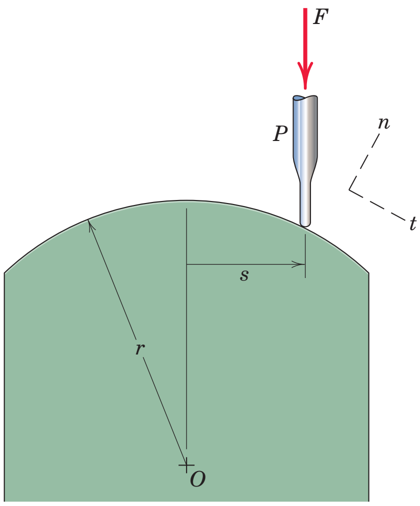
Problem 2/10

2/11 The two forces shown act at point A of the bent bar. Determine the resultant $\mathbf{R}$ of the two forces.



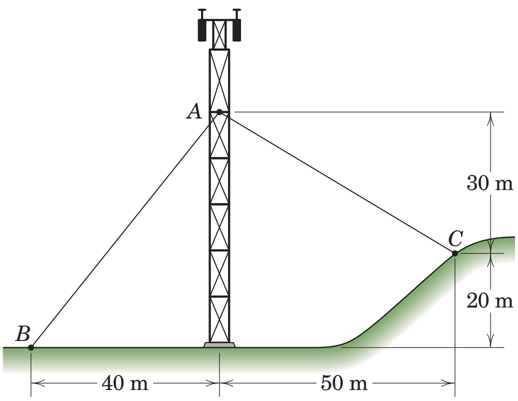
Problem 2/11

2/12 A small probe P is gently forced against the circular surface with a vertical force $\mathbf{F}$ as shown. Determine the n- and t-components of this force as functions of the horizontal position s.



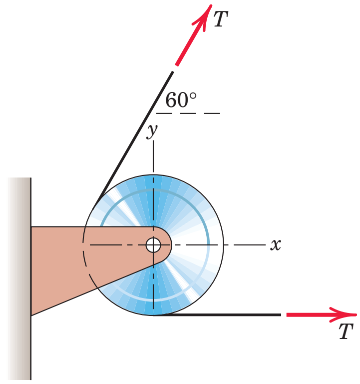
Problem 2/12

2/13 The guy cables AB and AC are attached to the top of the transmission tower. The tension in cable AB is 8 kN. Determine the required tension T in cable AC such that the net effect of the two cable tensions is a downward force at point A. Determine the magnitude R of this downward force.



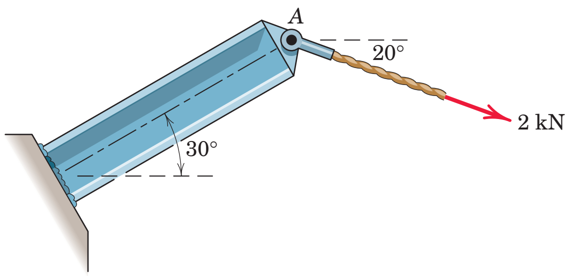
Problem 2/13

2/14 If the equal tensions T in the pulley cable are 400 N, express in vector notation the force $\mathbf{R}$ exerted on the pulley by the two tensions. Determine the magnitude of $\mathbf{R}$.



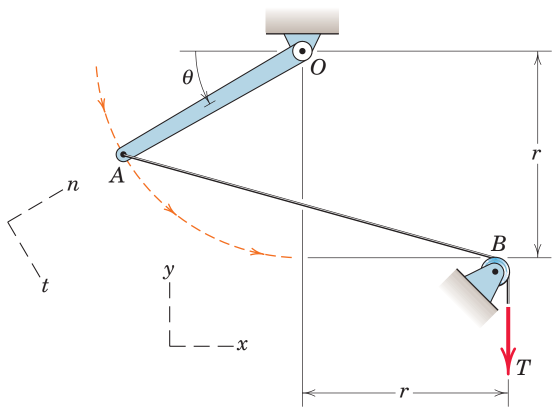
Problem 2/14

2/15 To satisfy design limitations it is necessary to determine the effect of the 2-kN tension in the cable on the shear, tension, and bending of the fixed I-beam. For this purpose replace this force by its equivalent of two forces at A, F_t parallel and F_n perpendicular to the beam. Determine F_t and F_n.



Problem 2/15

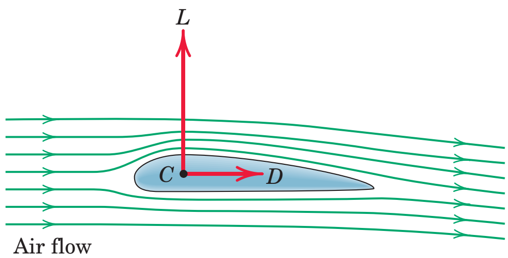
2/16 Determine the x- and y-components of the tension T which is applied to point A of the bar OA. Neglect the effects of the small pulley at B. Assume that r and θ are known.



Problem 2/16

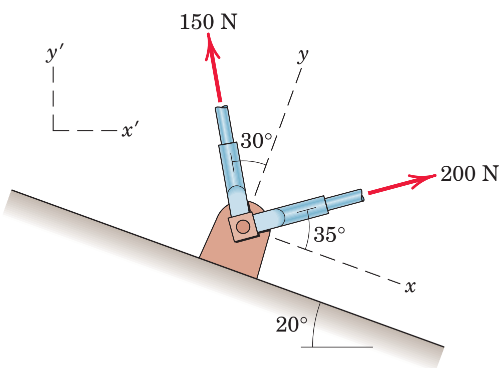
2/17 Refer to the mechanism of the previous problem. Develop general expressions for the n- and t-components of the tension T applied to point A. Then evaluate your expressions for $T = 100$ N and $\theta = 35°$.

2/18 The ratio of the lift force L to the drag force D for the simple airfoil is $L/D = 10$. If the lift force on a short section of the airfoil is 50 lb, compute the magnitude of the resultant force $\mathbf{R}$ and the angle θ which it makes with the horizontal.



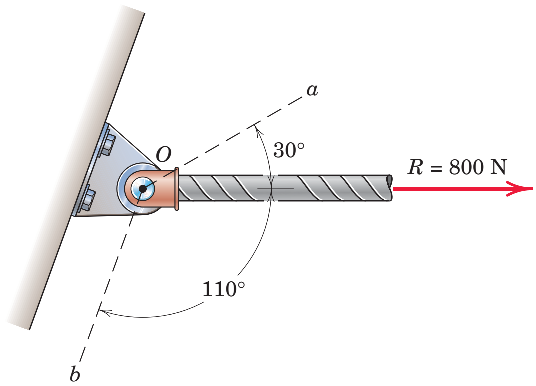
Problem 2/18

2/19 Determine the resultant $\mathbf{R}$ of the two forces applied to the bracket. Write $\mathbf{R}$ in terms of unit vectors along the x- and y-axes shown.



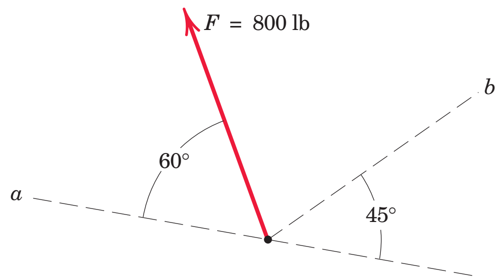
Problem 2/19

2/20 Determine the scalar components R_a and R_b of the force $\mathbf{R}$ along the nonrectangular axes a and b. Also determine the orthogonal projection P_a of $\mathbf{R}$ onto axis a.



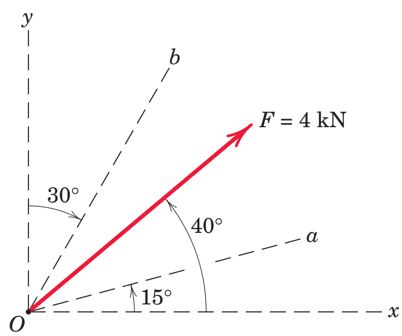
Problem 2/20

2/21 Determine the components of the 800-lb force $\mathbf{F}$ along the oblique axes a and b. Also, determine the projections of $\mathbf{F}$ onto the a- and b-axes.



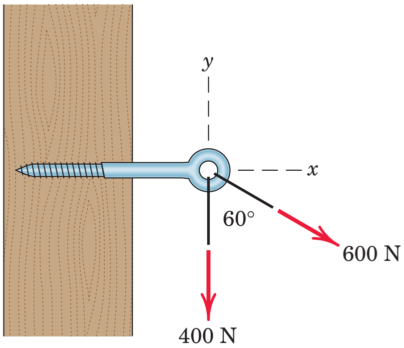
Problem 2/21

2/22 Determine the components F_a and F_b of the 4-kN force along the oblique axes a and b. Determine the projections P_a and P_b of $\mathbf{F}$ onto the a- and b-axes.



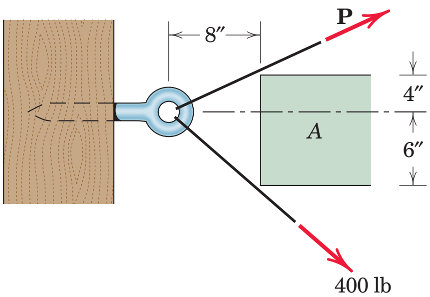
Problem 2/22

2/23 Determine the resultant **R** of the two forces shown by (*a*) applying the parallelogram rule for vector addition and (*b*) summing scalar components.



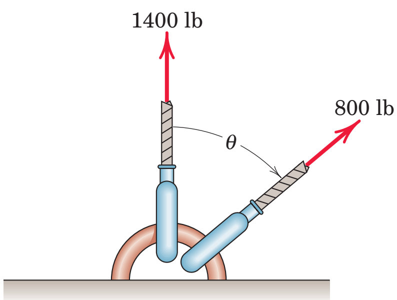
Problem 2/23

2/24 It is desired to remove the spike from the timber by applying force along its horizontal axis. An obstruction *A* prevents direct access, so that two forces, one 400 lb and the other **P**, are applied by cables as shown. Compute the magnitude of **P** necessary to ensure a resultant **T** directed along the spike. Also find *T*.



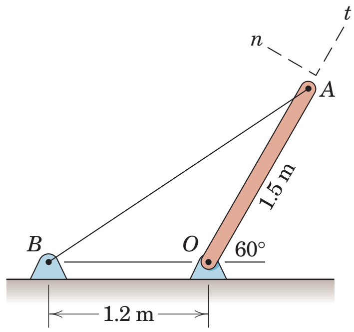
Problem 2/24

2/25 At what angle θ must the 800-lb force be applied in order that the resultant **R** of the two forces have a magnitude of 2000 lb? For this condition, determine the angle β between **R** and the vertical.



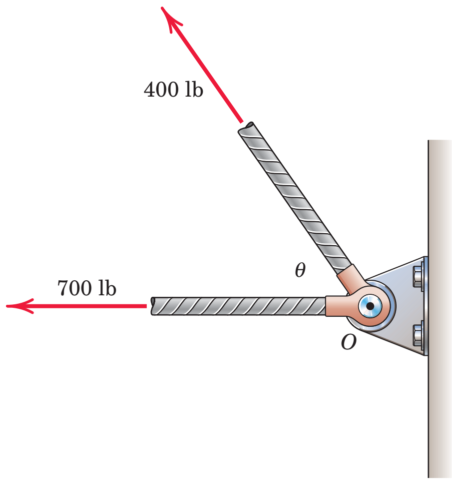
Problem 2/25

2/26 The cable *AB* prevents bar *OA* from rotating clockwise about the pivot *O*. If the cable tension is 750 N, determine the *n*- and *t*-components of this force acting on point *A* of the bar.



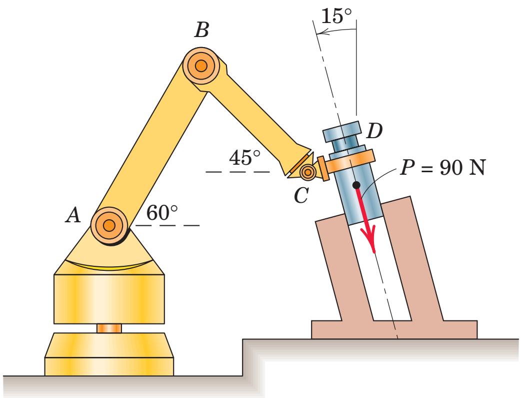
Problem 2/26

2/27 At what angle θ must the 400-lb force be applied in order that the resultant **R** of the two forces have a magnitude of 1000 lb? For this condition what will be the angle β between **R** and the horizontal?



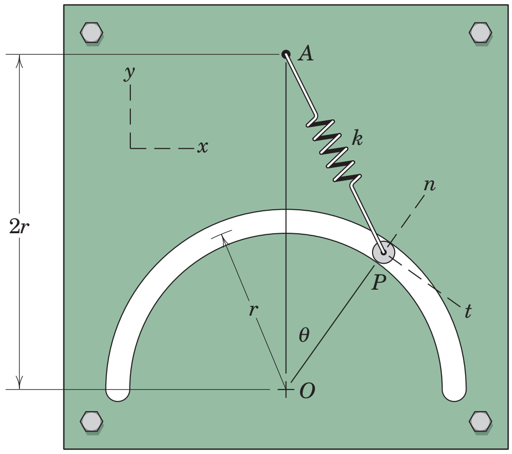
Problem 2/27

2/28 In the design of the robot to insert the small cylindrical part into a close-fitting circular hole, the robot arm must exert a 90-N force P on the part parallel to the axis of the hole as shown. Determine the components of the force which the part exerts *on* the robot along axes (a) parallel and perpendicular to the arm AB, and (b) parallel and perpendicular to the arm BC.



Problem 2/28

▶2/29 The unstretched length of the spring is r. When pin P is in an arbitrary position θ, determine the x- and y-components of the force which the spring exerts on the pin. Evaluate your general expressions for $r = 400$ mm, $k = 1.4$ kN/m, and $\theta = 40°$. (*Note*: The force in a spring is given by $F = k\delta$, where δ is the extension from the unstretched length.)



Problem 2/29

▶2/30 Refer to the figure and statement of Prob. 2/29. When pin P is in the position $\theta = 20°$, determine the n- and t-components of the force F which the spring of modulus $k = 1.4$ kN/m exerts on the pin. The distance $r = 400$ mm.

2/4 Moment

In addition to the tendency to move a body in the direction of its application, a force can also tend to rotate a body about an axis. The axis may be any line which neither intersects nor is parallel to the line of action of the force. This rotational tendency is known as the *moment* **M** of the force. Moment is also referred to as *torque*.

As a familiar example of the concept of moment, consider the pipe wrench of Fig. 2/8*a*. One effect of the force applied perpendicular to the handle of the wrench is the tendency to rotate the pipe about its vertical axis. The magnitude of this tendency depends on both the magnitude F of the force and the effective length d of the wrench handle. Common experience shows that a pull which is not perpendicular to the wrench handle is less effective than the right-angle pull shown.

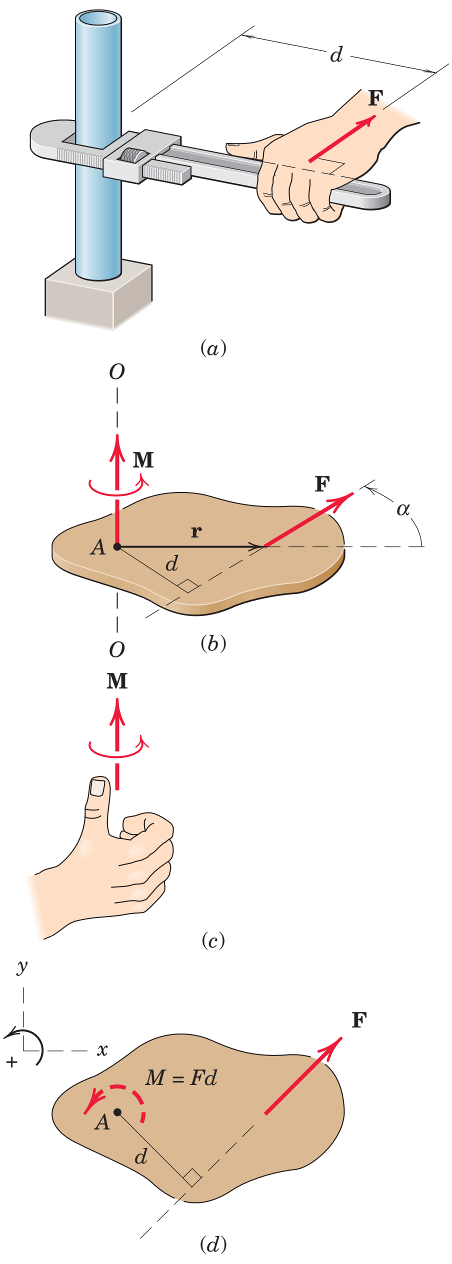


Figure 2/8

Moment about a Point

Figure 2/8*b* shows a two-dimensional body acted on by a force **F** in its plane. The magnitude of the moment or tendency of the force to rotate the body about the axis *O-O* perpendicular to the plane of the body is proportional both to the magnitude of the force and to the *moment arm* d, which is the perpendicular distance from the axis to the line of action of the force. Therefore, the magnitude of the moment is defined as

$$M = Fd \tag{2/5}$$

The moment is a vector **M** perpendicular to the plane of the body. The sense of **M** depends on the direction in which **F** tends to rotate the body. The right-hand rule, Fig. 2/8*c*, is used to identify this sense. We represent the moment of **F** about *O-O* as a vector pointing in the direction of the thumb, with the fingers curled in the direction of the rotational tendency.

The moment **M** obeys all the rules of vector combination and may be considered a sliding vector with a line of action coinciding with the moment axis. The basic units of moment in SI units are newton-meters (N·m), and in the U.S. customary system are pound-feet (lb-ft).

When dealing with forces which all act in a given plane, we customarily speak of the moment *about a point*. By this we mean the moment with respect to an axis normal to the plane and passing through the point. Thus, the moment of force **F** about point A in Fig. 2/8*d* has the magnitude $M = Fd$ and is counterclockwise.

Moment directions may be accounted for by using a stated sign convention, such as a plus sign (+) for counterclockwise moments and a minus sign (−) for clockwise moments, or vice versa. Sign consistency within a given problem is essential. For the sign convention of Fig. 2/8*d*, the moment of **F** about point A (or about the z-axis passing through point A) is positive. The curved arrow of the figure is a convenient way to represent moments in two-dimensional analysis.

The Cross Product

In some two-dimensional and many of the three-dimensional problems to follow, it is convenient to use a vector approach for moment calculations. The moment of **F** about point A of Fig. 2/8*b* may be represented by the cross-product expression

$$\mathbf{M} = \mathbf{r} \times \mathbf{F} \tag{2/6}$$

where **r** is a position vector which runs from the moment reference point A to *any* point on the line of action of **F**. The magnitude of this expression is given by*

$$M = Fr \sin \alpha = Fd \tag{2/7}$$

which agrees with the moment magnitude as given by Eq. 2/5. Note that the moment arm $d = r \sin \alpha$ does not depend on the particular point on the line of action of **F** to which the vector **r** is directed. We establish the direction and sense of **M** by applying the right-hand rule to the sequence $\mathbf{r} \times \mathbf{F}$. If the fingers of the right hand are curled in the direction of rotation from the positive sense of **r** to the positive sense of **F**, then the thumb points in the positive sense of **M**.

We must maintain the sequence $\mathbf{r} \times \mathbf{F}$, because the sequence $\mathbf{F} \times \mathbf{r}$ would produce a vector with a sense opposite to that of the correct moment. As was the case with the scalar approach, the moment **M** may be thought of as the moment about point A or as the moment about the line O-O which passes through point A and is perpendicular to the plane containing the vectors **r** and **F**. When we evaluate the moment of a force about a given point, the choice between using the vector cross product or the scalar expression depends on how the geometry of the problem is specified. If we know or can easily determine the perpendicular distance between the line of action of the force and the moment center, then the scalar approach is generally simpler. If, however, **F** and **r** are not perpendicular and are easily expressible in vector notation, then the cross-product expression is often preferable.

In Section B of this chapter, we will see how the vector formulation of the moment of a force is especially useful for determining the moment of a force about a point in three-dimensional situations.

Varignon's Theorem

One of the most useful principles of mechanics is *Varignon's theorem*, which states that the moment of a force about any point is equal to the sum of the moments of the components of the force about the same point.

*See item 7 in Art. C/7 of Appendix C for additional information concerning the cross product.

To prove this theorem, consider the force **R** acting in the plane of the body shown in Fig. 2/9*a*. The forces **P** and **Q** represent any two nonrectangular components of **R**. The moment of **R** about point O is

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R}$$

Because $\mathbf{R} = \mathbf{P} + \mathbf{Q}$, we may write

$$\mathbf{r} \times \mathbf{R} = \mathbf{r} \times (\mathbf{P} + \mathbf{Q})$$

Using the distributive law for cross products, we have

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{R} = \mathbf{r} \times \mathbf{P} + \mathbf{r} \times \mathbf{Q} \qquad \textbf{(2/8)}$$

which says that the moment of **R** about O equals the sum of the moments about O of its components **P** and **Q**. This proves the theorem.

Varignon's theorem need not be restricted to the case of two components, but it applies equally well to three or more. Thus we could have used any number of concurrent components of **R** in the foregoing proof.*

Figure 2/9*b* illustrates the usefulness of Varignon's theorem. The moment of **R** about point O is Rd. However, if d is more difficult to determine than p and q, we can resolve **R** into the components **P** and **Q**, and compute the moment as

$$M_O = Rd = -pP + qQ$$

where we take the clockwise moment sense to be positive.

Sample Problem 2/5 shows how Varignon's theorem can help us to calculate moments.

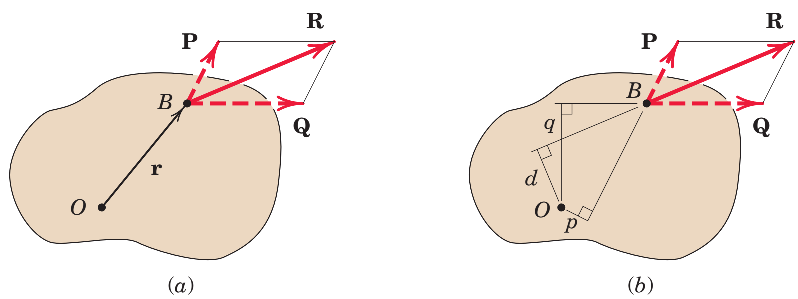
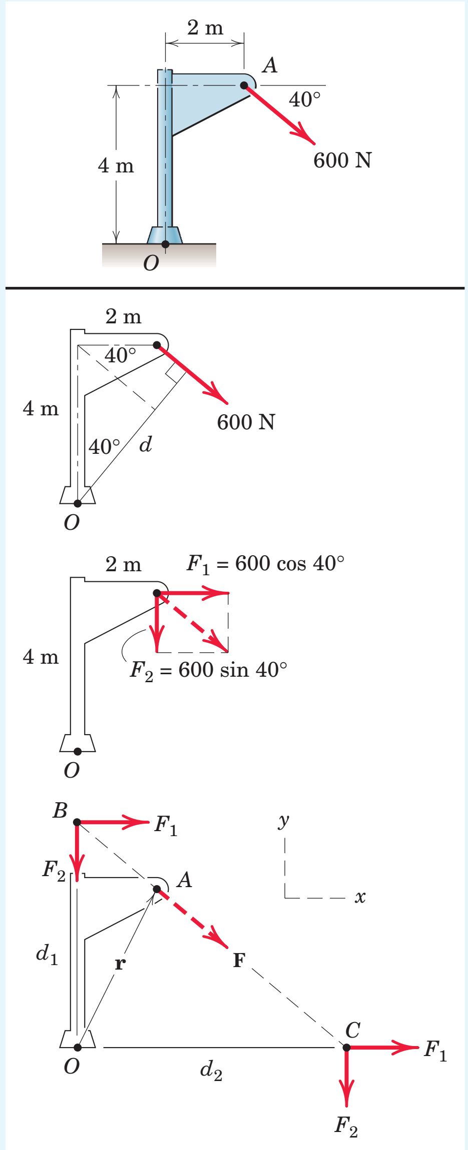


Figure 2/9

*As originally stated, Varignon's theorem was limited to the case of two concurrent components of a given force. See *The Science of Mechanics*, by Ernst Mach, originally published in 1883.

SAMPLE PROBLEM 2/5

Calculate the magnitude of the moment about the base point O of the 600-N force in five different ways.



Solution. *(I)* The moment arm to the 600-N force is

$$d = 4 \cos 40° + 2 \sin 40° = 4.35 \text{ m}$$

1 By $M = Fd$ the moment is clockwise and has the magnitude

$$M_O = 600(4.35) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(II) Replace the force by its rectangular components at A,

$$F_1 = 600 \cos 40° = 460 \text{ N}, \qquad F_2 = 600 \sin 40° = 386 \text{ N}$$

By Varignon's theorem, the moment becomes

2 $$M_O = 460(4) + 386(2) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(III) By the principle of transmissibility, move the 600-N force along its line of action to point B, which eliminates the moment of the component F_2. The moment arm of F_1 becomes

$$d_1 = 4 + 2 \tan 40° = 5.68 \text{ m}$$

and the moment is

$$M_O = 460(5.68) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

3 *(IV)* Moving the force to point C eliminates the moment of the component F_1. The moment arm of F_2 becomes

$$d_2 = 2 + 4 \cot 40° = 6.77 \text{ m}$$

and the moment is

$$M_O = 386(6.77) = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

(V) By the vector expression for a moment, and by using the coordinate system indicated on the figure together with the procedures for evaluating cross products, we have

4 $$\mathbf{M}_O = \mathbf{r} \times \mathbf{F} = (2\mathbf{i} + 4\mathbf{j}) \times 600(\mathbf{i} \cos 40° - \mathbf{j} \sin 40°)$$
$$= -2610\mathbf{k} \text{ N}\cdot\text{m}$$

The minus sign indicates that the vector is in the negative z-direction. The magnitude of the vector expression is

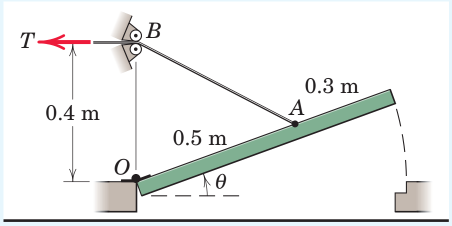
$$M_O = 2610 \text{ N}\cdot\text{m} \qquad \textit{Ans.}$$

Helpful Hints

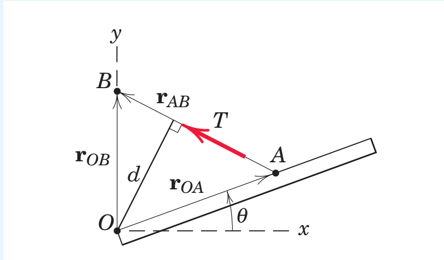
1. The required geometry here and in similar problems should not cause difficulty if the sketch is carefully drawn.
2. This procedure is frequently the shortest approach.
3. The fact that points B and C are not on the body proper should not cause concern, as the mathematical calculation of the moment of a force does not require that the force be on the body.
4. Alternative choices for the position vector $\mathbf{r}$ are $\mathbf{r} = d_1\mathbf{j} = 5.68\mathbf{j}$ m and $\mathbf{r} = d_2\mathbf{i} = 6.77\mathbf{i}$ m.

SAMPLE PROBLEM 2/6

The trap door OA is raised by the cable AB, which passes over the small frictionless guide pulleys at B. The tension everywhere in the cable is T, and this tension applied at A causes a moment M_O about the hinge at O. Plot the quantity M_O/T as a function of the door elevation angle θ over the range $0 \le \theta \le 90°$ and note minimum and maximum values. What is the physical significance of this ratio?



Solution. We begin by constructing a figure which shows the tension force $\mathbf{T}$ acting directly on the door, which is shown in an arbitrary angular position θ. It should be clear that the direction of $\mathbf{T}$ will vary as θ varies. In order to deal with this variation, we write a unit vector $\mathbf{n}_{AB}$ which "aims" $\mathbf{T}$:



1

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{\mathbf{r}_{OB} - \mathbf{r}_{OA}}{r_{AB}}$$

Using the x-y coordinates of our figure, we can write

2

$$\mathbf{r}_{OB} = 0.4\mathbf{j} \text{ m and } \mathbf{r}_{OA} = 0.5(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \text{ m}$$

So

$$\begin{aligned}\mathbf{r}_{AB} = \mathbf{r}_{OB} - \mathbf{r}_{OA} &= 0.4\mathbf{j} - (0.5)(\cos\theta\mathbf{i} + \sin\theta\mathbf{j}) \\ &= -0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j} \text{ m}\end{aligned}$$

and

$$\begin{aligned}r_{AB} &= \sqrt{(0.5\cos\theta)^2 + (0.4 - 0.5\sin\theta)^2} \\ &= \sqrt{0.41 - 0.4\sin\theta} \text{ m}\end{aligned}$$

The desired unit vector is

$$\mathbf{n}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}$$

Our tension vector can now be written as

$$\mathbf{T} = T\mathbf{n}_{AB} = T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right]$$

3 The moment of $\mathbf{T}$ about point O, as a vector, is $\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{T}$, where $\mathbf{r}_{OB} = 0.4\mathbf{j}$ m, or

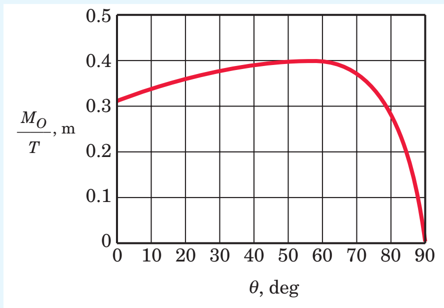
$$\begin{aligned}\mathbf{M}_O &= 0.4\mathbf{j} \times T\left[\frac{-0.5\cos\theta\mathbf{i} + (0.4 - 0.5\sin\theta)\mathbf{j}}{\sqrt{0.41 - 0.4\sin\theta}}\right] \\ &= \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}\mathbf{k}\end{aligned}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \frac{0.2T\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}}$$

and the requested ratio is

$$\frac{M_O}{T} = \frac{0.2\cos\theta}{\sqrt{0.41 - 0.4\sin\theta}} \qquad \textit{Ans.}$$



which is plotted in the accompanying graph. The expression M_O/T is the moment arm d (in meters) which runs from O to the line of action of $\mathbf{T}$. It has a maximum value of 0.4 m at $\theta = 53.1°$ (at which point $\mathbf{T}$ is horizontal) and a minimum value of 0 at $\theta = 90°$ (at which point $\mathbf{T}$ is vertical). The expression is valid even if T varies.

This sample problem treats moments in two-dimensional force systems, and it also points out the advantages of carrying out a solution for an arbitrary position, so that behavior over a range of positions can be examined.

Helpful Hints

1 Recall that any unit vector can be written as a vector divided by its magnitude. In this case the vector in the numerator is a position vector.

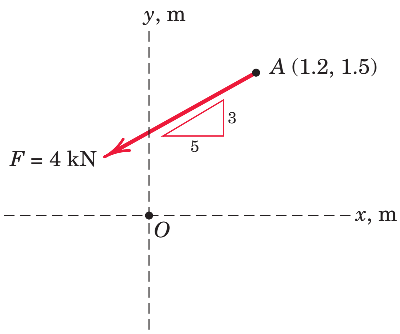
2 Recall that any vector may be written as a magnitude times an "aiming" unit vector.

3 In the expression $\mathbf{M} = \mathbf{r} \times \mathbf{F}$, the position vector $\mathbf{r}$ runs *from* the moment center *to* any point on the line of action of $\mathbf{F}$. Here, $\mathbf{r}_{OB}$ is more convenient than $\mathbf{r}_{OA}$.

PROBLEMS

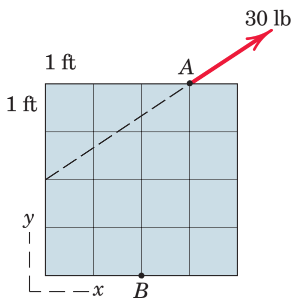
Introductory Problems

2/31 The 4-kN force **F** is applied at point A. Compute the moment of **F** about point O, expressing it both as a scalar and as a vector quantity. Determine the coordinates of the points on the x- and y-axes about which the moment of **F** is zero.



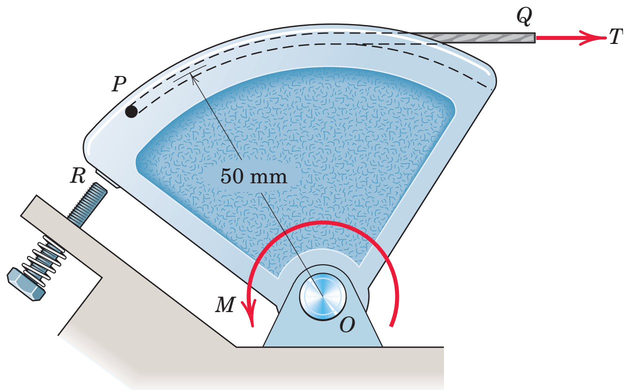
Problem 2/31

2/32 The rectangular plate is made up of 1-ft squares as shown. A 30-lb force is applied at point A in the direction shown. Calculate the moment M_B of the force about point B by at least two different methods.



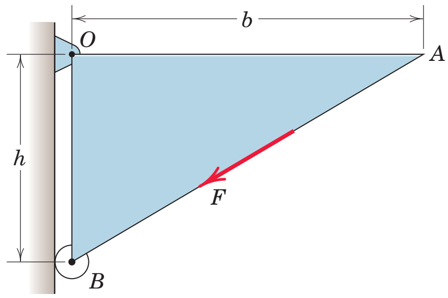
Problem 2/32

2/33 The throttle-control sector pivots freely at O. If an internal torsional spring exerts a return moment $M = 1.8$ N·m on the sector when in the position shown, for design purposes determine the necessary throttle-cable tension T so that the net moment about O is zero. Note that when T is zero, the sector rests against the idle-control adjustment screw at R.



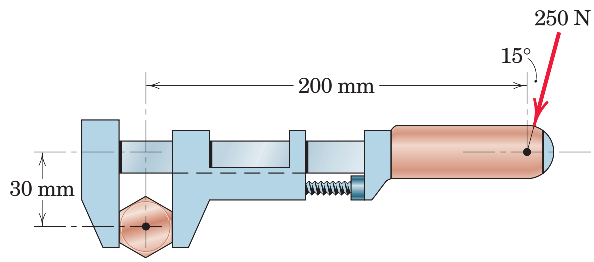
Problem 2/33

2/34 The force of magnitude F acts along the edge of the triangular plate. Determine the moment of **F** about point O.



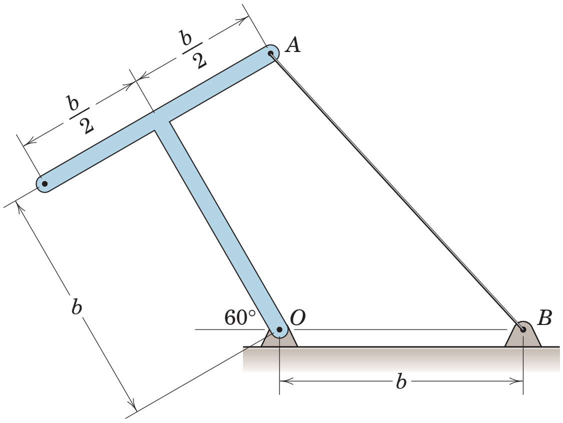
Problem 2/34

2/35 Calculate the moment of the 250-N force on the handle of the monkey wrench about the center of the bolt.



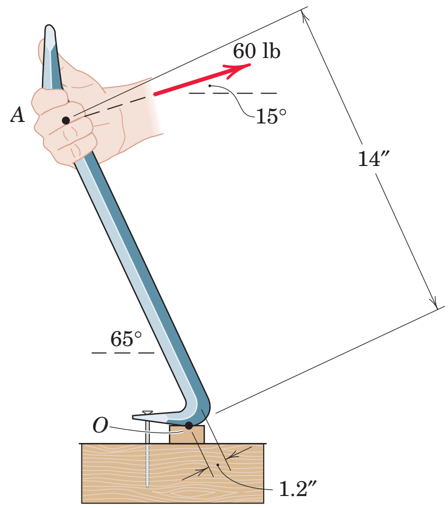
Problem 2/35

2/36 The tension in cable AB is 100 N. Determine the moment about O of this tension as applied to point A of the T-shaped bar. The dimension b is 600 mm.



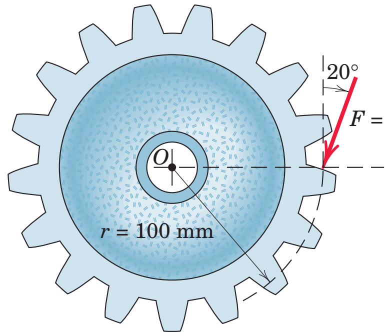
Problem 2/36

2/37 A prybar is used to remove a nail as shown. Determine the moment of the 60-lb force about the point O of contact between the prybar and the small support block.



Problem 2/37

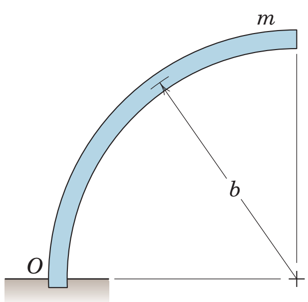
2/38 A force **F** of magnitude 60 N is applied to the gear. Determine the moment of **F** about point O.



Problem 2/38

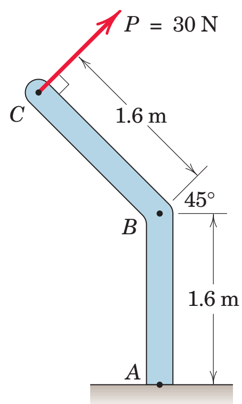
Representative Problems

2/39 The slender quarter-circular member of mass m is built-in at its support O. Determine the moment of its weight about point O. Use Table D/3 as necessary to determine the location of the mass center of the body.



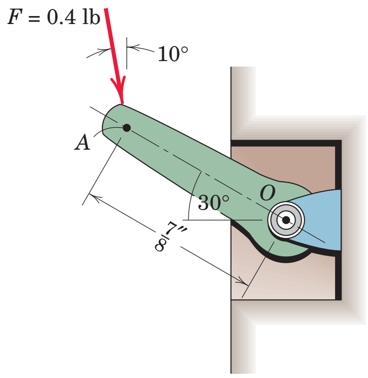
Problem 2/39

2/40 The 30-N force **P** is applied perpendicular to the portion BC of the bent bar. Determine the moment of **P** about point B and about point A.



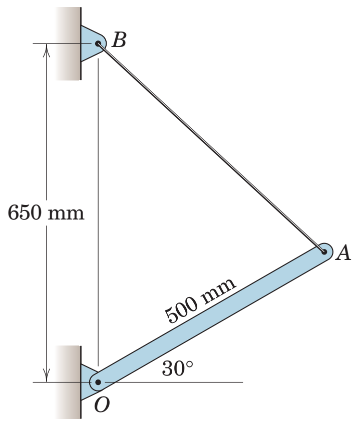
Problem 2/40

2/41 Compute the moment of the 0.4-lb force about the pivot O of the wall-switch toggle.



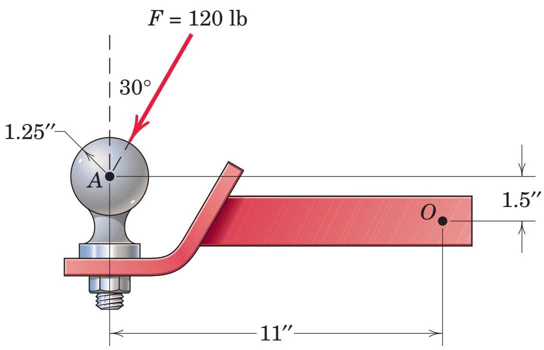
Problem 2/41

2/42 The cable AB carries a tension of 400 N. Determine the moment about O of this tension as applied to point A of the slender bar.



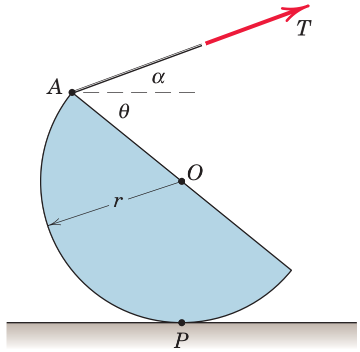
Problem 2/42

2/43 As a trailer is towed in the forward direction, the force $F = 120$ lb is applied as shown to the ball of the trailer hitch. Determine the moment of this force about point O.



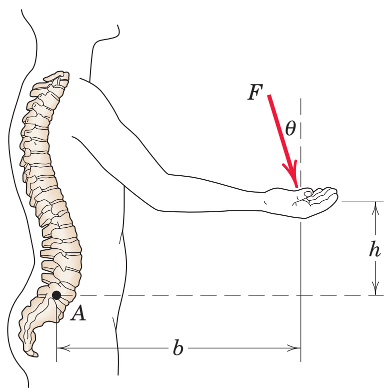
Problem 2/43

2/44 Determine the moments of the tension T about point P and about point O.



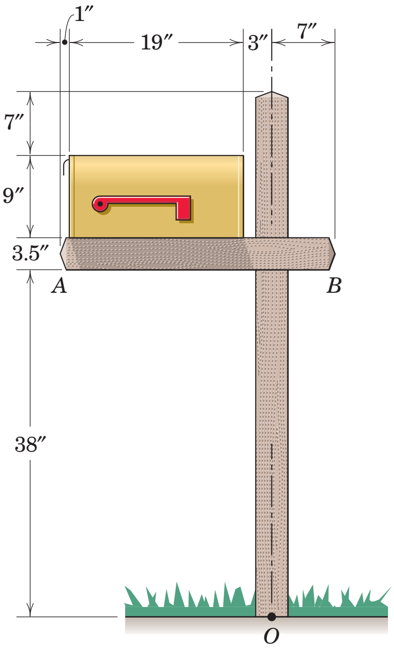
Problem 2/44

2/45 The lower lumbar region A of the spine is the part of the spinal column most susceptible to abuse while resisting excessive bending caused by the moment about A of a force F. For given values of F, b, and h, determine the angle θ which causes the most severe bending strain.



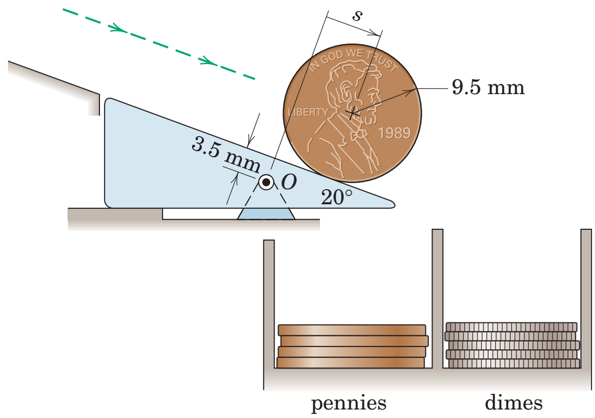
Problem 2/45

2/46 Determine the combined moment about O due to the weight of the mailbox and the cross member AB. The mailbox weighs 4 lb and the uniform cross member weighs 10 lb. Both weights act at the geometric centers of the respective items.



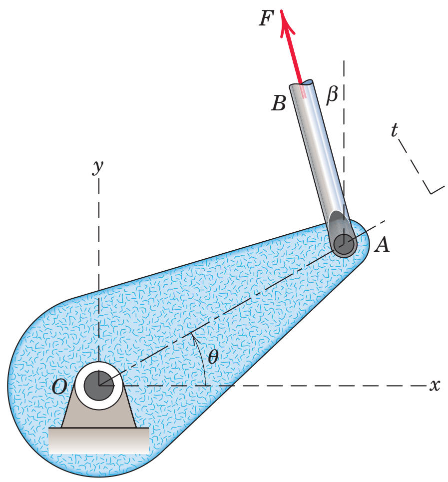
Problem 2/46

2/47 A portion of a mechanical coin sorter works as follows: Pennies and dimes roll down the 20° incline, the last triangular portion of which pivots freely about a horizontal axis through O. Dimes are light enough (2.28 grams each) so that the triangular portion remains stationary, and the dimes roll into the right collection column. Pennies, on the other hand, are heavy enough (3.06 grams each) so that the triangular portion pivots clockwise, and the pennies roll into the left collection column. Determine the moment about O of the weight of the penny in terms of the slant distance s in millimeters.



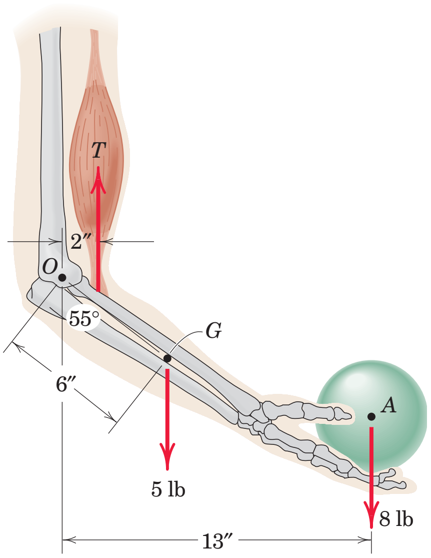
Problem 2/47

2/48 The crank of Prob. 2/10 is repeated here. If $\overline{OA} = 50$ mm, $\theta = 25°$, and $\beta = 55°$, determine the moment of the force $\mathbf{F}$ of magnitude $F = 20$ N about point O.



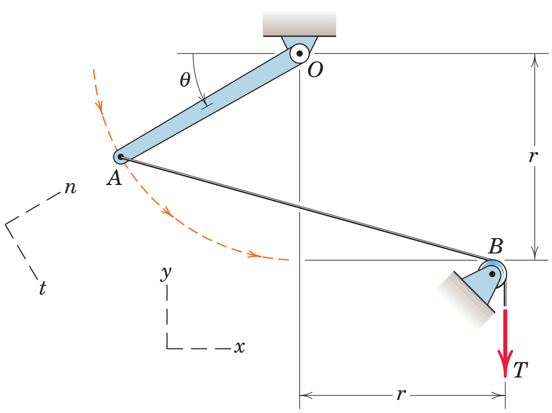
Problem 2/48

2/49 Elements of the lower arm are shown in the figure. The weight of the forearm is 5 lb with mass center at G. Determine the combined moment about the elbow pivot O of the weights of the forearm and the sphere. What must the biceps tension force be so that the overall moment about O is zero?



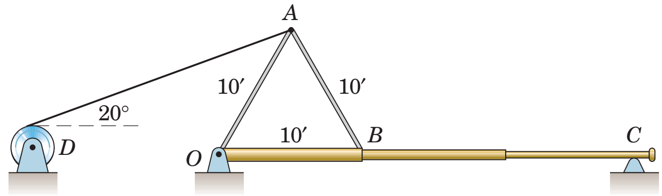
Problem 2/49

2/50 The mechanism of Prob. 2/16 is repeated here. For the conditions $\theta = 40°$, $T = 150$ N, and $r = 200$ mm, determine the moment about O of the tension T applied by cable AB to point A.



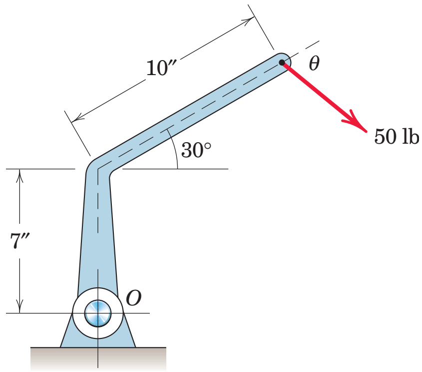
Problem 2/50

2/51 In order to raise the flagpole OC, a light frame OAB is attached to the pole and a tension of 780 lb is developed in the hoisting cable by the power winch D. Calculate the moment M_O of this tension about the hinge point O.



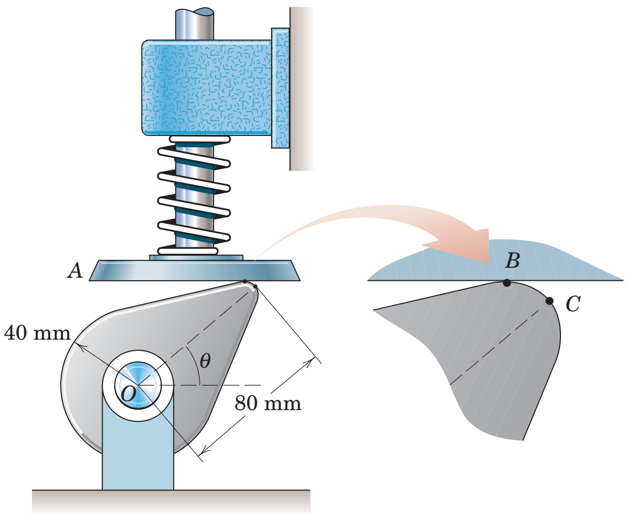
Problem 2/51

2/52 Determine the angle θ which will maximize the moment M_O of the 50-lb force about the shaft axis at O. Also compute M_O.



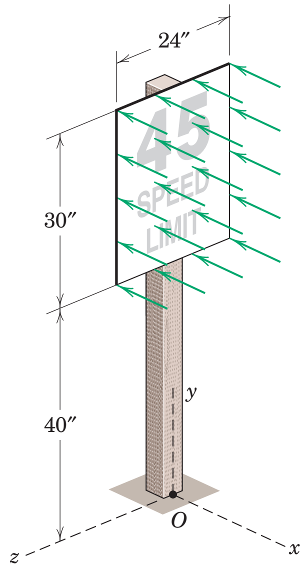
Problem 2/52

2/53 The spring-loaded follower A bears against the circular portion of the cam until the lobe of the cam lifts the plunger. The force required to lift the plunger is proportional to its vertical movement h from its lowest position. For design purposes determine the angle θ for which the moment of the contact force on the cam about the bearing O is a maximum. In the enlarged view of the contact, neglect the small distance between the actual contact point B and the end C of the lobe.



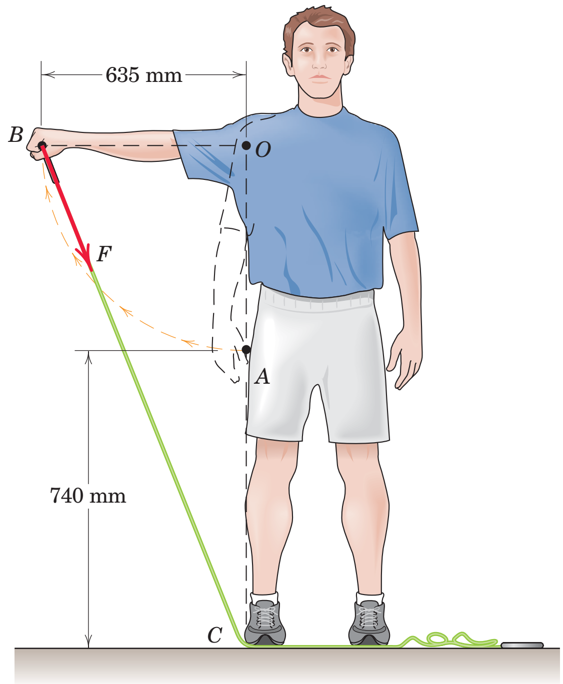
Problem 2/53

2/54 As the result of a wind blowing normal to the plane of the rectangular sign, a uniform pressure of 3.5 lb/ft^2 is exerted in the direction shown in the figure. Determine the moment of the resulting force about point O. Express your result as a vector using the coordinates shown.



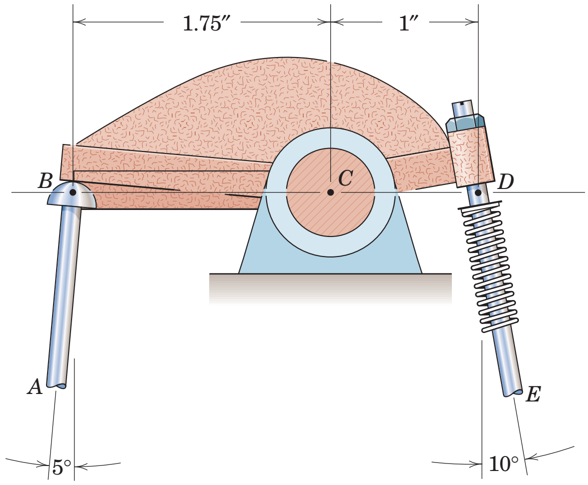
Problem 2/54

2/55 An exerciser begins with his arm in the relaxed vertical position OA, at which the elastic band is unstretched. He then rotates his arm to the horizontal position OB. The elastic modulus of the band is $k = 60$ N/m—that is, 60 N of force is required to stretch the band each additional meter of elongation. Determine the moment about O of the force which the band exerts on the hand B.



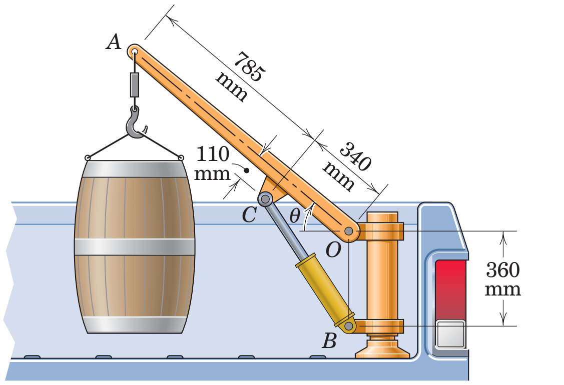
Problem 2/55

2/56 The rocker arm BD of an automobile engine is supported by a nonrotating shaft at C. If the design value of the force exerted by the pushrod AB on the rocker arm is 80 lb, determine the force which the valve stem DE must exert at D in order for the combined moment about point C to be zero. Compute the resultant of these two forces exerted on the rocker arm. Note that the points B, C, and D lie on a horizontal line and that both the pushrod and valve stem exert forces along their axes.



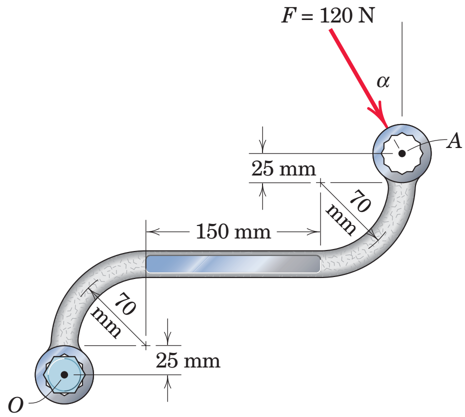
Problem 2/56

2/57 The small crane is mounted along the side of a pickup bed and facilitates the handling of heavy loads. When the boom elevation angle is $\theta = 40°$, the force in the hydraulic cylinder BC is 4.5 kN, and this force applied at point C is in the direction from B to C (the cylinder is in compression). Determine the moment of this 4.5-kN force about the boom pivot point O.



Problem 2/57

2/58 The 120-N force is applied as shown to one end of the curved wrench. If $\alpha = 30°$, calculate the moment of F about the center O of the bolt. Determine the value of α which would maximize the moment about O; state the value of this maximum moment.



Problem 2/58

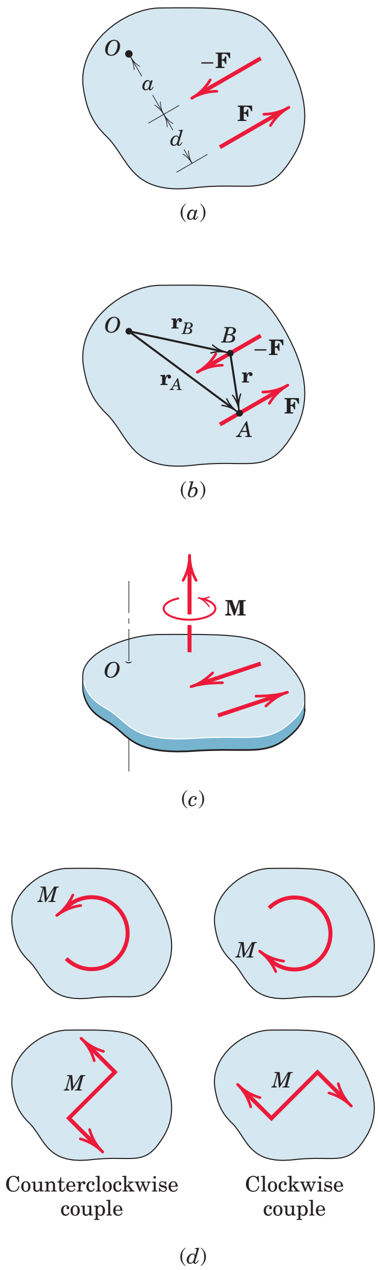


Figure 2/10

2/5 Couple

The moment produced by two equal, opposite, and noncollinear forces is called a *couple*. Couples have certain unique properties and have important applications in mechanics.

Consider the action of two equal and opposite forces **F** and −**F** a distance d apart, as shown in Fig. 2/10a. These two forces cannot be combined into a single force because their sum in every direction is zero. Their only effect is to produce a tendency of rotation. The combined moment of the two forces about an axis normal to their plane and passing through any point such as O in their plane is the couple **M**. This couple has a magnitude

$$M = F(a + d) - Fa$$

or

$$M = Fd$$

Its direction is counterclockwise when viewed from above for the case illustrated. Note especially that the magnitude of the couple is independent of the distance a which locates the forces with respect to the moment center O. It follows that the moment of a couple has the same value for all moment centers.

Vector Algebra Method

We may also express the moment of a couple by using vector algebra. With the cross-product notation of Eq. 2/6, the combined moment about point O of the forces forming the couple of Fig. 2/10b is

$$\mathbf{M} = \mathbf{r}_A \times \mathbf{F} + \mathbf{r}_B \times (-\mathbf{F}) = (\mathbf{r}_A - \mathbf{r}_B) \times \mathbf{F}$$

where $\mathbf{r}_A$ and $\mathbf{r}_B$ are position vectors which run from point O to arbitrary points A and B on the lines of action of **F** and −**F**, respectively. Because $\mathbf{r}_A - \mathbf{r}_B = \mathbf{r}$, we can express **M** as

$$\mathbf{M} = \mathbf{r} \times \mathbf{F}$$

Here again, the moment expression contains no reference to the moment center O and, therefore, is the same for all moment centers. Thus, we may represent **M** by a free vector, as shown in Fig. 2/10c, where the direction of **M** is normal to the plane of the couple and the sense of **M** is established by the right-hand rule.

Because the couple vector **M** is always perpendicular to the plane of the forces which constitute the couple, in two-dimensional analysis we can represent the sense of a couple vector as clockwise or counterclockwise by one of the conventions shown in Fig. 2/10d. Later, when we deal with couple vectors in three-dimensional problems, we will make full use of vector notation to represent them, and the mathematics will automatically account for their sense.

Equivalent Couples

Changing the values of F and d does not change a given couple as long as the product Fd remains the same. Likewise, a couple is not affected if the forces act in a different but parallel plane. Figure 2/11

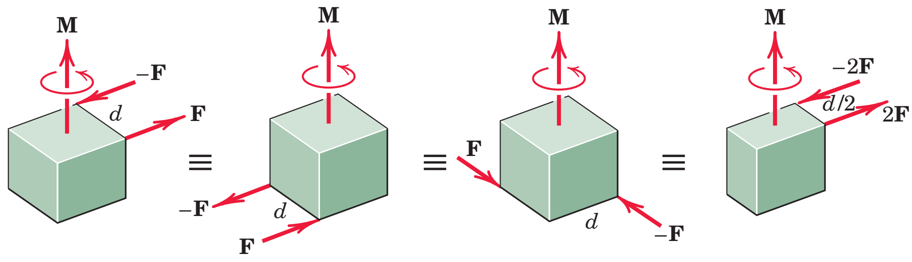


Figure 2/11

shows four different configurations of the same couple **M**. In each of the four cases, the couples are equivalent and are described by the same free vector which represents the identical tendencies to rotate the bodies.

Force–Couple Systems

The effect of a force acting on a body is the tendency to push or pull the body in the direction of the force, and to rotate the body about any fixed axis which does not intersect the line of the force. We can represent this dual effect more easily by replacing the given force by an equal parallel force and a couple to compensate for the change in the moment of the force.

The replacement of a force by a force and a couple is illustrated in Fig. 2/12, where the given force **F** acting at point A is replaced by an equal force **F** at some point B and the counterclockwise couple $M = Fd$. The transfer is seen in the middle figure, where the equal and opposite forces **F** and $-$**F** are added at point B without introducing any net external effects on the body. We now see that the original force at A and the equal and opposite one at B constitute the couple $M = Fd$, which is counterclockwise for the sample chosen, as shown in the right-hand part of the figure. Thus, we have replaced the original force at A by the same force acting at a different point B and a couple, without altering the external effects of the original force on the body. The combination of the force and couple in the right-hand part of Fig. 2/12 is referred to as a *force–couple system*.

By reversing this process, we can combine a given couple and a force which lies in the plane of the couple (normal to the couple vector) to produce a single, equivalent force. Replacement of a force by an equivalent force–couple system, and the reverse procedure, have many applications in mechanics and should be mastered.

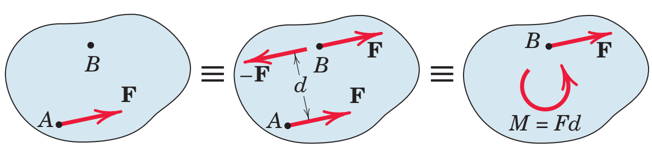
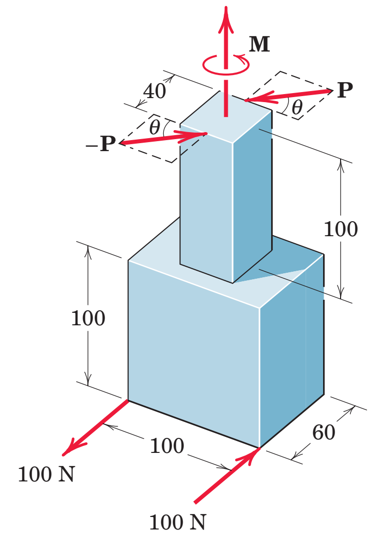


Figure 2/12

SAMPLE PROBLEM 2/7

The rigid structural member is subjected to a couple consisting of the two 100-N forces. Replace this couple by an equivalent couple consisting of the two forces **P** and −**P**, each of which has a magnitude of 400 N. Determine the proper angle θ.



Dimensions in millimeters

Solution. The original couple is counterclockwise when the plane of the forces is viewed from above, and its magnitude is

$[M = Fd]$ $\qquad M = 100(0.1) = 10 \text{ N}\cdot\text{m}$

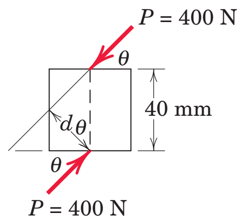
The forces **P** and −**P** produce a counterclockwise couple

$$M = 400(0.040) \cos \theta$$

① Equating the two expressions gives

$$10 = (400)(0.040) \cos \theta$$

$$\theta = \cos^{-1} \frac{10}{16} = 51.3° \qquad \textit{Ans.}$$

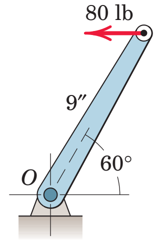


Helpful Hint

① Since the two equal couples are parallel free vectors, the only dimensions which are relevant are those which give the perpendicular distances between the forces of the couples.

SAMPLE PROBLEM 2/8

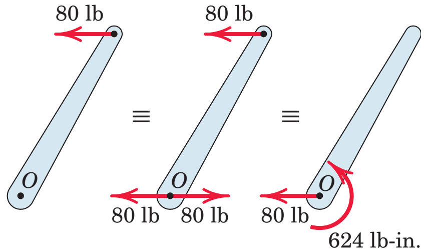
Replace the horizontal 80-lb force acting on the lever by an equivalent system consisting of a force at O and a couple.



Solution. We apply two equal and opposite 80-lb forces at O and identify the counterclockwise couple

$[M = Fd]$ $\qquad M = 80(9 \sin 60°) = 624 \text{ lb-in.} \qquad \textit{Ans.}$

① Thus, the original force is equivalent to the 80-lb force at O and the 624-lb-in. couple as shown in the third of the three equivalent figures.



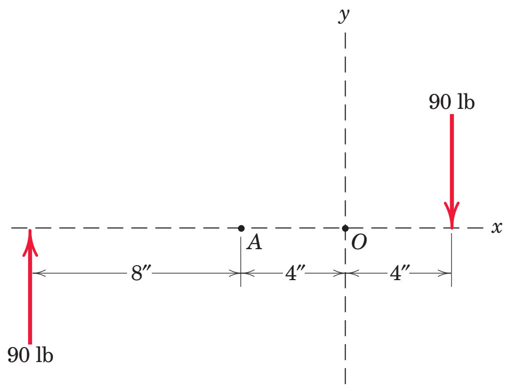
Helpful Hint

① The reverse of this problem is often encountered, namely, the replacement of a force and a couple by a single force. Proceeding in reverse is the same as replacing the couple by two forces, one of which is equal and opposite to the 80-lb force at O. The moment arm to the second force would be $M/F = 624/80 = 7.79$ in., which is 9 sin 60°, thus determining the line of action of the single resultant force of 80 lb.

PROBLEMS

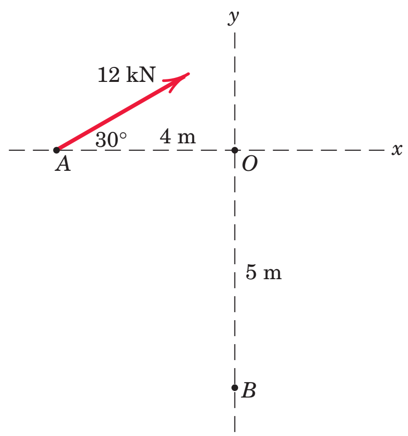
Introductory Problems

2/59 Compute the combined moment of the two 90-lb forces about (*a*) point *O* and (*b*) point *A*.



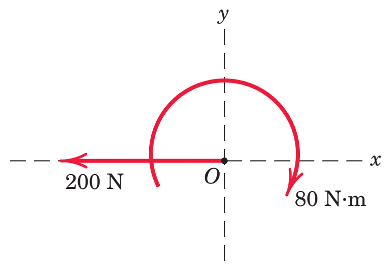
Problem 2/59

2/60 Replace the 12-kN force acting at point *A* by a force–couple system at (*a*) point *O* and (*b*) point *B*.



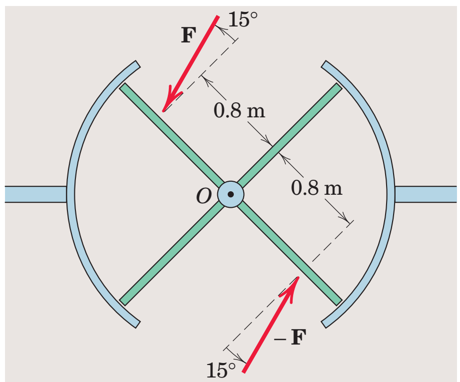
Problem 2/60

2/61 Replace the force–couple system at point *O* by a single force. Specify the coordinate y_A of the point on the *y*-axis through which the line of action of this resultant force passes.



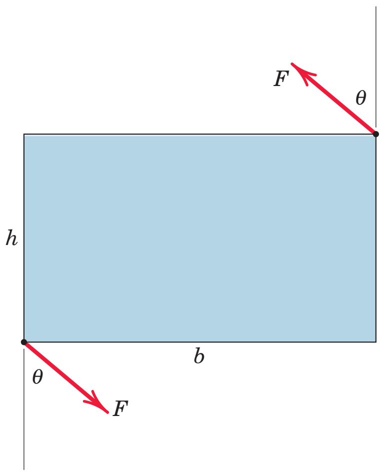
Problem 2/61

2/62 The top view of a revolving entrance door is shown. Two persons simultaneously approach the door and exert force of equal magnitudes as shown. If the resulting moment about the door pivot axis at *O* is 25 N·m, determine the force magnitude *F*.



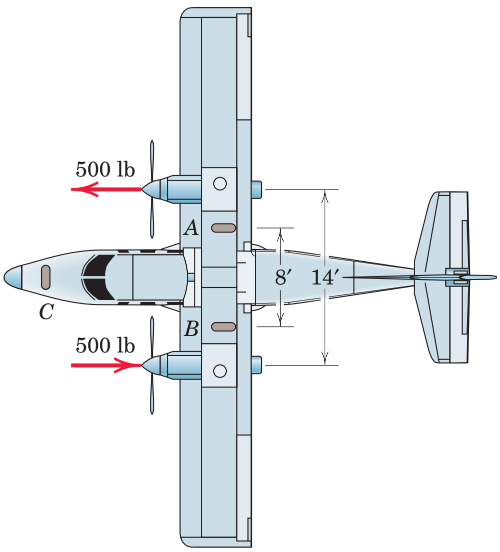
Problem 2/62

2/63 Determine the moment associated with the couple applied to the rectangular plate. Reconcile the results with those for the individual special cases of $\theta = 0$, $b = 0$, and $h = 0$.



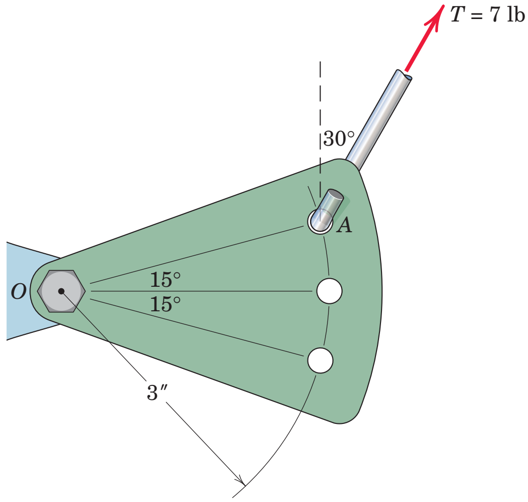
Problem 2/63

2/64 As part of a test, the two aircraft engines are revved up and the propeller pitches are adjusted so as to result in the fore and aft thrusts shown. What force F must be exerted by the ground on each of the main braked wheels at A and B to counteract the turning effect of the two propeller thrusts? Neglect any effects of the nose wheel C, which is turned 90° and unbraked.



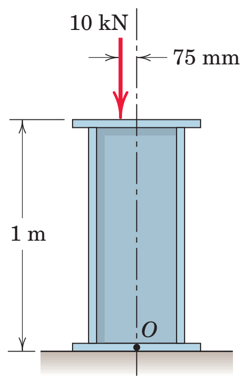
Problem 2/64

2/65 The 7-lb force is applied by the control rod on the sector as shown. Determine the equivalent force–couple system at O.



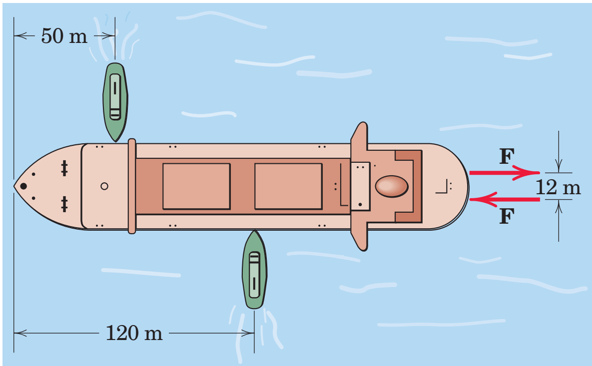
Problem 2/65

2/66 Replace the 10-kN force acting on the steel column by an equivalent force–couple system at point O. This replacement is frequently done in the design of structures.



Problem 2/66

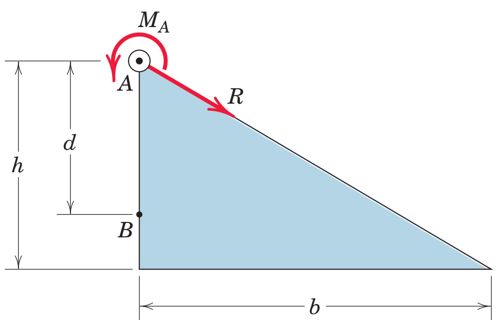
2/67 Each propeller of the twin-screw ship develops a full-speed thrust of 300 kN. In maneuvering the ship, one propeller is turning full speed ahead and the other full speed in reverse. What thrust P must each tug exert on the ship to counteract the effect of the ship's propellers?



Problem 2/67

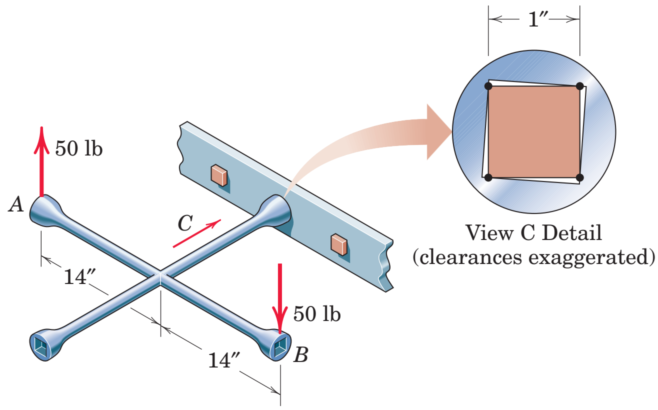
Representative Problems

2/68 The force–couple system at A is to be replaced by a single equivalent force acting at a point B on the vertical edge (or its extension) of the triangular plate. Determine the distance d between A and B.



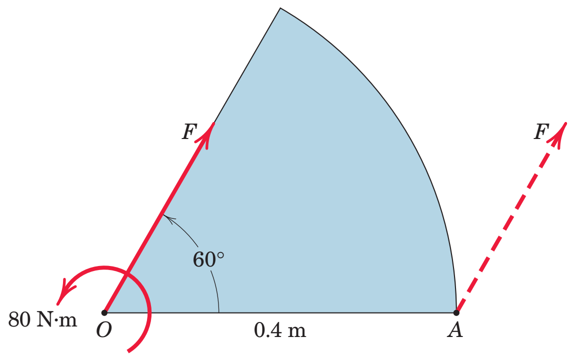
Problem 2/68

2/69 A lug wrench is used to tighten a square-head bolt. If 50-lb forces are applied to the wrench as shown, determine the magnitude F of the equal forces exerted on the four contact points on the 1-in. bolt head so that their external effect on the bolt is equivalent to that of the two 50-lb forces. Assume that the forces are perpendicular to the flats of the bolt head.



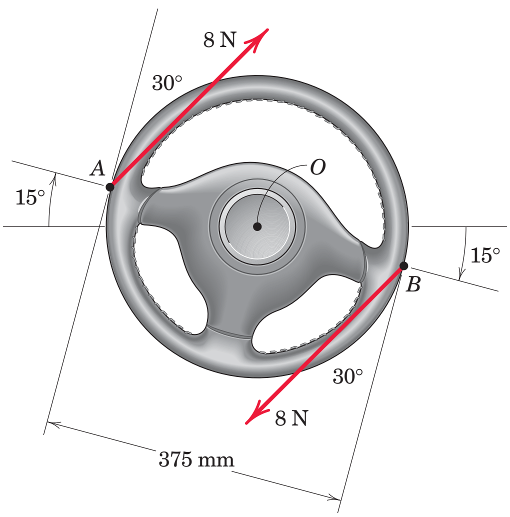
Problem 2/69

2/70 A force–couple system acts at O on the 60° circular sector. Determine the magnitude of the force F if the given system can be replaced by a stand-alone force at corner A of the sector.



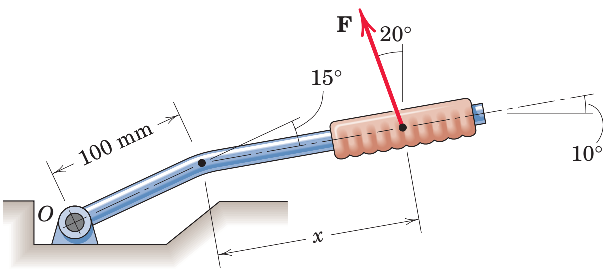
Problem 2/70

2/71 During a steady right turn, a person exerts the forces shown on the steering wheel. Note that each force consists of a tangential component and a radially-inward component. Determine the moment exerted about the steering column at O.



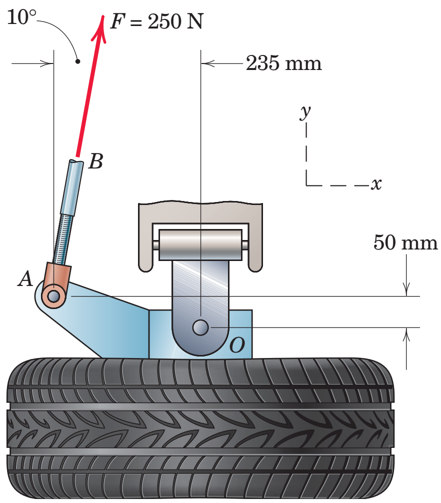
Problem 2/71

2/72 A force **F** of magnitude 50 N is exerted on the automobile parking-brake lever at the position $x = 250$ mm. Replace the force by an equivalent force–couple system at the pivot point O.



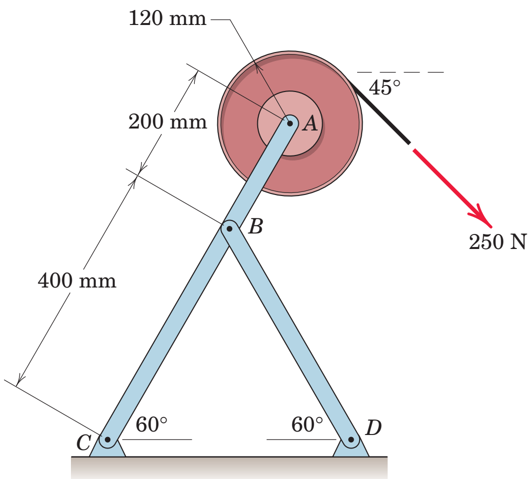
Problem 2/72

2/73 The tie-rod AB exerts the 250-N force on the steering knuckle AO as shown. Replace this force by an equivalent force–couple system at O.



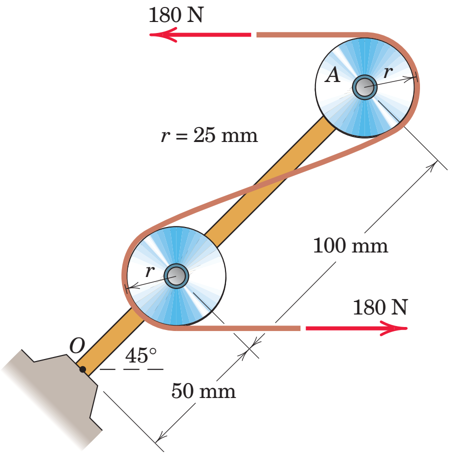
Problem 2/73

2/74 The 250-N tension is applied to a cord which is securely wrapped around the periphery of the disk. Determine the equivalent force–couple system at point C. Begin by finding the equivalent force–couple system at A.



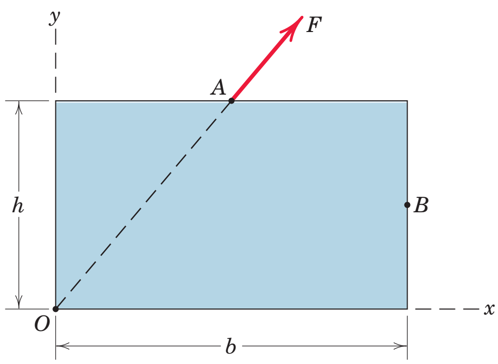
Problem 2/74

2/75 The system consisting of the bar OA, two identical pulleys, and a section of thin tape is subjected to the two 180-N tensile forces shown in the figure. Determine the equivalent force–couple system at point O.



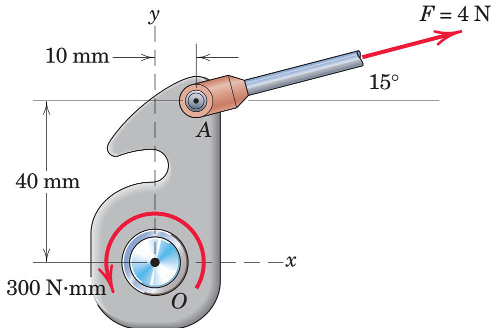
Problem 2/75

2/76 Points A and B are the midpoints of the sides of the rectangle. Replace the given force F acting at A by a force–couple system at B.



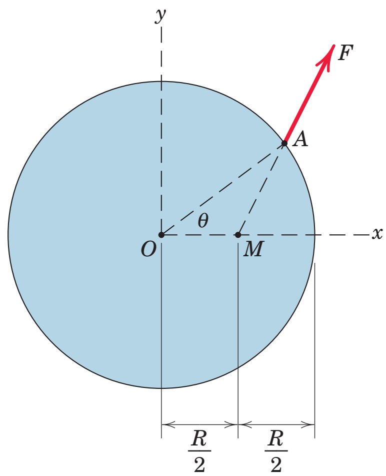
Problem 2/76

2/77 The device shown is a part of an automobile seat-back-release mechanism. The part is subjected to the 4-N force exerted at A and a 300-N·mm restoring moment exerted by a hidden torsional spring. Determine the y-intercept of the line of action of the single equivalent force.



Problem 2/77

2/78 The force F acts along line MA, where M is the midpoint of the radius along the x-axis. Determine the equivalent force–couple system at O if $\theta = 40°$.



Problem 2/78

2/6 Resultants

The properties of force, moment, and couple were developed in the previous four articles. Now we are ready to describe the resultant action of a group or *system* of forces. Most problems in mechanics deal with a system of forces, and it is usually necessary to reduce the system to its simplest form to describe its action. The *resultant* of a system of forces is the simplest force combination which can replace the original forces without altering the external effect on the rigid body to which the forces are applied.

Equilibrium of a body is the condition in which the resultant of all forces acting on the body is zero. This condition is studied in statics. When the resultant of all forces on a body is not zero, the acceleration of the body is obtained by equating the force resultant to the product of the mass and acceleration of the body. This condition is studied in dynamics. Thus, the determination of resultants is basic to both statics and dynamics.

The most common type of force system occurs when the forces all act in a single plane, say, the *x-y* plane, as illustrated by the system of three forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ in Fig. 2/13*a*. We obtain the magnitude and direction of the resultant force $\mathbf{R}$ by forming the *force polygon* shown in part *b* of the figure, where the forces are added head-to-tail in any sequence. Thus, for any system of coplanar forces we may write

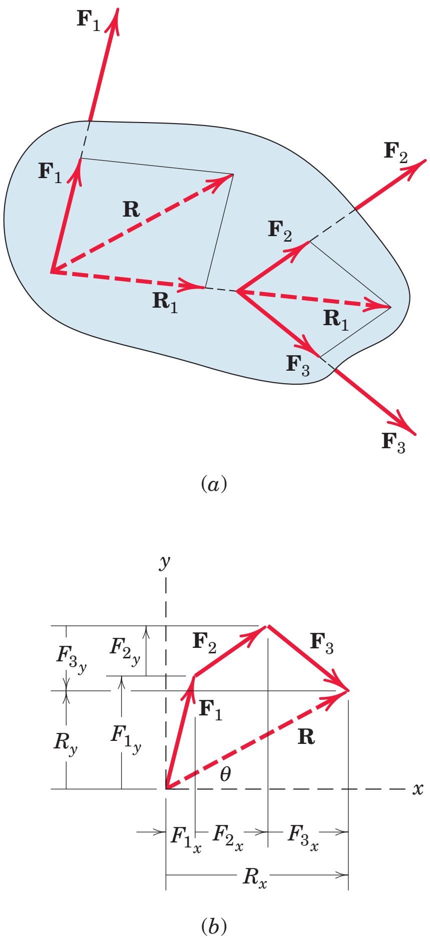


Figure 2/13

$$
\begin{gathered}
\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \cdots = \Sigma\mathbf{F} \\
R_x = \Sigma F_x \qquad R_y = \Sigma F_y \qquad R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} \\
\theta = \tan^{-1}\frac{R_y}{R_x} = \tan^{-1}\frac{\Sigma F_y}{\Sigma F_x}
\end{gathered}
\tag{2/9}
$$

Graphically, the correct line of action of $\mathbf{R}$ may be obtained by preserving the correct lines of action of the forces and adding them by the parallelogram law. We see this in part *a* of the figure for the case of three forces where the sum $\mathbf{R}_1$ of $\mathbf{F}_2$ and $\mathbf{F}_3$ is added to $\mathbf{F}_1$ to obtain $\mathbf{R}$. The principle of transmissibility has been used in this process.

Algebraic Method

We can use algebra to obtain the resultant force and its line of action as follows:

1. Choose a convenient reference point and move all forces to that point. This process is depicted for a three-force system in Figs. 2/14*a* and *b*, where M_1, M_2, and M_3 are the couples resulting from the transfer of forces $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ from their respective original lines of action to lines of action through point O.

2. Add all forces at O to form the resultant force $\mathbf{R}$, and add all couples to form the resultant couple M_O. We now have the single force–couple system, as shown in Fig. 2/14*c*.

3. In Fig. 2/14*d*, find the line of action of $\mathbf{R}$ by requiring $\mathbf{R}$ to have a moment of M_O about point O. Note that the force systems of Figs. 2/14*a* and 2/14*d* are equivalent, and that $\Sigma(Fd)$ in Fig. 2/14*a* is equal to Rd in Fig. 2/14*d*.

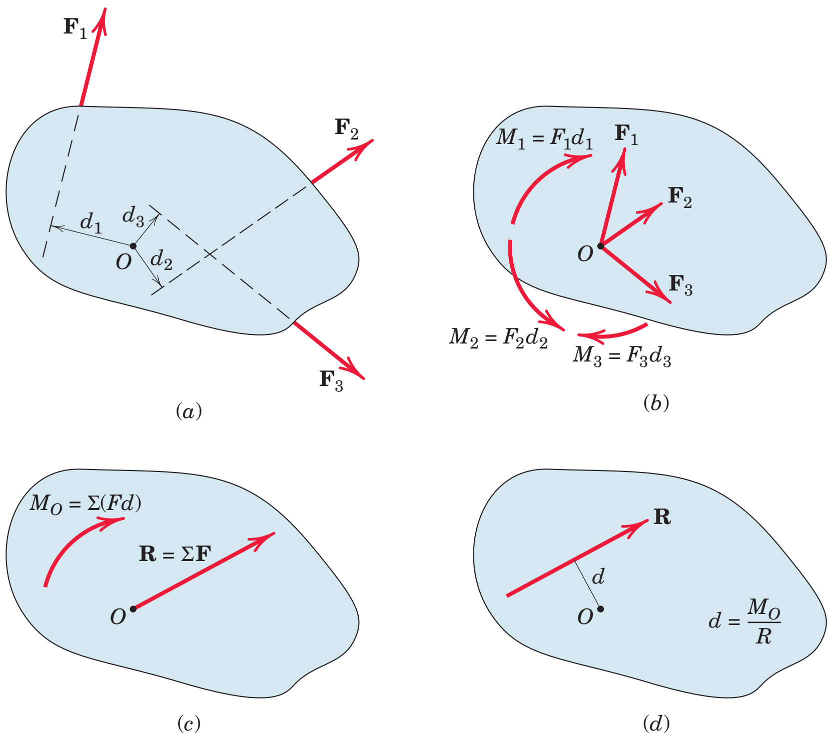


Figure 2/14

Principle of Moments

This process is summarized in equation form by

$$\mathbf{R} = \Sigma\mathbf{F}$$
$$M_O = \Sigma M = \Sigma(Fd) \tag{2/10}$$
$$Rd = M_O$$

The first two of Eqs. 2/10 reduce a given system of forces to a force–couple system at an arbitrarily chosen but convenient point O. The last equation specifies the distance d from point O to the line of action of $\mathbf{R}$, and states that the moment of the resultant force about any point O equals the sum of the moments of the original forces of the system about the same point. This extends Varignon's theorem to the case of *noncurrent* force systems; we call this extension the *principle of moments*.

For a concurrent system of forces where the lines of action of all forces pass through a common point O, the moment sum ΣM_O about that point is zero. Thus, the line of action of the resultant $\mathbf{R} = \Sigma\mathbf{F}$, determined by the first of Eqs. 2/10, passes through point O. For a parallel force system, select a coordinate axis in the direction of the forces. If the resultant force $\mathbf{R}$ for a given force system is zero, the resultant of the system need not be zero because the resultant may be a couple. The three forces in Fig. 2/15, for instance, have a zero resultant force but have a resultant clockwise couple $M = F_3d$.

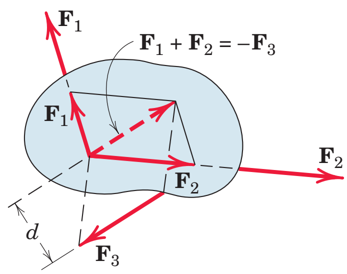
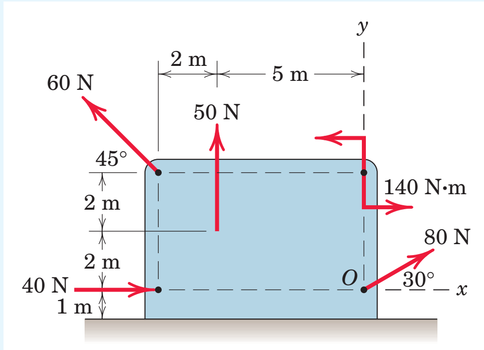


Figure 2/15

SAMPLE PROBLEM 2/9

Determine the resultant of the four forces and one couple which act on the plate shown.



Solution. Point O is selected as a convenient reference point for the force–couple system which is to represent the given system.

$[R_x = \Sigma F_x]$ $\qquad R_x = 40 + 80 \cos 30° - 60 \cos 45° = 66.9 \text{ N}$

$[R_y = \Sigma F_y]$ $\qquad R_y = 50 + 80 \sin 30° + 60 \cos 45° = 132.4 \text{ N}$

$[R = \sqrt{R_x{}^2 + R_y{}^2}]$ $\qquad R = \sqrt{(66.9)^2 + (132.4)^2} = 148.3 \text{ N}$ *Ans.*

$\left[\theta = \tan^{-1} \dfrac{R_y}{R_x}\right]$ $\qquad \theta = \tan^{-1} \dfrac{132.4}{66.9} = 63.2°$ *Ans.*

1 $[M_O = \Sigma(Fd)]$ $\qquad M_O = 140 - 50(5) + 60 \cos 45°(4) - 60 \sin 45°(7)$

$$= -237 \text{ N·m}$$

The force–couple system consisting of **R** and M_O is shown in Fig. *a*.

We now determine the final line of action of **R** such that **R** alone represents the original system.

$[Rd = |M_O|]$ $\qquad 148.3d = 237 \qquad d = 1.600 \text{ m}$ *Ans.*

Hence, the resultant **R** may be applied at any point on the line which makes a 63.2° angle with the x-axis and is tangent at point A to a circle of 1.600-m radius with center O, as shown in part b of the figure. We apply the equation $Rd = M_O$ in an absolute-value sense (ignoring any sign of M_O) and let the physics of the situation, as depicted in Fig. a, dictate the final placement of **R**. Had M_O been counterclockwise, the correct line of action of **R** would have been the tangent at point B.

The resultant **R** may also be located by determining its intercept distance b to point C on the x-axis, Fig. c. With R_x and R_y acting through point C, only R_y exerts a moment about O so that

$$R_y b = |M_O| \qquad \text{and} \qquad b = \frac{237}{132.4} = 1.792 \text{ m}$$

Alternatively, the y-intercept could have been obtained by noting that the moment about O would be due to R_x only.

A more formal approach in determining the final line of action of **R** is to use the vector expression

$$\mathbf{r} \times \mathbf{R} = \mathbf{M}_O$$

where $\mathbf{r} = x\mathbf{i} + y\mathbf{j}$ is a position vector running from point O to any point on the line of action of **R**. Substituting the vector expressions for **r**, **R**, and $\mathbf{M}_O$ and carrying out the cross product result in

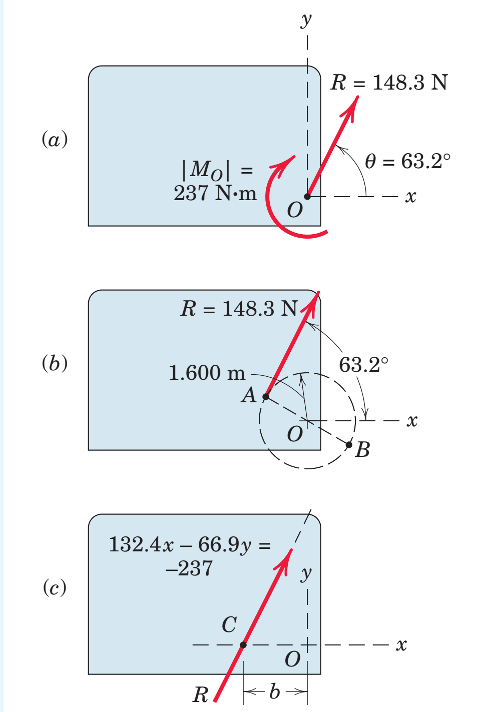
$$(x\mathbf{i} + y\mathbf{j}) \times (66.9\mathbf{i} + 132.4\mathbf{j}) = -237\mathbf{k}$$

$$(132.4x - 66.9y)\mathbf{k} = -237\mathbf{k}$$

Thus, the desired line of action, Fig. c, is given by

$$132.4x - 66.9y = -237$$

2 By setting $y = 0$, we obtain $x = -1.792$ m, which agrees with our earlier calculation of the distance b.



Helpful Hints

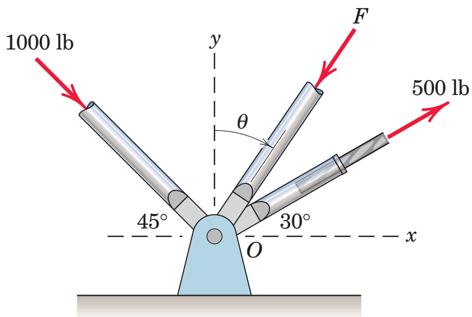
1 We note that the choice of point O as a moment center eliminates any moments due to the two forces which pass through O. Had the clockwise sign convention been adopted, M_O would have been +237 N·m, with the plus sign indicating a sense which agrees with the sign convention. Either sign convention, of course, leads to the conclusion of a clockwise moment M_O.

2 Note that the vector approach yields sign information automatically, whereas the scalar approach is more physically oriented. You should master both methods.

PROBLEMS

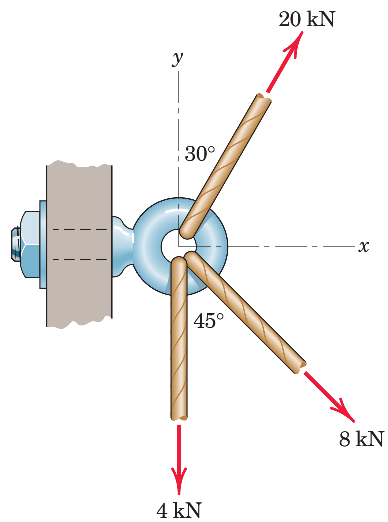
Introductory Problems

2/79 Two rods and one cable are attached to the support at O. If two of the forces are as shown, determine the magnitude F and direction θ of the third force so that the resultant of the three forces is vertically downward with a magnitude of 1200 lb.



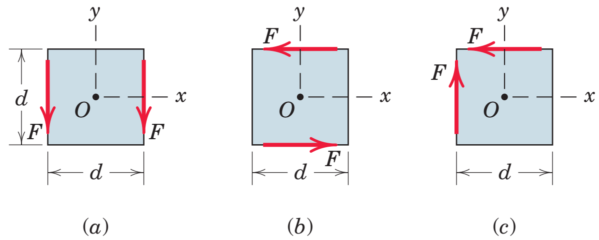
Problem 2/79

2/80 Determine the resultant $\mathbf{R}$ of the three tension forces acting on the eye bolt. Find the magnitude of $\mathbf{R}$ and the angle θ_x which $\mathbf{R}$ makes with the positive x-axis.



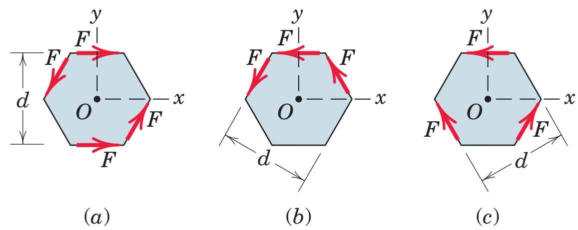
Problem 2/80

2/81 Determine the equivalent force–couple system at the center O for each of the three cases of forces being applied along the edges of a square plate of side d.



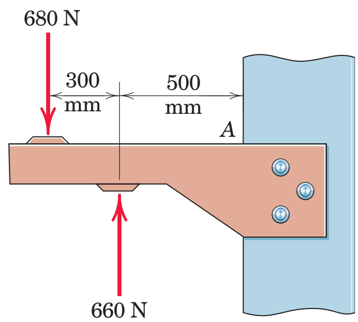
Problem 2/81

2/82 Determine the equivalent force–couple system at the origin O for each of the three cases of forces being applied along the edges of a regular hexagon of width d. If the resultant can be so expressed, replace this force–couple system with a single force.



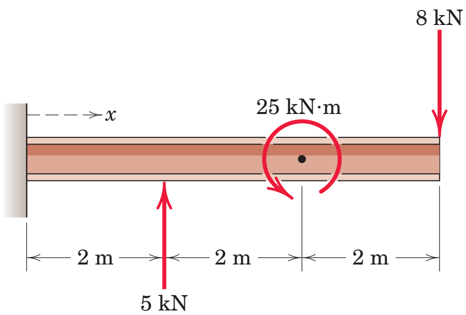
Problem 2/82

2/83 Where does the resultant of the two forces act?



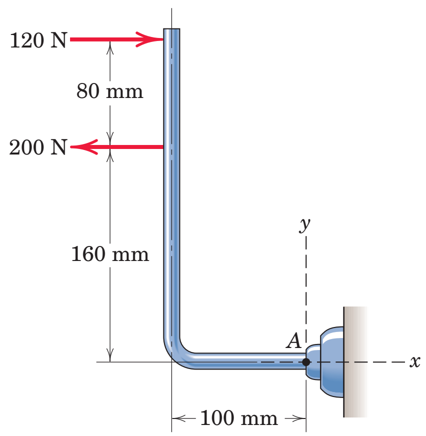
Problem 2/83

2/84 Determine and locate the resultant **R** of the two forces and one couple acting on the I-beam.



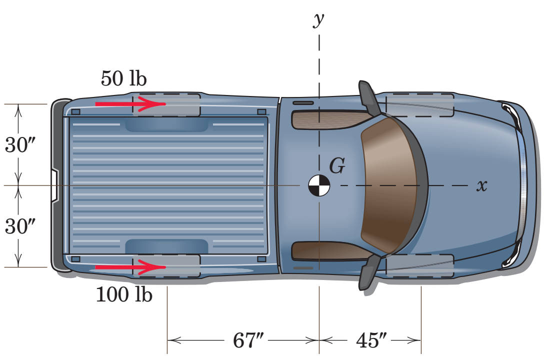
Problem 2/84

2/85 Replace the two forces acting on the bent pipe by a single equivalent force **R**. Specify the distance y from point A to the line of action of **R**.



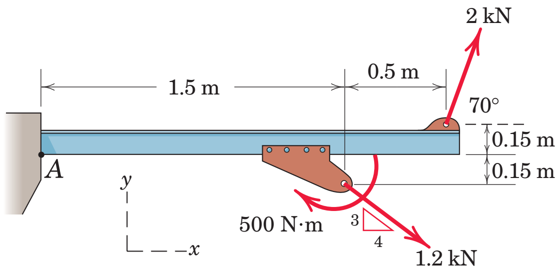
Problem 2/85

2/86 Under nonuniform and slippery road conditions, the two forces shown are exerted on the two rear-drive wheels of the pickup truck, which has a limited-slip rear differential. Determine the y-intercept of the resultant of this force system.



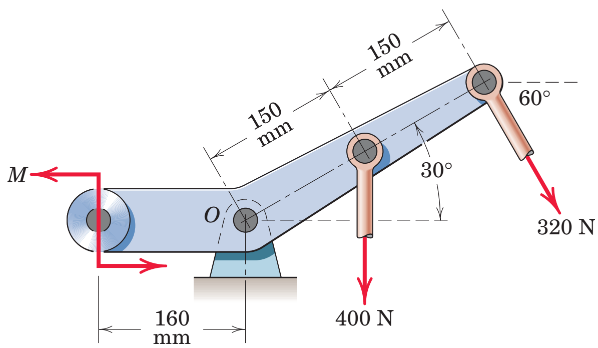
Problem 2/86

2/87 The flanged steel cantilever beam with riveted bracket is subjected to the couple and two forces shown, and their effect on the design of the attachment at A must be determined. Replace the two forces and couple by an equivalent couple M and resultant force **R** at A.



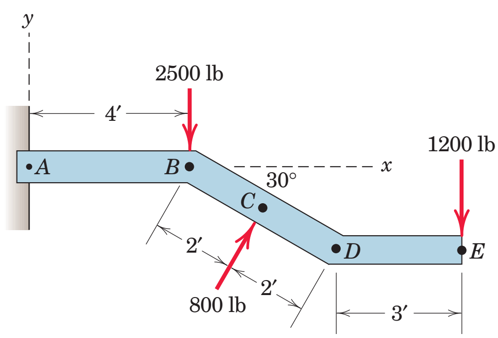
Problem 2/87

2/88 If the resultant of the two forces and couple M passes through point O, determine M.



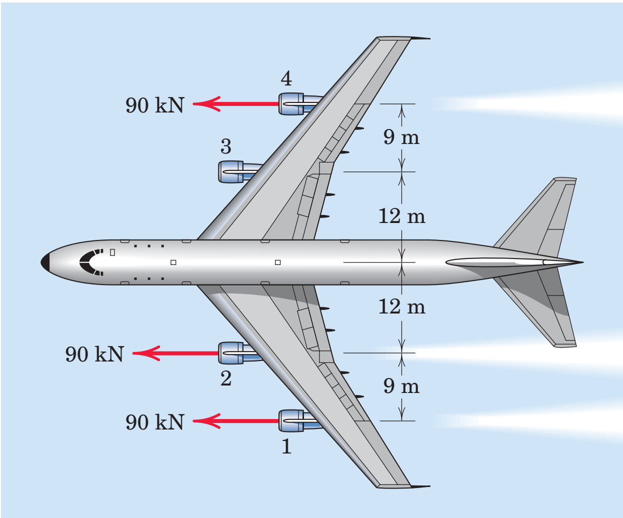
Problem 2/88

2/89 Replace the three forces which act on the bent bar by a force–couple system at the support point A. Then determine the x-intercept of the line of action of the stand-alone resultant force **R**.



Problem 2/89

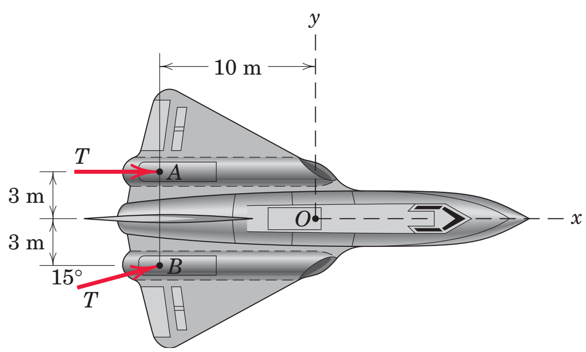
2/90 A commercial airliner with four jet engines, each producing 90 kN of forward thrust, is in a steady, level cruise when engine number 3 suddenly fails. Determine and locate the resultant of the three remaining engine thrust vectors. Treat this as a two-dimensional problem.



Problem 2/90

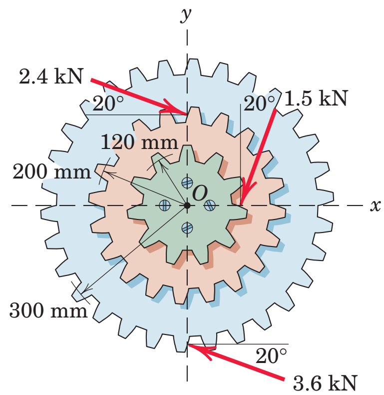
Representative Problems

2/91 The directions of the two thrust vectors of an experimental aircraft can be independently changed from the conventional forward direction within limits. For the thrust configuration shown, determine the equivalent force–couple system at point O. Then replace this force–couple system by a single force and specify the point on the x-axis through which the line of action of this resultant passes. These results are vital to assessing design performance.



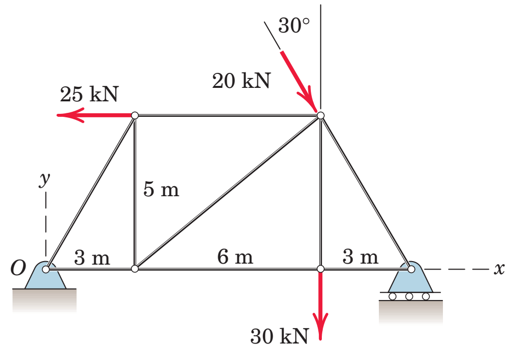
Problem 2/91

2/92 Determine the x- and y-axis intercepts of the line of action of the resultant of the three loads applied to the gearset.



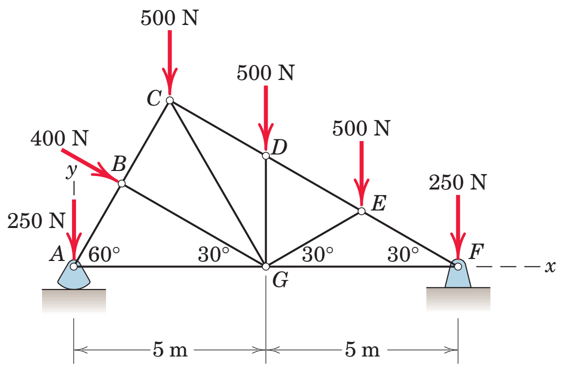
Problem 2/92

2/93 Determine the resultant $\mathbf{R}$ of the three forces acting on the simple truss. Specify the points on the x- and y-axes through which $\mathbf{R}$ must pass.



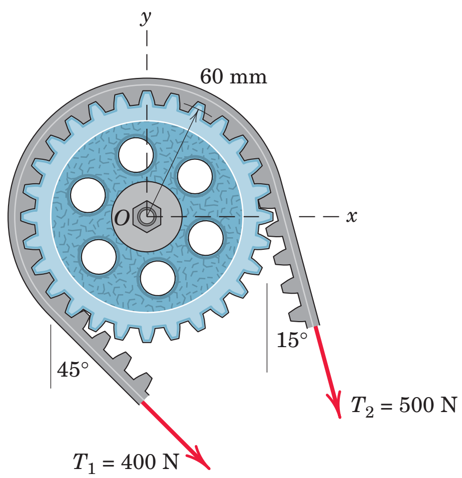
Problem 2/93

2/94 The asymmetric roof truss is of the type used when a near normal angle of incidence of sunlight onto the south-facing surface *ABC* is desirable for solar energy purposes. The five vertical loads represent the effect of the weights of the truss and supported roofing materials. The 400-N load represents the effect of wind pressure. Determine the equivalent force–couple system at *A*. Also, compute the *x*-intercept of the line of action of the system resultant treated as a single force **R**.



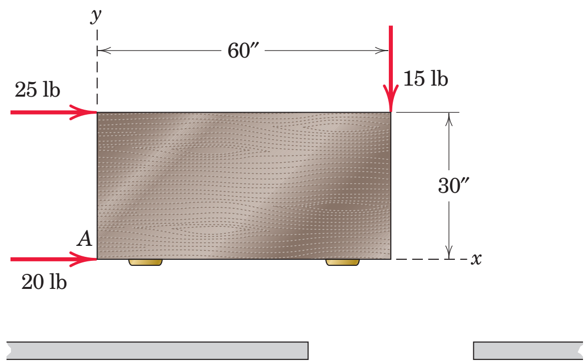
Problem 2/94

2/95 As part of a design test, the camshaft–drive sprocket is fixed and then the two forces shown are applied to a length of belt wrapped around the sprocket. Find the resultant of this system of two forces and determine where its line of action intersects both the *x*- and *y*-axes.



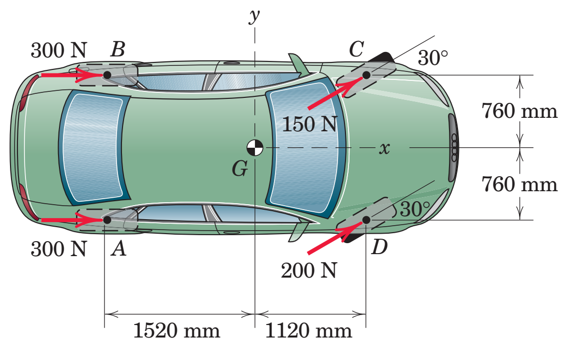
Problem 2/95

2/96 While sliding a desk toward the doorway, three students exert the forces shown in the overhead view. Determine the equivalent force–couple system at point *A*. Then determine the equation of the line of action of the resultant force.



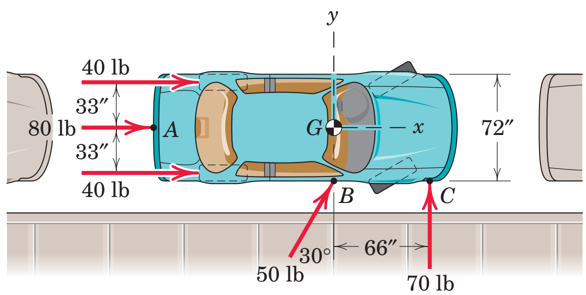
Problem 2/96

2/97 Under nonuniform and slippery road conditions, the four forces shown are exerted on the four drive wheels of the all-wheel-drive vehicle. Determine the resultant of this system and the *x*- and *y*-intercepts of its line of action. Note that the front and rear tracks are equal (i.e., $\overline{AB} = \overline{CD}$).



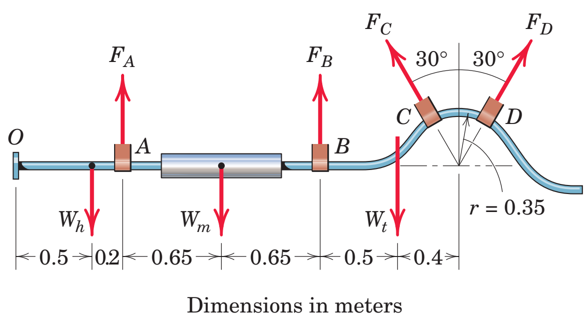
Problem 2/97

2/98 A rear-wheel-drive car is stuck in the snow between other parked cars as shown. In an attempt to free the car, three students exert forces on the car at points A, B, and C while the driver's actions result in a forward thrust of 40 lb acting parallel to the plane of rotation of each rear wheel. Treating the problem as two-dimensional, determine the equivalent force–couple system at the car center of mass G and locate the position x of the point on the car centerline through which the resultant passes. Neglect all forces not shown.



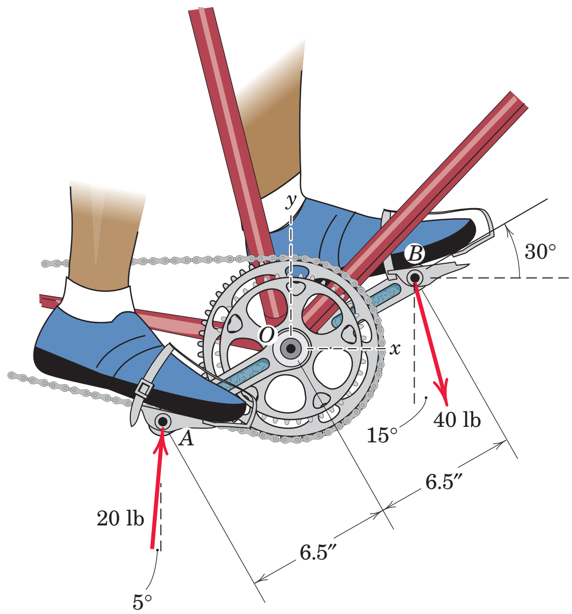
Problem 2/98

2/99 An exhaust system for a pickup truck is shown in the figure. The weights W_h, W_m, and W_t of the headpipe, muffler, and tailpipe are 10, 100, and 50 N, respectively, and act at the indicated points. If the exhaust-pipe hanger at point A is adjusted so that its tension F_A is 50 N, determine the required forces in the hangers at points B, C, and D so that the force–couple system at point O is zero. Why is a zero force–couple system at O desirable?



Problem 2/99

2/100 The pedal–chainwheel unit of a bicycle is shown in the figure. The left foot of the rider exerts the 40-lb force, while the use of toe clips allows the right foot to exert the nearly upward 20-lb force. Determine the equivalent force–couple system at point O. Also, determine the equation of the line of action of the system resultant treated as a single force $\mathbf{R}$. Treat the problem as two-dimensional.



Problem 2/100