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4th CLASS
LECTURE No. 4

CHAPTER

14

Shafts

14.1 Introduction

A shaft is a rotating machine element which is used to transmit power from one place to another. The power is delivered to the shaft by some tangential force and the resultant torque (or twisting moment) set up within the shaft permits the power to be transferred to various machines linked up to the shaft. In order to transfer the power from one shaft to another, the various members such as pulleys, gears etc., are mounted on it. These members along with the forces exerted upon them causes the shaft to bending. In other words, we may say that a shaft is used for the transmission of torque and bending moment. The various members are mounted on the shaft by means of keys or splines.

Notes: 1. The shafts are usually cylindrical, but may be square or cross-shaped in section. They are solid in cross-section but sometimes hollow shafts are also used.

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2. An ***axle,*** though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.

3. A ***spindle*** is a short shaft that imparts motion either to a cutting tool (*e.g.* drill press spindles) or to a work piece (*e.g.* lathe spindles).

14.2 Material Used for Shafts

The material used for shafts should have the following properties :

1. It should have high strength.
2. It should have good machinability.
3. It should have low notch sensitivity factor.
4. It should have good heat treatment properties.
5. It should have high wear resistant properties.

The material used for ordinary shafts is carbon steel of grades 40 C 8, 45 C 8, 50 C 4 and 50 C 12.

The mechanical properties of these grades of carbon steel are given in the following table.

Table 14.1. Mechanical properties of steels used for shafts.

Indian standard designation	*Ultimate tensile strength, MPa*	*Yield strength, MPa*
40 C 8	560 - 670	320
45 C 8	610 - 700	350
50 C 4	640 - 760	370
50 C 12	700 Min.	390

When a shaft of high strength is required, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

14.3 Manufacturing of Shafts

Shafts are generally manufactured by hot rolling and finished to size by cold drawing or turning and grinding. The cold rolled shafts are stronger than hot rolled shafts but with higher residual stresses. The residual stresses may cause distortion of the shaft when it is machined, especially when slots or keyways are cut. Shafts of larger diameter are usually forged and turned to size in a lathe.

14.4 Types of Shafts

The following two types of shafts are important from the subject point of view :

1. ***Transmission shafts.*** These shafts transmit power between the source and the machines absorbing power. The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts. Since these shafts carry machine parts such as pulleys, gears etc., therefore they are subjected to bending in addition to twisting.

2. ***Machine shafts.*** These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

14.5 Standard Sizes of Transmission Shafts

The standard sizes of transmission shafts are :

25 mm to 60 mm with 5 mm steps; 60 mm to 110 mm with 10 mm steps ; 110 mm to 140 mm with 15 mm steps ; and 140 mm to 500 mm with 20 mm steps.

The standard length of the shafts are 5 m, 6 m and 7 m.

14.6 Stresses in Shafts

The following stresses are induced in the shafts :

1. Shear stresses due to the transmission of torque (*i.e.* due to torsional load).

2. Bending stresses (tensile or compressive) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.

3. Stresses due to combined torsional and bending loads.

14.7 Maximum Permissible Working Stresses for Transmission Shafts

According to American Society of Mechanical Engineers (ASME) code for the design of transmission shafts, the maximum permissible working stresses in tension or compression may be taken as

(*a*) 112 MPa for shafts without allowance for keyways.

(*b*) 84 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible tensile stress (σ_t) may be taken as 60 per cent of the elastic limit in tension (σ_{el}), but not more than 36 per cent of the ultimate tensile strength (σ_u). In other words, the permissible tensile stress,

$$\sigma_t = 0.6\ \sigma_{el} \text{ or } 0.36\ \sigma_u, \text{ whichever is less.}$$

The maximum permissible shear stress may be taken as

(*a*) 56 MPa for shafts without allowance for key ways.

(*b*) 42 MPa for shafts with allowance for keyways.

For shafts purchased under definite physical specifications, the permissible shear stress (τ) may be taken as 30 per cent of the elastic limit in tension (σ_{el}) but not more than 18 per cent of the ultimate tensile strength (σ_u). In other words, the permissible shear stress,

$$\tau = 0.3\ \sigma_{el} \text{ or } 0.18\ \sigma_u, \text{ whichever is less.}$$

14.8 Design of Shafts

The shafts may be designed on the basis of

1. Strength, and **2.** Rigidity and stiffness.

In designing shafts on the basis of strength, the following cases may be considered :

(*a*) Shafts subjected to twisting moment or torque only,

(*b*) Shafts subjected to bending moment only,

(*c*) Shafts subjected to combined twisting and bending moments, and

(*d*) Shafts subjected to axial loads in addition to combined torsional and bending loads.

We shall now discuss the above cases, in detail, in the following pages.

14.9 Shafts Subjected to Twisting Moment Only

When the shaft is subjected to a twisting moment (or torque) only, then the diameter of the shaft may be obtained by using the torsion equation. We know that

$$\frac{T}{J} = \frac{\tau}{r} \qquad \text{...}(i)$$

where T = Twisting moment (or torque) acting upon the shaft,

J = Polar moment of inertia of the shaft about the axis of rotation,

τ = Torsional shear stress, and

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r = Distance from neutral axis to the outer most fibre

= $d / 2$; where d is the diameter of the shaft.

We know that for round solid shaft, polar moment of inertia,

$$J = \frac{\pi}{32} \times d^4$$

The equation ***(i)*** may now be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \times d^3 \qquad \text{...(ii)}$$

From this equation, we may determine the diameter of round solid shaft (d).

We also know that for hollow shaft, polar moment of inertia,

$$J = \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]$$

where d_o and d_i = Outside and inside diameter of the shaft, and $r = d_o / 2$.

Substituting these values in equation ***(i)***, we have

$$\frac{T}{\frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right]} = \frac{\tau}{\frac{d_o}{2}} \quad \text{or} \quad T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] \qquad \text{...(iii)}$$

Let k = Ratio of inside diameter and outside diameter of the shaft

= d_i / d_o

Now the equation ***(iii)*** may be written as

$$T = \frac{\pi}{16} \times \tau \times \frac{(d_o)^4}{d_o}\left[1 - \left(\frac{d_i}{d_o}\right)^4\right] = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) \qquad \text{...(iv)}$$

Shafts inside generators and motors are made to bear high torsional stresses.

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From the equations **(*iii*)** or **(*iv*)**, the outside and inside diameter of a hollow shaft may be determined.

It may be noted that

1. The hollow shafts are usually used in marine work. These shafts are stronger per kg of material and they may be forged on a mandrel, thus making the material more homogeneous than would be possible for a solid shaft.

When a hollow shaft is to be made equal in strength to a solid shaft, the twisting moment of both the shafts must be same. In other words, for the same material of both the shafts,

$$T = \frac{\pi}{16} \times \tau \left[\frac{(d_o)^4 - (d_i)^4}{d_o}\right] = \frac{\pi}{16} \times \tau \times d^3$$

$$\therefore \quad \frac{(d_o)^4 - (d_i)^4}{d_o} = d^3 \quad \text{or} \quad (d_o)^3 (1 - k^4) = d^3$$

2. The twisting moment (T) may be obtained by using the following relation :

We know that the power transmitted (in watts) by the shaft,

$$P = \frac{2\pi N \times T}{60} \quad \text{or} \quad T = \frac{P \times 60}{2\pi N}$$

where T = Twisting moment in N-m, and

N = Speed of the shaft in r.p.m.

3. In case of belt drives, the twisting moment (T) is given by

$$T = (T_1 - T_2)\, R$$

where T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively, and

R = Radius of the pulley.

Example 14.1. *A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.*

Solution. Given : N = 200 r.p.m. ; P = 20 kW = 20×10^3 W; τ = 42 MPa = 42 N/mm^2

Let d = Diameter of the shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 955 \times 10^3 / 8.25 = 115\ 733$ or d = 48.7 say 50 mm **Ans.**

Example 14.2. *A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.*

Solution. Given : P = 1 MW = 1×10^6 W ; N = 240 r.p.m. ; $T_{max} = 1.2\, T_{mean}$; τ = 60 MPa = 60 N/mm^2

Let d = Diameter of the shaft.

We know that mean torque transmitted by the shaft,

$$T_{mean} = \frac{P \times 60}{2\pi N} = \frac{1 \times 10^6 \times 60}{2\pi \times 240} = 39\ 784 \text{ N-m} = 39\ 784 \times 10^3 \text{ N-mm}$$

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∴ Maximum torque transmitted,

$$T_{max} = 1.2\, T_{mean} = 1.2 \times 39\,784 \times 10^3 = 47\,741 \times 10^3 \text{ N-mm}$$

We know that maximum torque transmitted (T_{max}),

$$47\,741 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.78\, d^3$$

∴ $d^3 = 47\,741 \times 10^3 / 11.78 = 4053 \times 10^3$

or $d = 159.4$ say 160 mm **Ans.**

Example 14.3. *Find the diameter of a solid steel shaft to transmit 20 kW at 200 r.p.m. The ultimate shear stress for the steel may be taken as 360 MPa and a factor of safety as 8.*

If a hollow shaft is to be used in place of the solid shaft, find the inside and outside diameter when the ratio of inside to outside diameters is 0.5.

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 200 r.p.m. ; τ_u = 360 MPa = 360 N/mm^2 ; $F.S.$ = 8 ; $k = d_i / d_o = 0.5$

We know that the allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{360}{8} = 45 \text{ N/mm}^2$$

Diameter of the solid shaft

Let d = Diameter of the solid shaft.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

We also know that torque transmitted by the solid shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\, d^3$$

∴ $d^3 = 955 \times 10^3 / 8.84 = 108\,032$ or $d = 47.6$ say 50 mm **Ans.**

Diameter of hollow shaft

Let d_i = Inside diameter, and

d_o = Outside diameter.

We know that the torque transmitted by the hollow shaft (T),

$$955 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4)$$

$$= \frac{\pi}{16} \times 45\, (d_o)^3\, [1 - (0.5)^4] = 8.3\, (d_o)^3$$

∴ $(d_o)^3 = 955 \times 10^3 / 8.3 = 115\,060$ or $d_o = 48.6$ say 50 mm **Ans.**

and $d_i = 0.5\, d_o = 0.5 \times 50 = 25$ mm **Ans.**

14.10 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation. We know that

$$\frac{M}{I} = \frac{\sigma_b}{y} \qquad \ldots(i)$$

where M = Bending moment,

I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation,

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σ_b = Bending stress, and

y = Distance from neutral axis to the outer-most fibre.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \quad \text{and} \quad y = \frac{d}{2}$$

Substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b \times d^3$$

From this equation, diameter of the solid shaft (d) may be obtained.

We also know that for a hollow shaft, moment of inertia,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right] = \frac{\pi}{64}(d_o)^4 (1 - k^4) \qquad \text{...(where } k = d_i / d_o\text{)}$$

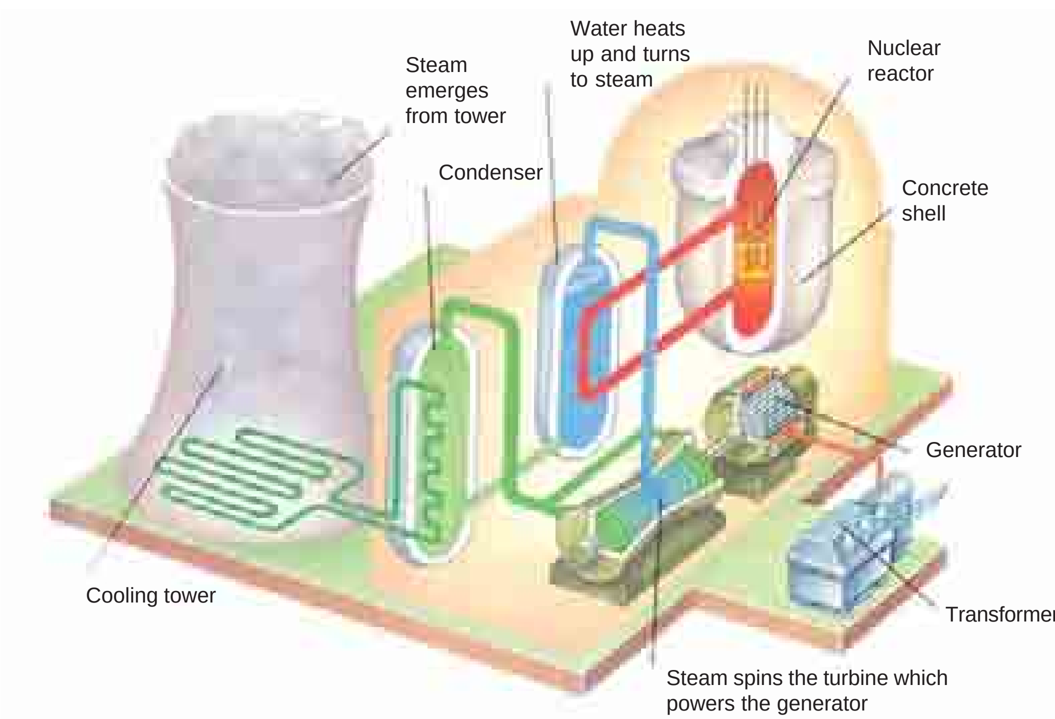
and $$y = d_o / 2$$

Again substituting these values in equation (*i*), we have

$$\frac{M}{\frac{\pi}{64}(d_o)^4 (1 - k^4)} = \frac{\sigma_b}{\frac{d_o}{2}} \quad \text{or} \quad M = \frac{\pi}{32} \times \sigma_b (d_o)^3 (1 - k^4)$$

From this equation, the outside diameter of the shaft (d_o) may be obtained.

Note: We have already discussed in Art. 14.1 that the axles are used to transmit bending moment only. Thus, axles are designed on the basis of bending moment only, in the similar way as discussed above.



In a neuclear power plant, stearm is generated using the heat of nuclear reactions. Remaining function of steam turbines and generators is same as in theraml power plants.

Example 14.4. *A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.*

Solution. Given : $W = 50$ kN $= 50 \times 10^3$ N ; $L = 100$ mm ; $x = 1.4$ m ; $\sigma_b = 100$ MPa $= 100$ N/mm^2

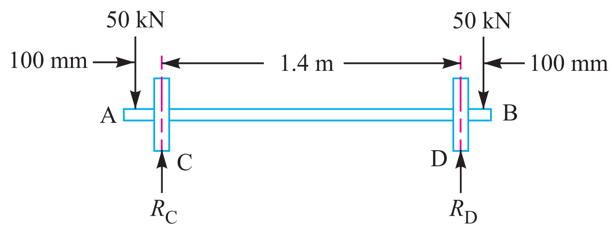


Fig. 14.1

The axle with wheels is shown in Fig. 14.1.

A little consideration will show that the maximum bending moment acts on the wheels at *C* and *D*. Therefore maximum bending moment,

$$*M = W.L = 50 \times 10^3 \times 100 = 5 \times 10^6 \text{ N-mm}$$

Let d = Diameter of the axle.

We know that the maximum bending moment (M),

$$5 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\, d^3$$

$$\therefore \quad d^3 = 5 \times 10^6 / 9.82 = 0.51 \times 10^6 \text{ or } d = 79.8 \text{ say } 80 \text{ mm}$$ **Ans.**

14.11 Shafts Subjected to Combined Twisting Moment and Bending Moment

When the shaft is subjected to combined twisting moment and bending moment, then the shaft must be designed on the basis of the two moments simultaneously. Various theories have been suggested to account for the elastic failure of the materials when they are subjected to various types of combined stresses. The following two theories are important from the subject point of view :

1. Maximum shear stress theory or Guest's theory. It is used for ductile materials such as mild steel.

2. Maximum normal stress theory or Rankine's theory. It is used for brittle materials such as cast iron.

Let τ = Shear stress induced due to twisting moment, and

σ_b = Bending stress (tensile or compressive) induced due to bending moment.

According to maximum shear stress theory, the maximum shear stress in the shaft,

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

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* The maximum B.M. may be obtained as follows :

$R_C = R_D = 50$ kN $= 50 \times 10^3$ N

B.M. at *A*, $M_A = 0$

B.M. at *C*, $M_C = 50 \times 10^3 \times 100 = 5 \times 10^6$ N-mm

B.M. at *D*, $M_D = 50 \times 10^3 \times 1500 - 50 \times 10^3 \times 1400 = 5 \times 10^6$ N-mm

B.M. at *B*, $M_B = 0$

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Substituting the values of τ and σ_b from Art. 14.9 and Art. 14.10, we have

$$\tau_{max} = \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2} = \frac{16}{\pi\,d^3}\left[\sqrt{M^2 + T^2}\right]$$

or $$\frac{\pi}{16} \times \tau_{max} \times d^3 = \sqrt{M^2 + T^2} \qquad ...(i)$$

The expression $\sqrt{M^2 + T^2}$ is known as ***equivalent twisting moment*** and is denoted by T_e. The equivalent twisting moment may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment. By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation (*i*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3 \qquad ...(ii)$$

From this expression, diameter of the shaft (d) may be evaluated.

Now according to maximum normal stress theory, the maximum normal stress in the shaft,

$$\sigma_{b(max)} = \frac{1}{2}\,\sigma_b + \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2} \qquad ...(iii)$$

$$= \frac{1}{2} \times \frac{32\,M}{\pi\,d^3} + \frac{1}{2}\sqrt{\left(\frac{32\,M}{\pi\,d^3}\right)^2 + 4\left(\frac{16\,T}{\pi\,d^3}\right)^2}$$

$$= \frac{32}{\pi\,d^3}\left[\frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right)\right]$$

or $$\frac{\pi}{32} \times \sigma_{b\,(max)} \times d^3 = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] \qquad ...(iv)$$

The expression $\frac{1}{2}\left[\left(M + \sqrt{M^2 + T^2}\right)\right]$ is known as ***equivalent bending moment*** and is denoted by M_e. The equivalent bending moment may be defined as **that moment which when acting alone produces the same tensile or compressive stress (σ_b) as the actual bending moment.** By limiting the maximum normal stress [$\sigma_{b(max)}$] equal to the allowable bending stress (σ_b), then the equation (*iv*) may be written as

$$M_e = \frac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \frac{\pi}{32} \times \sigma_b \times d^3 \qquad ...(v)$$

From this expression, diameter of the shaft (d) may be evaluated.

Notes: 1. In case of a hollow shaft, the equations (*ii*) and (*v*) may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4)$$

and $$M_e = \tfrac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{\pi}{32} \times \sigma_b\,(d_o)^3\,(1 - k^4)$$

2. It is suggested that diameter of the shaft may be obtained by using both the theories and the larger of the two values is adopted.

Example 14.5. *A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.*

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Solution. Given : M = 3000 N-m = 3×10^6 N-mm ; T = 10 000 N-m = 10×10^6 N-mm ; σ_{tu} = 700 MPa = 700 N/mm^2 ; τ_u = 500 MPa = 500 N/mm^2

We know that the allowable tensile stress,

$$\sigma_t \text{ or } \sigma_b = \frac{\sigma_{tu}}{F.S.} = \frac{700}{6} = 116.7 \text{ N/mm}^2$$

and allowable shear stress,

$$\tau = \frac{\tau_u}{F.S.} = \frac{500}{6} = 83.3 \text{ N/mm}^2$$

Let d = Diameter of the shaft in mm.

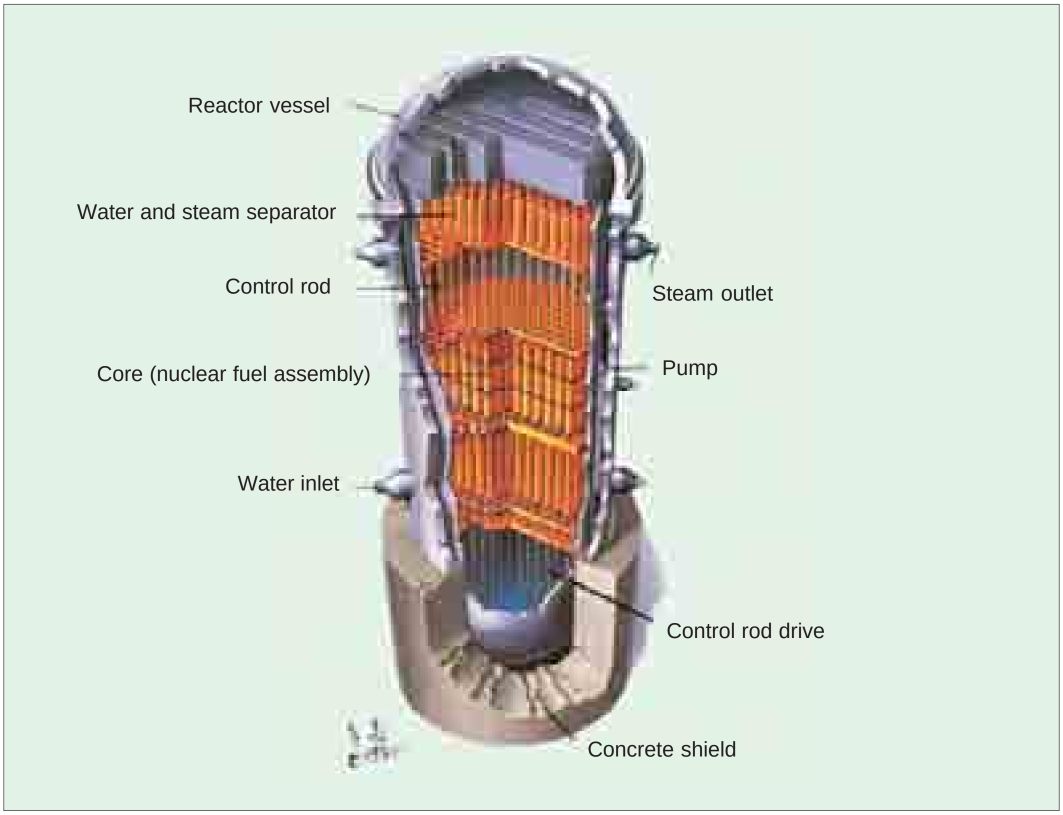
According to maximum shear stress theory, equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(3 \times 10^6)^2 + (10 \times 10^6)^2} = 10.44 \times 10^6 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$10.44 \times 10^6 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 83.3 \times d^3 = 16.36\, d^3$$

$\therefore$ $d^3 = 10.44 \times 10^6 / 16.36 = 0.636 \times 10^6$ or d = 86 mm



Nuclear Reactor

Note : This picture is given as additional information and is not a direct example of the current chapter.

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According to maximum normal stress theory, equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$

$$= \frac{1}{2}(3 \times 10^6 + 10.44 \times 10^6) = 6.72 \times 10^6 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$6.72 \times 10^6 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 116.7 \times d^3 = 11.46\, d^3$$

$$\therefore \quad d^3 = 6.72 \times 10^6 / 11.46 = 0.586 \times 10^6 \quad \text{or} \quad d = 83.7 \text{ mm}$$

Taking the larger of the two values, we have

d = 86 say 90 mm **Ans.**

Example 14.6. *A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm. The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.*

Solution. Given : P = 7.5 kW = 7500 W ; N = 300 r.p.m. ; D = 150 mm = 0.15 m ; L = 200 mm = 0.2 m ; τ = 45 MPa = 45 N/mm^2 ; α = 20°

Fig. 14.2 shows a shaft with a gear mounted on the bearings.

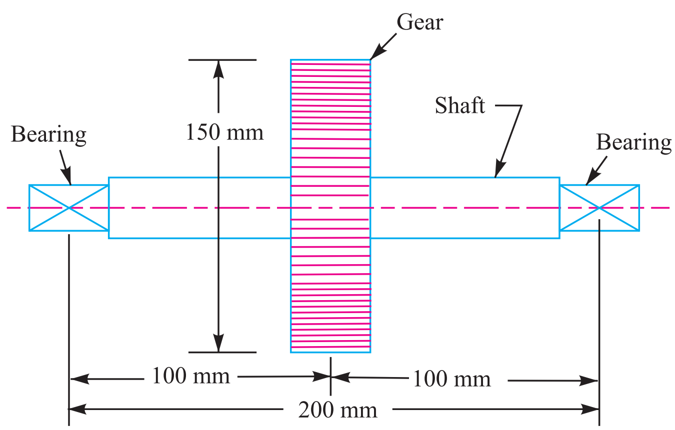

Fig. 14.2

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{7500 \times 60}{2\pi \times 300} = 238.7 \text{ N-m}$$

$\therefore$ Tangential force on the gear,

$$F_t = \frac{2T}{D} = \frac{2 \times 238.7}{0.15} = 3182.7 \text{ N}$$

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and the normal load acting on the tooth of the gear,

$$W = \frac{F_t}{\cos \alpha} = \frac{3182.7}{\cos 20°} = \frac{3182.7}{0.9397} = 3387 \text{ N}$$

Since the gear is mounted at the middle of the shaft, therefore maximum bending moment at the centre of the gear,

$$M = \frac{W.L}{4} = \frac{3387 \times 0.2}{4} = 169.4 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(169.4)^2 + (238.7)^2} = 292.7 \text{ N-m}$$
$$= 292.7 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$292.7 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 45 \times d^3 = 8.84\ d^3$$

$\therefore$ $d^3 = 292.7 \times 10^3 / 8.84 = 33 \times 10^3$ or $d = 32$ say 35 mm **Ans.**

Example 14.7. *A shaft made of mild steel is required to transmit 100 kW at 300 r.p.m. The supported length of the shaft is 3 metres. It carries two pulleys each weighing 1500 N supported at a distance of 1 metre from the ends respectively. Assuming the safe value of stress, determine the diameter of the shaft.*

Solution. Given : $P = 100$ kW $= 100 \times 10^3$ W ; $N = 300$ r.p.m. ; $L = 3$ m ; $W = 1500$ N

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{100 \times 10^3 \times 60}{2\pi \times 300} = 3183 \text{ N-m}$$

The shaft carrying the two pulleys is like a simply supported beam as shown in Fig. 14.3. The reaction at each support will be 1500 N, *i.e.*

$$R_A = R_B = 1500 \text{ N}$$

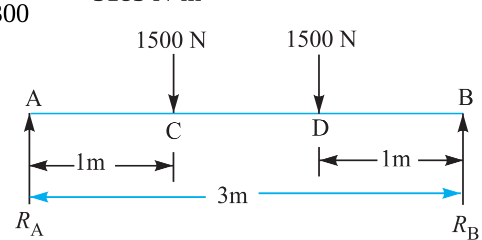

Fig. 14.3

A little consideration will show that the maximum bending moment lies at each pulley *i.e.* at *C* and *D*.

$\therefore$ Maximum bending moment,

$$M = 1500 \times 1 = 1500 \text{ N-m}$$

Let d = Diameter of the shaft in mm.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(1500)^2 + (3183)^2} = 3519 \text{ N-m}$$
$$= 3519 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$3519 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 60 \times d^3 = 11.8\ d^3 \quad \text{...(Assuming } \tau = 60 \text{ N/mm}^2\text{)}$$

$\therefore$ $d^3 = 3519 \times 10^3 / 11.8 = 298 \times 10^3$ or $d = 66.8$ say 70 mm **Ans.**

Example 14.8. *A line shaft is driven by means of a motor placed vertically below it. The pulley on the line shaft is 1.5 metre in diameter and has belt tensions 5.4 kN and 1.8 kN on the tight side and slack side of the belt respectively. Both these tensions may be assumed to be vertical. If the pulley be*

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overhang from the shaft, the distance of the centre line of the pulley from the centre line of the bearing being 400 mm, find the diameter of the shaft. Assuming maximum allowable shear stress of 42 MPa.

Solution . Given : D = 1.5 m or R = 0.75 m; T_1 = 5.4 kN = 5400 N ; T_2 = 1.8 kN = 1800 N ; L = 400 mm ; τ = 42 MPa = 42 N/mm^2

A line shaft with a pulley is shown in Fig 14.4.

We know that torque transmitted by the shaft,

$$T = (T_1 - T_2) R = (5400 - 1800)\, 0.75 = 2700 \text{ N-m}$$
$$= 2700 \times 10^3 \text{ N-mm}$$

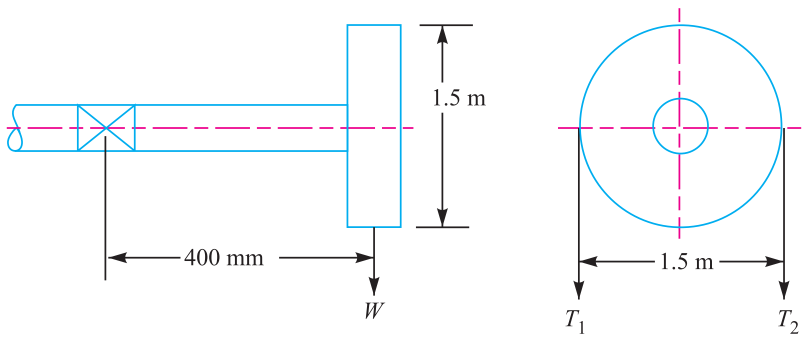

Fig. 14.4

Neglecting the weight of shaft, total vertical load acting on the pulley,

$$W = T_1 + T_2 = 5400 + 1800 = 7200 \text{ N}$$

∴ Bending moment, $M = W \times L = 7200 \times 400 = 2880 \times 10^3$ N-mm

Let d = Diameter of the shaft in mm.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(2880 \times 10^3)^2 + (2700 \times 10^3)^2}$$
$$= 3950 \times 10^3 \text{ N-mm}$$

Steel shaft

Top

We also know that equivalent twisting moment (T_e),

$$3950 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 3950 \times 10^3/8.25 = 479 \times 10^3$ or $d = 78$ say 80 mm **Ans.**

Example 14.9. *A shaft is supported by two bearings placed 1 m apart. A 600 mm diameter pulley is mounted at a distance of 300 mm to the right of left hand bearing and this drives a pulley directly below it with the help of belt having maximum tension of 2.25 kN. Another pulley 400 mm diameter is placed 200 mm to the left of right hand bearing and is driven with the help of electric motor and belt, which is placed horizontally to the right. The angle of contact for both the pulleys is 180° and μ = 0.24. Determine the suitable diameter for a solid shaft, allowing working stress of 63 MPa in tension and 42 MPa in shear for the material of shaft. Assume that the torque on one pulley is equal to that on the other pulley.*

Solution. Given : $AB = 1$ m ; $D_C = 600$ mm or $R_C = 300$ mm $= 0.3$ m ; $AC = 300$ mm $= 0.3$ m ; $T_1 = 2.25$ kN $= 2250$ N ; $D_D = 400$ mm or $R_D = 200$ mm $= 0.2$ m ; $BD = 200$ mm $= 0.2$ m ; $\theta = 180° = \pi$ rad ; $\mu = 0.24$; $\sigma_b = 63$ MPa $= 63$ N/mm² ; $\tau = 42$ MPa $= 42$ N/mm²

The space diagram of the shaft is shown in Fig. 14.5 (*a*).

Let T_1 = Tension in the tight side of the belt on pulley $C = 2250$ N ...(Given)

T_2 = Tension in the slack side of the belt on pulley C.

We know that

$$2.3 \log \left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.24 \times \pi = 0.754$$

$$\therefore \quad \log \left(\frac{T_1}{T_2}\right) = \frac{0.754}{2.3} = 0.3278 \quad \text{or} \quad \frac{T_1}{T_2} = 2.127 \quad \text{...(Taking antilog of 0.3278)}$$

and

$$T_2 = \frac{T_1}{2.127} = \frac{2250}{2.127} = 1058 \text{ N}$$

$\therefore$ Vertical load acting on the shaft at C,

$$W_C = T_1 + T_2 = 2250 + 1058 = 3308 \text{ N}$$

and vertical load on the shaft at D

$$= 0$$

The vertical load diagram is shown in Fig. 14.5 (*c*).

We know that torque acting on the pulley C,

$$T = (T_1 - T_2)\, R_C = (2250 - 1058)\, 0.3 = 357.6 \text{ N-m}$$

The torque diagram is shown in Fig. 14.5 (*b*).

Let T_3 = Tension in the tight side of the belt on pulley D, and

T_4 = Tension in the slack side of the belt on pulley D.

Since the torque on both the pulleys (*i.e.* C and D) is same, therefore

$$(T_3 - T_4)\, R_D = T = 357.6 \text{ N-m} \quad \text{or} \quad T_3 - T_4 = \frac{357.6}{R_D} = \frac{357.6}{0.2} = 1788 \text{ N} \quad \text{...(i)}$$

We know that

$$\frac{T_3}{T_4} = \frac{T_1}{T_2} = 2.127 \quad \text{or} \quad T_3 = 2.127\, T_4 \quad \text{...(ii)}$$

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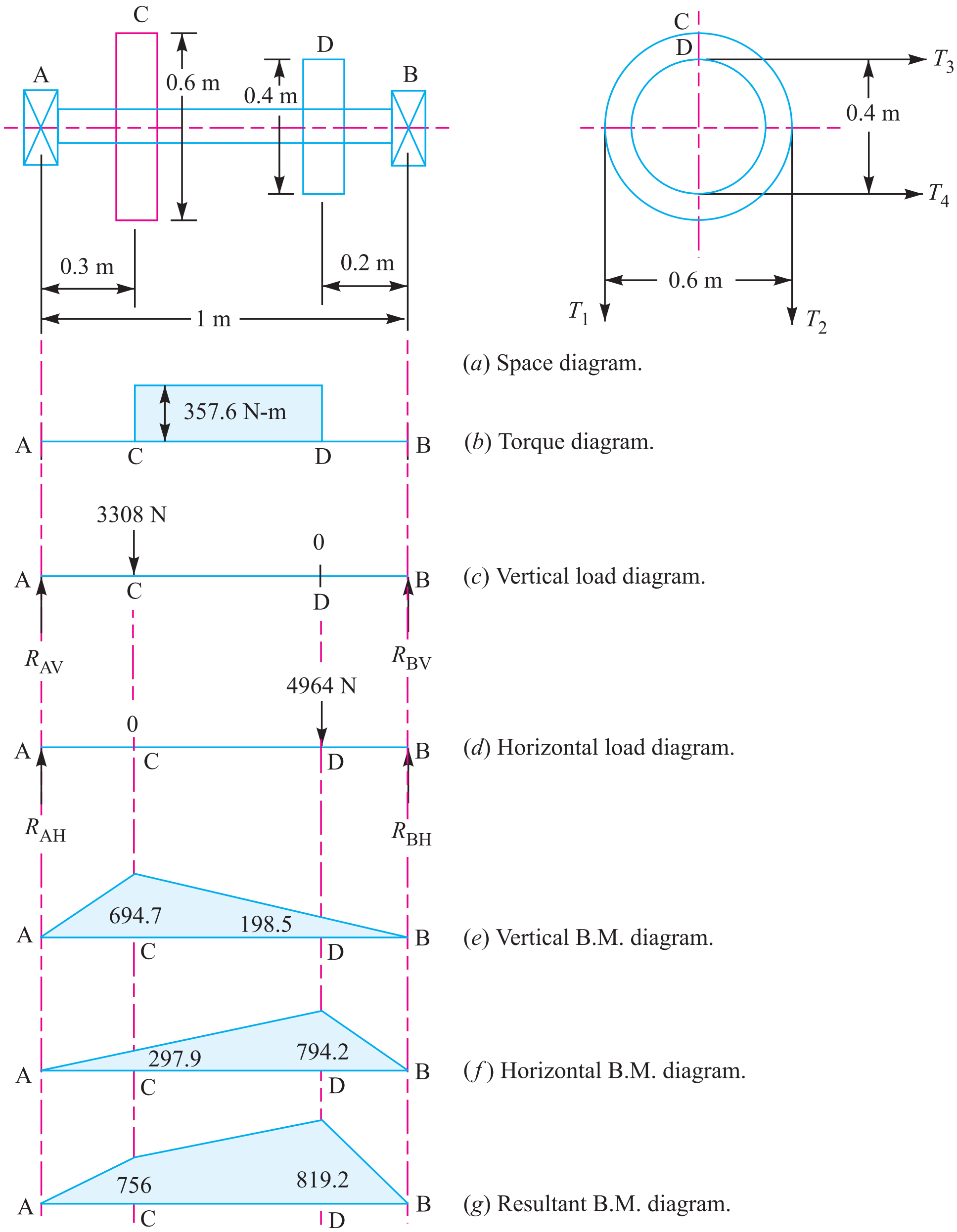

Fig. 14.5

From equations (*i*) and (*ii*), we find that

$$T_3 = 3376 \text{ N, and } T_4 = 1588 \text{ N}$$

∴ Horizontal load acting on the shaft at *D*,

$$W_D = T_3 + T_4 = 3376 + 1588 = 4964 \text{ N}$$

and horizontal load on the shaft at $C = 0$

The horizontal load diagram is shown in Fig. 14.5 (*d*).

Now let us find the maximum bending moment for vertical and horizontal loading.

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First of all, considering the vertical loading at *C*. Let R_{AV} and R_{BV} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AV} + R_{BV} = 3308 \text{ N}$$

Taking moments about *A*,

$$R_{BV} \times 1 = 3308 \times 0.3 \quad \text{or} \quad R_{BV} = 992.4 \text{ N}$$

and $$R_{AV} = 3308 - 992.4 = 2315.6 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

B.M. at *C*, $$M_{CV} = R_{AV} \times 0.3 = 2315.6 \times 0.3 = 694.7 \text{ N-m}$$

B.M. at *D*, $$M_{DV} = R_{BV} \times 0.2 = 992.4 \times 0.2 = 198.5 \text{ N-m}$$

The bending moment diagram for vertical loading in shown in Fig. 14.5 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 4964 \text{ N}$$

Taking moments about *A*,

$$R_{BH} \times 1 = 4964 \times 0.8 \quad \text{or} \quad R_{BH} = 3971 \text{ N}$$

and $$R_{AH} = 4964 - 3971 = 993 \text{ N}$$

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $$M_{CH} = R_{AH} \times 0.3 = 993 \times 0.3 = 297.9 \text{ N-m}$$

B.M. at *D*, $$M_{DH} = R_{BH} \times 0.2 = 3971 \times 0.2 = 794.2 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.5 (*f*).

Resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(694.7)^2 + (297.9)^2} = 756 \text{ N-m}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(198.5)^2 + (794.2)^2} = 819.2 \text{ N-m}$$

The resultant bending moment diagram is shown in Fig. 14.5 (*g*).

We see that bending moment is maximum at *D*.

∴ Maximum bending moment,

$$M = M_D = 819.2 \text{ N-m}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(819.2)^2 + (357.6)^2} = 894 \text{ N-m}$$
$$= 894 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$894 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

∴ $$d^3 = 894 \times 10^3 / 8.25 = 108 \times 10^3 \quad \text{or} \quad d = 47.6 \text{ mm}$$

Again we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left(M + \sqrt{M^2 + T^2}\right) = \frac{1}{2}(M + T_e)$$
$$= \frac{1}{2}(819.2 + 894) = 856.6 \text{ N-m} = 856.6 \times 10^3 \text{ N-mm}$$

Top

We also know that equivalent bending moment (M_e),

$$856.6 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 63 \times d^3 = 6.2\, d^3$$

$$\therefore \quad d^3 = 856.6 \times 10^3/6.2 = 138.2 \times 10^3 \quad \text{or} \quad d = 51.7 \text{ mm}$$

Taking larger of the two values, we have

$$d = 51.7 \text{ say } 55 \text{ mm} \quad \textbf{Ans.}$$

Example 14.10. *A shaft is supported on bearings A and B, 800 mm between centres. A 20° straight tooth spur gear having 600 mm pitch diameter, is located 200 mm to the right of the left hand bearing A, and a 700 mm diameter pulley is mounted 250 mm towards the left of bearing B. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of wrap. The pulley also serves as a flywheel and weighs 2000 N. The maximum belt tension is 3000 N and the tension ratio is 3 : 1. Determine the maximum bending moment and the necessary shaft diameter if the allowable shear stress of the material is 40 MPa.*

Solution. Given : AB = 800 mm ; α_C = 20° ; D_C = 600 mm or R_C = 300 mm ; AC = 200 mm ; D_D = 700 mm or R_D = 350 mm ; DB = 250 mm ; θ = 180° = π rad ; W = 2000 N ; T_1 = 3000 N ; $T_1/T_2 = 3$; τ = 40 MPa = 40 N/mm²

The space diagram of the shaft is shown in Fig. 14.6 (*a*).

We know that the torque acting on the shaft at D,

$$T = (T_1 - T_2)\, R_D = T_1 \left(1 - \frac{T_2}{T_1}\right) R_D$$

$$= 3000 \left(1 - \frac{1}{3}\right) 350 = 700 \times 10^3 \text{ N-mm} \qquad \ldots(\because\ T_1/T_2 = 3)$$

The torque diagram is shown in Fig. 14.6 (*b*).

Assuming that the torque at D is equal to the torque at C, therefore the tangential force acting on the gear C,

$$F_{tc} = \frac{T}{R_C} = \frac{700 \times 10^3}{300} = 2333 \text{ N}$$

and the normal load acting on the tooth of gear C,

$$W_C = \frac{F_{tc}}{\cos \alpha_C} = \frac{2333}{\cos 20°} = \frac{2333}{0.9397} = 2483 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.7. Resolving the normal load vertically and horizontally, we get

Vertical component of W_C *i.e.* the vertical load acting on the shaft at C,

$$W_{CV} = W_C \cos 20°$$
$$= 2483 \times 0.9397 = 2333 \text{ N}$$

and horizontal component of W_C *i.e.* the horizontal load acting on the shaft at C,

$$W_{CH} = W_C \sin 20°$$
$$= 2483 \times 0.342 = 849 \text{ N}$$

Since $T_1 / T_2 = 3$ and T_1 = 3000 N, therefore

$$T_2 = T_1 / 3 = 3000 / 3 = 1000 \text{ N}$$

Camshaft

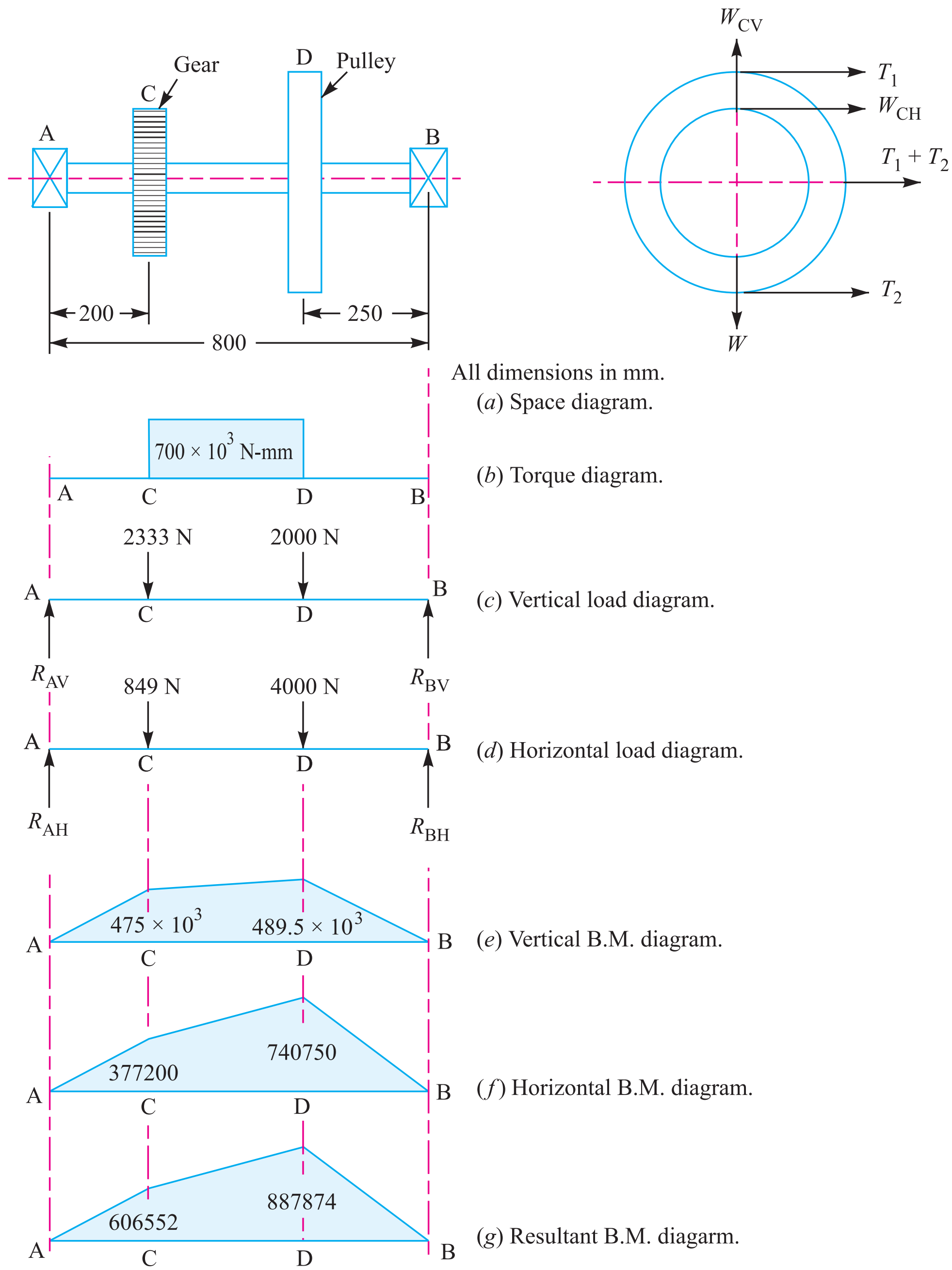

Fig. 14.6

∴ Horizontal load acting on the shaft at *D*,

$$W_{DH} = T_1 + T_2 = 3000 + 1000 = 4000 \text{ N}$$

and vertical load acting on the shaft at *D*,

$$W_{DV} = W = 2000 \text{ N}$$

The vertical and horizontal load diagram at C and D is shown in Fig. 14.6 (c) and (d) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all considering the vertical loading at C and D. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2333 + 2000 = 4333 \text{ N}$$

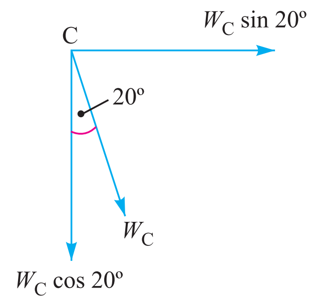

Fig. 14.7

Taking moments about A, we get

$$R_{BV} \times 800 = 2000\,(800 - 250) + 2333 \times 200 = 1\,566\,600$$

$$\therefore \quad R_{BV} = 1\,566\,600 / 800 = 1958 \text{ N}$$

and $$R_{AV} = 4333 - 1958 = 2375 \text{ N}$$

We know that B.M. at A and B,

$$M_{AV} = M_{BV} = 0$$

B.M. at C, $$M_{CV} = R_{AV} \times 200 = 2375 \times 200 = 475 \times 10^3 \text{ N-mm}$$

B.M. at D, $$M_{DV} = R_{BV} \times 250 = 1958 \times 250 = 489.5 \times 10^3 \text{ N-mm}$$

The bending moment diagram for vertical loading is shown in Fig. 14.6 (e).

Now consider the horizontal loading at C and D. Let R_{AH} and R_{BH} be the reactions at the bearings A and B respectively. We know that

$$R_{AH} + R_{BH} = 849 + 4000 = 4849 \text{ N}$$

Taking moments about A, we get

$$R_{BH} \times 800 = 4000\,(800 - 250) + 849 \times 200 = 2\,369\,800$$

$$\therefore \quad R_{BH} = 2\,369\,800 / 800 = 2963 \text{ N}$$

and $$R_{AH} = 4849 - 2963 = 1886 \text{ N}$$

We know that B.M. at A and B,

$$M_{AH} = M_{BH} = 0$$

B.M. at C, $$M_{CH} = R_{AH} \times 200 = 1886 \times 200 = 377\,200 \text{ N-mm}$$

B.M. at D, $$M_{DH} = R_{BH} \times 250 = 2963 \times 250 = 740\,750 \text{ N-mm}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.6 (f).

We know that resultant B.M. at C,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(475 \times 10^3)^2 + (377\,200)^2} = 606\,552 \text{ N-mm}$$

and resultant B.M. at D,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(489.5 \times 10^3)^2 + (740\,750)^2} = 887\,874 \text{ N-mm}$$

Maximum bending moment

The resultant B.M. diagram is shown in Fig. 14.6 (g). We see that the bending moment is maximum at D, therefore

Maximum B.M., $$M = M_D = 887\,874 \text{ N-mm}$$ **Ans.**

Diameter of the shaft

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(887\ 874)^2 + (700 \times 10^3)^2} = 1131 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1131 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 40 \times d^3 = 7.86\ d^3$$

$\therefore$ $d^3 = 1131 \times 10^3 / 7.86 = 144 \times 10^3$ or $d = 52.4$ say 55 mm **Ans.**

Example 14.11. *A steel solid shaft transmitting 15 kW at 200 r.p.m. is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 30 teeth of 5 mm module is located 100 mm to the left of the right hand bearing and delivers power horizontally to the right. The gear having 100 teeth of 5 mm module is located 150 mm to the right of the left hand bearing and receives power in a vertical direction from below. Using an allowable stress of 54 MPa in shear, determine the diameter of the shaft.*

Solution. Given : P = 15 kW = 15 × 10³ W ; N = 200 r.p.m. ; AB = 750 mm ; T_D = 30 ; m_D = 5 mm ; BD = 100 mm ; T_C = 100 ; m_C = 5 mm ; AC = 150 mm ; τ = 54 MPa = 54 N/mm²

The space diagram of the shaft is shown in Fig. 14.8 (*a*).

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{15 \times 10^3 \times 60}{2\pi \times 200} = 716 \text{ N-m} = 716 \times 10^3 \text{ N-mm}$$

The torque diagram is shown in Fig. 14.8 (*b*).

We know that diameter of gear

= No. of teeth on the gear × module

$\therefore$ Radius of gear C,

$$R_C = \frac{T_C \times m_C}{2} = \frac{100 \times 5}{2} = 250 \text{ mm}$$

and radius of pinion D,

$$R_D = \frac{T_D \times m_D}{2} = \frac{30 \times 5}{2} = 75 \text{ mm}$$

Assuming that the torque at C and D is same (*i.e.* 716 × 10³ N-mm), therefore tangential force on the gear C, acting downward,

$$F_{tC} = \frac{T}{R_C} = \frac{716 \times 10^3}{250} = 2870 \text{ N}$$

and tangential force on the pinion D, acting horizontally,

$$F_{tD} = \frac{T}{R_D} = \frac{716 \times 10^3}{75} = 9550 \text{ N}$$

The vertical and horizontal load diagram is shown in Fig. 14.8 (*c*) and (*d*) respectively.

Now let us find the maximum bending moment for vertical and horizontal loading.

First of all, considering the vertical loading at C. Let R_{AV} and R_{BV} be the reactions at the bearings A and B respectively. We know that

$$R_{AV} + R_{BV} = 2870 \text{ N}$$

Taking moments about A, we get

$$R_{BV} \times 750 = 2870 \times 150$$

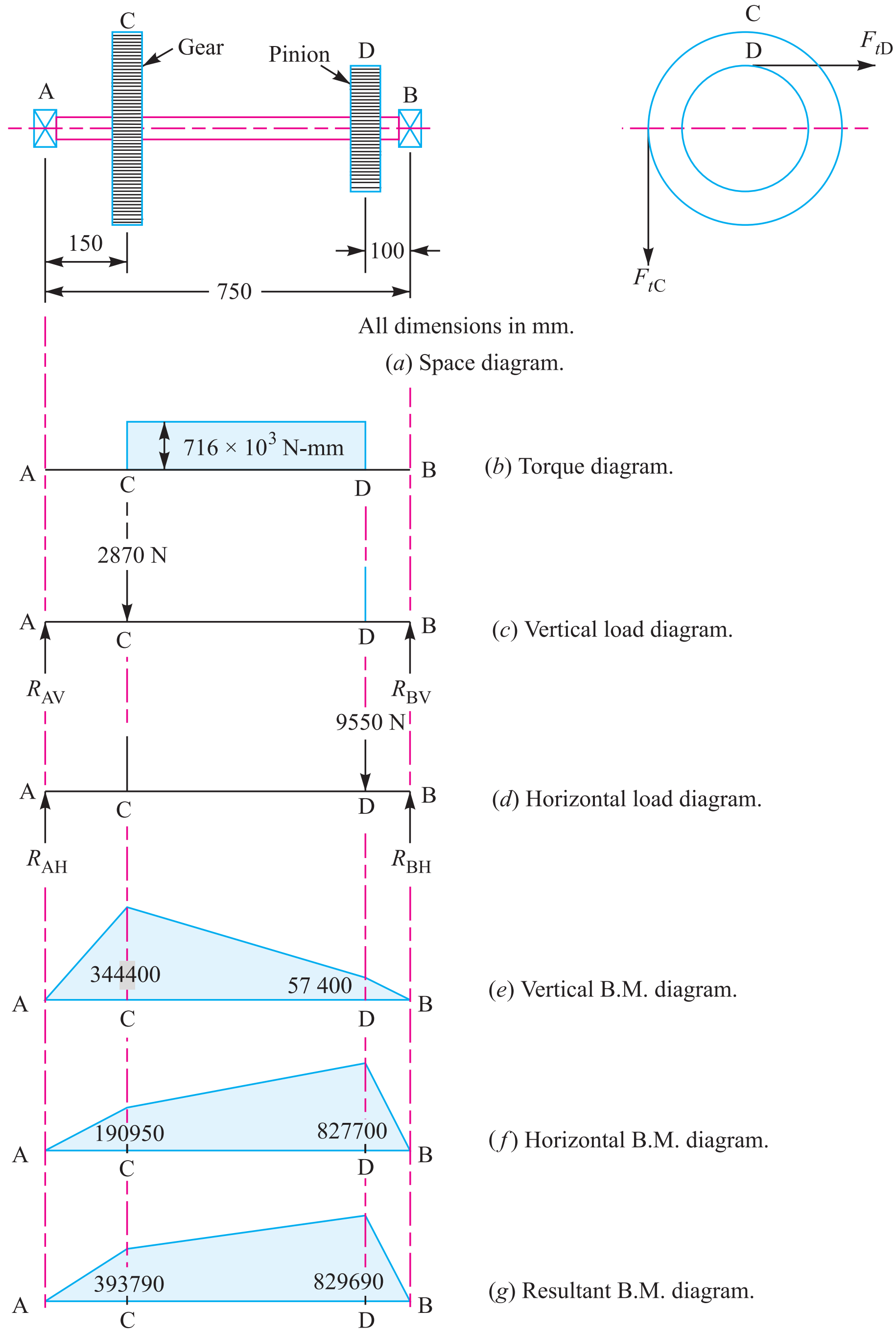

Fig. 14.8

$\therefore \qquad R_{BV} = 2870 \times 150 / 750 = 574$ N

and $\quad R_{AV} = 2870 - 574 = 2296$ N

We know that B.M. at *A* and *B*,

$$M_{AV} = M_{BV} = 0$$

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B.M. at *C*, $M_{CV} = R_{AV} \times 150 = 2296 \times 150 = 344\ 400$ N-mm

B.M. at *D*, $M_{DV} = R_{BV} \times 100 = 574 \times 100 = 57\ 400$ N-mm

The B.M. diagram for vertical loading is shown in Fig. 14.8 (*e*).

Now considering horizontal loading at *D*. Let R_{AH} and R_{BH} be the reactions at the bearings *A* and *B* respectively. We know that

$$R_{AH} + R_{BH} = 9550 \text{ N}$$

Taking moments about *A*, we get

$$R_{BH} \times 750 = 9550\ (750 - 100) = 9550 \times 650$$

$$\therefore \quad R_{BH} = 9550 \times 650 / 750 = 8277 \text{ N}$$

and $R_{AH} = 9550 - 8277 = 1273$ N

We know that B.M. at *A* and *B*,

$$M_{AH} = M_{BH} = 0$$

B.M. at *C*, $M_{CH} = R_{AH} \times 150 = 1273 \times 150 = 190\ 950$ N-mm

B.M. at *D*, $M_{DH} = R_{BH} \times 100 = 8277 \times 100 = 827\ 700$ N-mm

The B.M. diagram for horizontal loading is shown in Fig. 14.8 (*f*).

We know that resultant B.M. at *C*,

$$M_C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(344\ 400)^2 + (190\ 950)^2}$$
$$= 393\ 790 \text{ N-mm}$$

and resultant B.M. at *D*,

$$M_D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(57\ 400)^2 + (827\ 700)^2}$$
$$= 829\ 690 \text{ N-mm}$$

The resultant B.M. diagram is shown in Fig. 14.8 (*g*). We see that the bending moment is maximum at *D*.

$\therefore$ Maximum bending moment,

$$M = M_D = 829\ 690 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(829\ 690)^2 + (716 \times 10^3)^2} = 1096 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

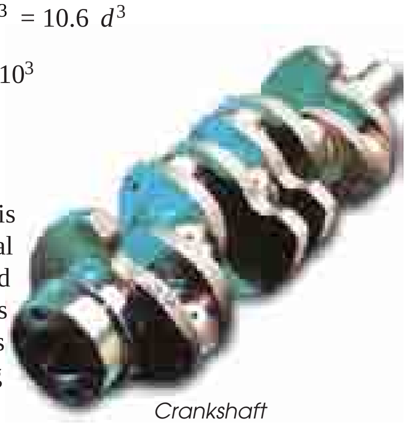
$$1096 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 54 \times d^3 = 10.6\ d^3$$

$$\therefore \quad d^3 = 1096 \times 10^3 / 10.6 = 103.4 \times 10^3$$

or d = 47 say 50 mm **Ans.**

14.12 Shafts Subjected to Fluctuating Loads

In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment. But in actual practice, the shafts are subjected to fluctuating torque and bending moments. In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment (*T*) and bending moment (*M*). Thus for a shaft



Crankshaft

Top

subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

where K_m = Combined shock and fatigue factor for bending, and

K_t = Combined shock and fatigue factor for torsion.

The following table shows the recommended values for K_m and K_t.

Table 14.2. Recommended values for K_m and K_t.

Nature of load	K_m	K_t
1. Stationary shafts		
(*a*) Gradually applied load	1.0	1.0
(*b*) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(*a*) Gradually applied or steady load	1.5	1.0
(*b*) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(*c*) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 14.12. *A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?*

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 200 r.p.m. ; W = 900 N ; L = 2.5 m ; τ = 42 MPa = 42 N/mm^2 ; σ_b = 56 MPa = 56 N/mm^2

Size of the shaft

Let d = Diameter of the shaft, in mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 200} = 955 \text{ N-m} = 955 \times 10^3 \text{ N-mm}$$

and maximum bending moment of a simply supported shaft carrying a central load,

$$M = \frac{W \times L}{4} = \frac{900 \times 2.5}{4} = 562.5 \text{ N-m} = 562.5 \times 10^3 \text{ N-mm}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{M^2 + T^2} = \sqrt{(562.5 \times 10^3)^2 + (955 \times 10^3)^2}$$

$$= 1108 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1108 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $d^3 = 1108 \times 10^3 / 8.25 = 134.3 \times 10^3$ or $d = 51.2$ mm

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[M + \sqrt{M^2 + T^2}\right] = \tfrac{1}{2}(M + T_e)$$

$$= \tfrac{1}{2}(562.5 \times 10^3 + 1108 \times 10^3) = 835.25 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$835.25 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$\therefore$ $d^3 = 835.25 \times 10^3 / 5.5 = 152 \times 10^3$ or $d = 53.4$ mm

Taking the larger of the two values, we have

$d = 53.4$ say 55 mm **Ans.**

Size of the shaft when subjected to gradually applied load

Let d = Diameter of the shaft.

From Table 14.2, for rotating shafts with gradually applied loads,

$K_m = 1.5$ and $K_t = 1$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 562.5 \times 10^3)^2 + (1 \times 955 \times 10^3)^2} = 1274 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$1274 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\, d^3$$

$\therefore$ $d^3 = 1274 \times 10^3 / 8.25 = 154.6 \times 10^3$ or $d = 53.6$ mm

We know that the equivalent bending moment,

$$M_e = \tfrac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \tfrac{1}{2}\left[K_m \times M + T_e\right]$$

$$= \tfrac{1}{2}\left[1.5 \times 562.5 \times 10^3 + 1274 \times 10^3\right] = 1059 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$1059 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 56 \times d^3 = 5.5\, d^3$$

$\therefore$ $d^3 = 1059 \times 10^3 / 5.5 = 192.5 \times 10^3 = 57.7$ mm

Taking the larger of the two values, we have

$d = 57.7$ say 60 mm **Ans.**

Example 14.13. *Design a shaft to transmit power from an electric motor to a lathe head stock through a pulley by means of a belt drive. The pulley weighs 200 N and is located at 300 mm from the centre of the bearing. The diameter of the pulley is 200 mm and the maximum power transmitted is 1 kW at 120 r.p.m. The angle of lap of the belt is 180° and coefficient of friction between the belt and the pulley is 0.3. The shock and fatigue factors for bending and twisting are 1.5 and 2.0 respectively. The allowable shear stress in the shaft may be taken as 35 MPa.*

Solution. Given : W = 200 N ; L = 300 mm ; D = 200 mm or R = 100 mm ; P = 1 kW = 1000 W ; N = 120 r.p.m. ; θ = 180° = π rad ; μ = 0.3 ; K_m = 1.5 ; K_t = 2 ; τ = 35 MPa = 35 N/mm^2

The shaft with pulley is shown in Fig. 14.9.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{1000 \times 60}{2\pi \times 120} = 79.6 \text{ N-m} = 79.6 \times 10^3 \text{ N-mm}$$

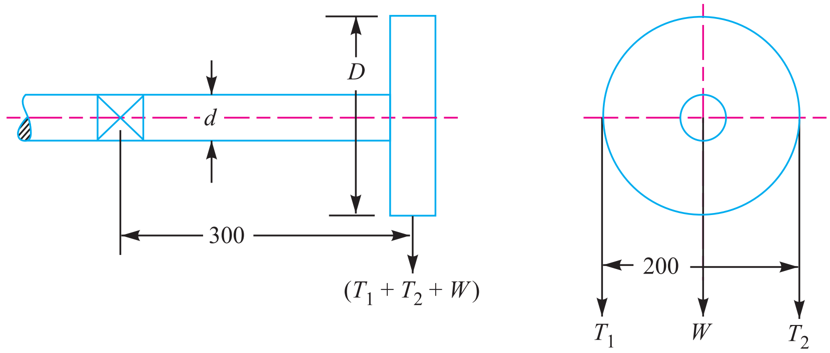


Fig. 14.9

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt respectively in newtons.

∴ Torque transmitted (T),

$$79.6 \times 10^3 = (T_1 - T_2) R = (T_1 - T_2)\, 100$$

$$\therefore \quad T_1 - T_2 = 79.6 \times 10^3 / 100 = 796 \text{ N} \qquad ...(i)$$

We know that

$$2.3 \log\left(\frac{T_1}{T_2}\right) = \mu.\theta = 0.3\,\pi = 0.9426$$

$$\therefore \quad \log\left(\frac{T_1}{T_2}\right) = \frac{0.9426}{2.3} = 0.4098 \text{ or } \frac{T_1}{T_2} = 2.57 \qquad ...(ii)$$

...(Taking antilog of 0.4098)

From equations ***(i)*** and ***(ii)***, we get,

$$T_1 = 1303 \text{ N, and } T_2 = 507 \text{ N}$$

We know that the total vertical load acting on the pulley,

$$W_T = T_1 + T_2 + W = 1303 + 507 + 200 = 2010 \text{ N}$$

∴ Bending moment acting on the shaft,

$$M = W_T \times L = 2010 \times 300 = 603 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t + T)^2}$$

$$= \sqrt{(1.5 \times 603 \times 10^3)^2 + (2 \times 79.6 \times 10^3)^2} = 918 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$918 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 35 \times d^3 = 6.87\, d^3$$

$$\therefore \quad d^3 = 918 \times 10^3 / 6.87 = 133.6 \times 10^3 \text{ or } d = 51.1 \text{ say } 55 \text{ mm}$$ **Ans.**

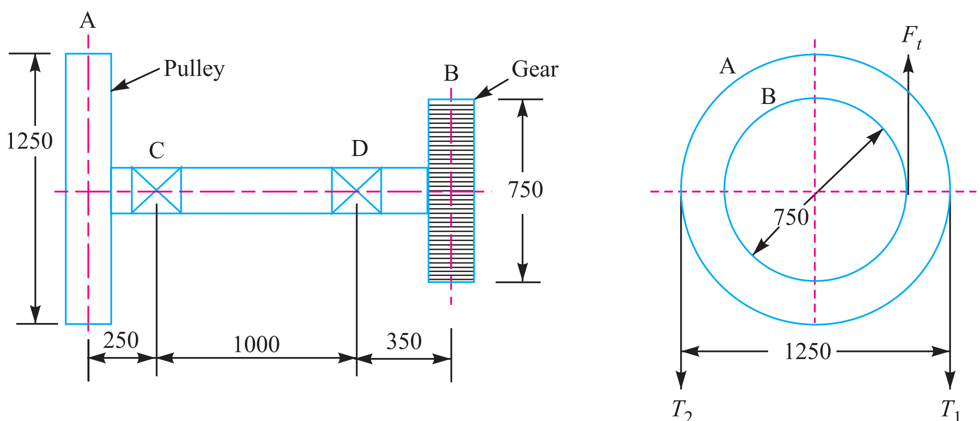
Example 14.14. *Fig. 14.10 shows a shaft carrying a pulley A and a gear B and supported in two bearings C and D. The shaft transmits 20 kW at 150 r.p.m. The tangential force F_t on the gear B acts vertically upwards as shown.*

The pulley delivers the power through a belt to another pulley of equal diameter vertically below the pulley A. The ratio of tensions T_1 / T_2 is equal to 2.5. The gear and the pulley weigh 900 N and 2700 N respectively. The permissible shear stress for the material of the shaft may be taken as 63 MPa. Assuming the weight of the shaft to be negligible in comparison with the other loads, determine its diameter. Take shock and fatigue factors for bending and torsion as 2 and 1.5 respectively.

Solution. Given : P = 20 kW = 20 × 10^3 W ; N = 150 r.p.m. ; T_1/T_2 = 2.5 ; W_B = 900 N ; W_A = 2700 N ; τ = 63 MPa = 63 N/mm^2 ; K_m = 2 ; K_t = 1.5 ; D_B = 750 mm or R_B = 375 mm ; D_A = 1250 mm or R_A = 625 mm.

We know that torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 150} = 1273 \text{ N-m} = 1273 \times 10^3 \text{ N-mm}$$



All dimensions in mm.

Fig. 14.10

Let T_1 and T_2 = Tensions in the tight side and slack side of the belt on pulley A.

Since the torque on the pulley is same as that of shaft (*i.e.* 1273 × 10^3 N-mm), therefore

$(T_1 - T_2) R_A$ = 1273 × 10^3 or $T_1 - T_2$ = 1273 × 10^3 / 625 = 2037 N ...(*i*)

Since T_1 / T_2 = 2.5 or T_1 = 2.5 T_2, therefore

2.5 $T_2 - T_2$ = 2037 or T_2 = 2037/1.5 = 1358 N ...[From equation (*i*)]

and T_1 = 2.5 × 1358 = 3395 N

∴ Total vertical load acting downward on the shaft at A

$$= T_1 + T_2 + W_A = 3395 + 1358 + 2700 = 7453 \text{ N}$$

Assuming that the torque on the gear B is same as that of the shaft, therefore the tangential force acting vertically upward on the gear B,

$$F_t = \frac{T}{R_B} = \frac{1273 \times 10^3}{375} = 3395 \text{ N}$$

Since the weight of gear B (W_B = 900 N) acts vertically downward, therefore the total vertical load acting upward on the shaft at B

$$= F_t - W_B = 3395 - 900 = 2495 \text{ N}$$

Now let us find the reactions at the bearings C and D. Let R_C and R_D be the reactions at C and D respectively. A little consideration will show that the reaction R_C will act upward while the reaction R_D act downward as shown in Fig. 14.11.

Taking moments about *D*, we get

$$R_C \times 1000 = 7453 \times 1250 + 2495 \times 350 = 10.2 \times 10^6$$

$$\therefore \quad R_C = 10.2 \times 10^6 / 1000 = 10\ 200 \text{ N}$$

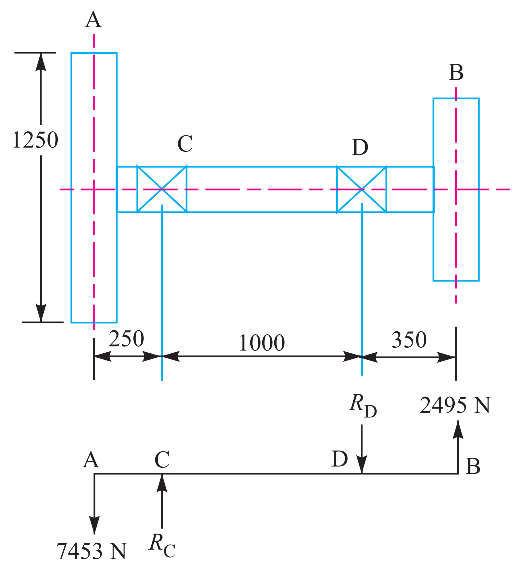


Fig. 14.11

For the equilibrium of the shaft,

$$R_D + 7453 = R_C + 2495 = 10\ 200 + 2495 = 12\ 695$$

$$\therefore \quad R_D = 12\ 695 - 7453 = 5242 \text{ N}$$

We Know that B.M. at *A* and *B*

$$= 0$$

B.M. at *C* $= 7453 \times 250 = 1863 \times 10^3$ N-mm

B.M. at *D* $= 2495 \times 350 = 873 \times 10^3$ N-mm

We see that the bending moment is maximum at *C*.

$\therefore$ Maximum B.M. $= M = M_C = 1863 \times 10^3$ N-mm

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1863 \times 10^3)^2 + (1.5 \times 1273 \times 10^3)^2}$$

$$= 4187 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$4187 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 63 \times d^3 = 12.37\ d^3.$$

$$\therefore \quad d^3 = 4187 \times 10^3 / 12.37 = 338 \times 10^3$$

or $\quad d = 69.6$ say 70 mm **Ans.**

Example 14.15. *A horizontal nickel steel shaft rests on two bearings, A at the left and B at the right end and carries two gears C and D located at distances of 250 mm and 400 mm respectively from the centre line of the left and right bearings. The pitch diameter of the gear C is 600 mm and that of gear D is 200 mm. The distance between the centre line of the bearings is 2400 mm. The shaft*

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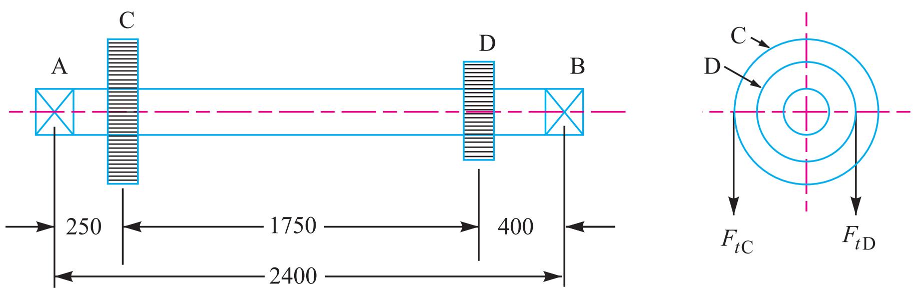
transmits 20 kW at 120 r.p.m. The power is delivered to the shaft at gear C and is taken out at gear D in such a manner that the tooth pressure F_{tC} of the gear C and F_{tD} of the gear D act vertically downwards.

Find the diameter of the shaft, if the working stress is 100 MPa in tension and 56 MPa in shear. The gears C and D weighs 950 N and 350 N respectively. The combined shock and fatigue factors for bending and torsion may be taken as 1.5 and 1.2 respectively.

Solution. Given : AC = 250 mm ; BD = 400 mm ; D_C = 600 mm or R_C = 300 mm ; D_D = 200 mm or R_D = 100 mm ; AB = 2400 mm ; P = 20 kW = 20 × 10^3 W ; N = 120 r.p.m ; σ_t = 100 MPa = 100 N/mm^2 ; τ = 56 MPa = 56 N/mm^2 ; W_C = 950 N ; W_D = 350 N ; K_m = 1.5 ; K_t = 1.2

The shaft supported in bearings and carrying gears is shown in Fig. 14.12.



All dimensions in mm.

Fig. 14.12

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2\pi N} = \frac{20 \times 10^3 \times 60}{2\pi \times 120} = 1590 \text{ N-m} = 1590 \times 10^3 \text{ N-mm}$$

Since the torque acting at gears C and D is same as that of the shaft, therefore the tangential force acting at gear C,

$$F_{tC} = \frac{T}{R_C} = \frac{1590 \times 10^3}{300} = 5300 \text{ N}$$

Car rear axle.

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and total load acting downwards on the shaft at *C*

$$= F_{tC} + W_C = 5300 + 950 = 6250 \text{ N}$$

Similarly tangential force acting at gear *D*,

$$F_{tD} = \frac{T}{R_D} = \frac{1590 \times 10^3}{100} = 15\,900 \text{ N}$$

and total load acting downwards on the shaft at *D*

$$= F_{tD} + W_D = 15\,900 + 350 = 16\,250 \text{ N}$$

Now assuming the shaft as a simply supported beam as shown in Fig. 14.13, the maximum bending moment may be obtained as discussed below :

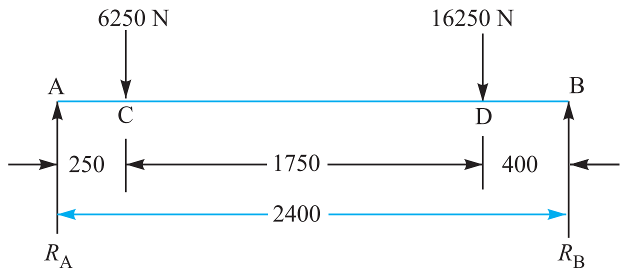

Fig. 14.13

Let R_A and R_B = Reactions at *A* and *B* respectively.

∴ $R_A + R_B$ = Total load acting downwards at *C* and *D*

$= 6250 + 16\,250 = 22\,500$ N

Now taking moments about *A*,

$$R_B \times 2400 = 16\,250 \times 2000 + 6250 \times 250 = 34\,062.5 \times 10^3$$

∴ $R_B = 34\,062.5 \times 10^3 / 2400 = 14\,190$ N

and $R_A = 22\,500 - 14\,190 = 8310$ N

A little consideration will show that the maximum bending moment will be either at *C* or *D*.

We know that bending moment at *C*,

$$M_C = R_A \times 250 = 8310 \times 250 = 2077.5 \times 10^3 \text{ N-mm}$$

Bending moment at *D*,

$${}^*M_D = R_B \times 400 = 14\,190 \times 400 = 5676 \times 10^3 \text{ N-mm}$$

∴ Maximum bending moment transmitted by the shaft,

$$M = M_D = 5676 \times 10^3 \text{ N-mm}$$

Let d = Diameter of the shaft.

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(1.5 \times 5676 \times 10^3)^2 + (1.2 \times 1590 \times 10^3)^2}$$

$$= 8725 \times 10^3 \text{ N-mm}$$

* The bending moment at *D* may also be calculated as follows :
$M_D = R_A \times 2000 - (\text{Total load at } C)\ 1750$

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We also know that the equivalent twisting moment (T_e),

$$8725 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 56 \times d^3 = 11\ d^3$$

$$\therefore \quad d^3 = 8725 \times 10^3 / 11 = 793 \times 10^3 \text{ or } d = 92.5 \text{ mm}$$

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}(K_m \times M + T_e)$$

$$= \frac{1}{2}\left[1.5 \times 5676 \times 10^3 + 8725 \times 10^3\right] = 8620 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$8620 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 100 \times d^3 = 9.82\ d^3 \quad \text{...(Taking } \sigma_b = \sigma_t\text{)}$$

$$\therefore \quad d^3 = 8620 \times 10^3 / 9.82 = 878 \times 10^3 \text{ or } d = 95.7 \text{ mm}$$

Taking the larger of the two values, we have

$$d = 95.7 \text{ say } 100 \text{ mm} \quad \textbf{Ans.}$$

Example 14.16. *A hoisting drum 0.5 m in diameter is keyed to a shaft which is supported in two bearings and driven through a 12 : 1 reduction ratio by an electric motor. Determine the power of the driving motor, if the maximum load of 8 kN is hoisted at a speed of 50 m/min and the efficiency of the drive is 80%. Also determine the torque on the drum shaft and the speed of the motor in r.p.m. Determine also the diameter of the shaft made of machinery steel, the working stresses of which are 115 MPa in tension and 50 MPa in shear. The drive gear whose diameter is 450 mm is mounted at the end of the shaft such that it overhangs the nearest bearing by 150 mm. The combined shock and fatigue factors for bending and torsion may be taken as 2 and 1.5 respectively.*

Solution. Given : D = 0.5 m or R = 0.25 m ; Reduction ratio = 12 : 1 ; W = 8 kN = 8000 N ; v = 50 m/min ; η = 80% = 0.8 ; σ_t = 115 MPa = 115 N/mm^2 ; τ = 50 MPa = 50 N/mm^2 ; D_1 = 450 mm or R_1 = 225 mm = 0.225 m ; Overhang = 150 mm = 0.15 m ; K_m = 2 ; K_t = 1.5

Power of the driving motor

We know that the energy supplied to the hoisting drum per minute

$$= W \times v = 8000 \times 50 = 400 \times 10^3 \text{ N-m/min}$$

$\therefore$ Power supplied to the hoisting drum

$$= \frac{400 \times 10^3}{60} = 6670 \text{ W} = 6.67 \text{ kW} \quad \text{...}(\because 1 \text{ N-m/s} = 1 \text{ W})$$

Since the efficiency of the drive is 0.8, therefore power of the driving motor

$$= \frac{6.67}{0.8} = 8.33 \text{ kW} \quad \textbf{Ans.}$$

Torque on the drum shaft

We know that the torque on the drum shaft,

$$T = W.R = 8000 \times 0.25 = 2000 \text{ N-m} \quad \textbf{Ans.}$$

Speed of the motor

Let N = Speed of the motor in r.p.m.

We know that angular speed of the hoisting drum

$$= \frac{\text{Linear speed}}{\text{Radius of the drum}} = \frac{v}{R} = \frac{50}{0.25} = 200 \text{ rad / min}$$

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Since the reduction ratio is 12 : 1, therefore the angular speed of the electric motor,

$$\omega = 200 \times 12 = 2400 \text{ rad/min}$$

and speed of the motor in r.p.m.,

$$N = \frac{\omega}{2\pi} = \frac{2400}{2\pi} = 382 \text{ r.p.m. } \textbf{Ans.}$$

Diameter of the shaft

Let d = Diameter of the shaft.

Since the torque on the drum shaft is 2000 N-m, therefore the tangential tooth load on the drive gear,

$$F_t = \frac{T}{R_1} = \frac{2000}{0.225} = 8900 \text{ N}$$

Assuming that the pressure angle of the drive gear in 20°, therefore the maximum bending load on the shaft due to tooth load

$$= \frac{F_t}{\cos 20°} = \frac{8900}{0.9397} = 9470 \text{ N}$$

Since the overhang of the shaft is 150 mm = 0.15 m, therefore bending moment at the bearing,

$$M = 9470 \times 0.15 = 1420 \text{ N-m}$$

We know that the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

$$= \sqrt{(2 \times 1420)^2 + (1.5 \times 2000)^2} = 4130 \text{ N-m} = 4130 \times 10^3 \text{ N-mm}$$

We also know that equivalent twisting moment (T_e),

$$4130 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 50 \times d^3 = 9.82\ d^3$$

$\therefore$ $d^3 = 4130 \times 10^3 / 9.82 = 420.6 \times 10^3$ or $d = 75$ mm

Again we know that the equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}(K_m \times M + T_e)$$

$$= \frac{1}{2}(2 \times 1420 + 4130) = 3485 \text{ N-m} = 3485 \times 10^3 \text{ N-mm}$$

We also know that equivalent bending moment (M_e),

$$3485 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 115 \times d^3 = 11.3\ d^3$$

$\therefore$ $d^3 = 3485 \times 10^3 / 11.3 = 308.4 \times 10^3$ or $d = 67.5$ mm

Taking the larger of the two values, we have

$$d = 75 \text{ mm } \textbf{Ans.}$$

Example 14.17. *A solid steel shaft is supported on two bearings 1.8 m apart and rotates at 250 r.p.m. A 20° involute gear D, 300 mm diameter is keyed to the shaft at a distance of 150 mm to the left on the right hand bearing. Two pulleys B and C are located on the shaft at distances of 600 mm and 1350 mm respectively to the right of the left hand bearing. The diameters of the pulleys B and C are 750 mm and 600 mm respectively. 30 kW is supplied to the gear, out of which 18.75 kW is taken off at the pulley C and 11.25 kW from pulley B. The drive from B is vertically downward while from C the drive is downward at an angle of 60° to the horizontal. In both cases the belt tension ratio is 2 and the angle of lap is 180°. The combined fatigue and shock factors for torsion and bending may be taken as 1.5 and 2 respectively.*

Design a suitable shaft taking working stress to be 42 MPa in shear and 84 MPa in tension.

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Solution. Given : PQ = 1.8 m ; N = 250 r.p.m ; α_D = 20° ; D_D = 300 mm or R_D = 150 mm = 0.15 m ; QD = 150 mm = 0.15 m ; PB = 600 mm = 0.6 m ; PC = 1350 mm = 1.35 m ; D_B = 750 mm or R_B = 375 mm = 0.375 m ; D_C = 600 mm or R_C = 300 mm = 0.3 m ; P_D = 30 kW = 30 × 10^3 W ; P_C = 18.75 kW = 18.75 × 10^3 W ; P_B = 11.25 kW = 11.25 × 10^3 W ; $T_{B1}/T_{B2} = T_{C1}/T_{C2}$ = 2 ; θ = 180° = π rad ; K_t = 1.5 ; K_m = 2 ; τ = 42 MPa = 42 N/mm^2 ; σ_t = 84 MPa = 84 N/mm^2

First of all, let us find the total loads acting on the gear D and pulleys C and B respectively.

For gear D

We know that torque transmitted by the gear D,

$$T_D = \frac{P_D \times 60}{2\pi N} = \frac{30 \times 10^3 \times 60}{2\pi \times 250} = 1146 \text{ N-m}$$

∴ Tangential force acting on the gear D,

$$F_{tD} = \frac{T_D}{R_D} = \frac{1146}{0.15} = 7640 \text{ N}$$

and the normal load acting on the gear tooth,

$$W_D = \frac{F_{tD}}{\cos 20°} = \frac{7640}{0.9397} = 8130 \text{ N}$$

The normal load acts at 20° to the vertical as shown in Fig. 14.14. Resolving the normal load vertically and horizontally, we have

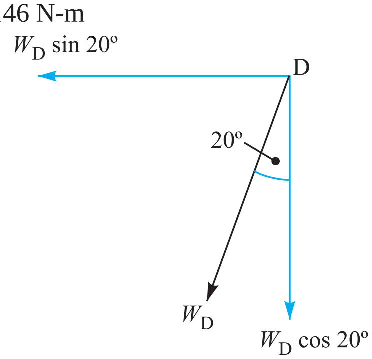


Fig. 14.14

Vertical component of W_D

$$= W_D \cos 20° = 8130 \times 0.9397 = 7640 \text{ N}$$

Horizontal component of W_D

$$= W_D \sin 20° = 8130 \times 0.342 = 2780 \text{ N}$$

For pulley C

We know that torque transmitted by pulley C,

$$T_C = \frac{P_C \times 60}{2\pi N} = \frac{18.75 \times 10^3 \times 60}{2\pi \times 250} = 716 \text{ N-m}$$

Let T_{C1} and T_{C2} = Tensions in the tight side and slack side of the belt for pulley C.

We know that torque transmitted by pulley C (T_C),

$$716 = (T_{C1} - T_{C2})\, R_C = (T_{C1} - T_{C2})\, 0.3$$

$$\therefore \quad T_{C1} - T_{C2} = 716 / 0.3 = 2387 \text{ N} \qquad ...(i)$$

Since T_{C1} / T_{C2} = 2 or $T_{C1} = 2\, T_{C2}$, therefore from equation (i), we have

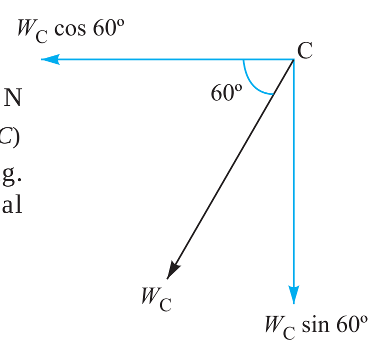
$$T_{C2} = 2387 \text{ N ; and } T_{C1} = 4774 \text{ N}$$

∴ Total load acting on pulley C,

$$W_C = T_{C1} + T_{C2} = 4774 + 2387 = 7161 \text{ N}$$

...(Neglecting weight of pulley C)

This load acts at 60° to the horizontal as shown in Fig. 14.15. Resolving the load W_C into vertical and horizontal components, we have



Vertical component of W_C

$$= W_C \sin 60° = 7161 \times 0.866 = 6200 \text{ N}$$

Trainwheels and Axles

and horizontal component of W_C

$$= W_C \cos 60° = 7161 \times 0.5$$
$$= 3580 \text{ N}$$

For pulley B

We know that torque transmitted by pulley *B*,

$$T_B = \frac{P_B \times 60}{2\pi N} = \frac{11.25 \times 10^3 \times 60}{2\pi \times 250} = 430 \text{ N-m}$$

Let T_{B1} and T_{B2} = Tensions in the tight side and slack side of the belt for pulley *B*.

We know that torque transmitted by pulley *B* (T_B),

$$430 = (T_{B1} - T_{B2}) R_B = (T_{B1} - T_{B2})\, 0.375$$

$\therefore$ $T_{B1} - T_{B2} = 430 / 0.375 = 1147$ N ...(*ii*)

Since $T_{B1} / T_{B2} = 2$ or $T_{B1} = 2T_{B2}$, therefore from equation (*ii*), we have

$$T_{B2} = 1147 \text{ N, and } T_{B1} = 2294 \text{ N}$$

$\therefore$ Total load acting on pulley *B*,

$$W_B = T_{B1} + T_{B2} = 2294 + 1147 = 3441 \text{ N}$$

This load acts vertically downwards.

From above, we may say that the shaft is subjected to the vertical and horizontal loads as follows :

Type of loading	*Load in N*		
	At D	*At C*	*At B*
Vertical	7640	6200	3441
Horizontal	2780	3580	0

The vertical and horizontal load diagrams are shown in Fig. 14.16 (*c*) and (*d*).

First of all considering vertical loading on the shaft. Let R_{PV} and R_{QV} be the reactions at bearings *P* and *Q* respectively for vertical loading. We know that

$$R_{PV} + R_{QV} = 7640 + 6200 + 3441 = 17\ 281\ \text{N}$$

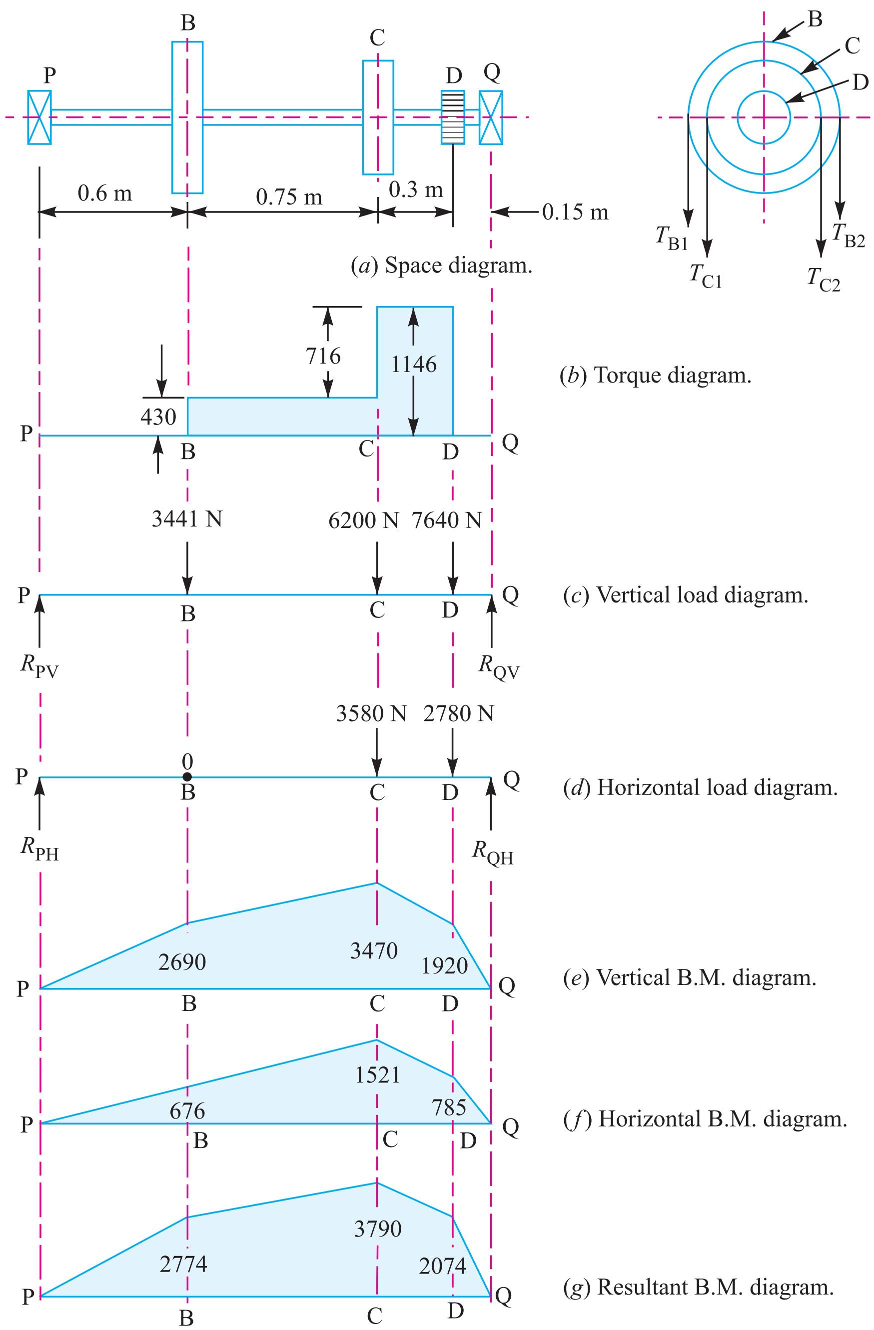


Fig. 14.16

Taking moments about P, we get

$$R_{QV} \times 1.8 = 7640 \times 1.65 + 6200 \times 1.35 + 3441 \times 0.6 = 23\ 041$$

$\therefore$ $$R_{QV} = 23\ 041 / 1.8 = 12\ 800 \text{ N}$$

and $$R_{PV} = 17\ 281 - 12\ 800 = 4481 \text{ N}$$

We know that B.M. at P and Q,

$$M_{PV} = M_{QV} = 0$$

B.M. at B, $$M_{BV} = 4481 \times 0.6 = 2690 \text{ N-m}$$

B.M. at C, $$M_{CV} = 4481 \times 1.35 - 3441 \times 0.75 = 3470 \text{ N-m}$$

and B.M. at D, $$M_{DV} = 12\ 800 \times 0.15 = 1920 \text{ N-m}$$

The bending moment diagram for vertical loading is shown in Fig. 14.16 (*e*).

Now considering horizontal loading. Let R_{PH} and R_{QH} be the reactions at the bearings P and Q respectively for horizontal loading. We know that

$$R_{PH} + R_{QH} = 2780 + 3580 = 6360 \text{ N}$$

Taking moments about P, we get

$$R_{QH} \times 1.8 = 2780 \times 1.65 + 3580 \times 1.35 = 9420 \text{ N}$$

$\therefore$ $$R_{QH} = 9420 / 1.8 = 5233 \text{ N}$$

and $$R_{PH} = 6360 - 5233 = 1127 \text{ N}$$

We know that B.M. at P and Q,

$$M_{PH} = M_{QH} = 0$$

B.M. at B, $$M_{BH} = 1127 \times 0.6 = 676 \text{ N-m}$$

B.M. at C, $$M_{CH} = 1127 \times 1.35 = 1521 \text{ N-m}$$

and B.M. at D, $$M_{DH} = 5233 \times 0.15 = 785 \text{ N-m}$$

The bending moment diagram for horizontal loading is shown in Fig. 14.16 (*f*).

The resultant bending moments for the points B, C and D are as follows :

$$\text{Resultant B.M. at } B = \sqrt{(M_{BV})^2 + (M_{BH})^2} = \sqrt{(2690)^2 + (676)^2} = 2774 \text{ N-m}$$

$$\text{Resultant B.M. at } C = \sqrt{(M_{CV})^2 + (M_{CH})^2} = \sqrt{(3470)^2 + (1521)^2} = 3790 \text{ N-m}$$

$$\text{Resultant B.M. at } D = \sqrt{(M_{DV})^2 + (M_{DH})^2} = \sqrt{(1920)^2 + (785)^2} = 2074 \text{ N-m}$$

From above we see that the resultant bending moment is maximum at C.

$\therefore$ $$M = M_C = 3790 \text{ N-m}$$

and maximum torque at C,

$$T = \text{Torque corresponding to 30 kW} = T_D = 1146 \text{ N-m}$$

Let $$d = \text{Diameter of the shaft in mm.}$$

We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \sqrt{(2 \times 3790)^2 + (1.5 \times 1146)^2}$$

$$= 7772 \text{ N-m} = 7772 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment (T_e),

$$7772 \times 10^3 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times 42 \times d^3 = 8.25\ d^3$$

$\therefore$ $$d^3 = 7772 \times 10^3 / 8.25 = 942 \times 10^3 \quad \text{or} \quad d = 98 \text{ mm}$$

Again, we know that equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right] = \frac{1}{2}(K_m \times M + T_e)$$

$$= \frac{1}{2}(2 \times 3790 + 7772) = 7676 \text{ N-m} = 7676 \times 10^3 \text{ N-mm}$$

We also know that the equivalent bending moment (M_e),

$$7676 \times 10^3 = \frac{\pi}{32} \times \sigma_b \times d^3 = \frac{\pi}{32} \times 84 \times d^3 = 8.25\ d^3$$

$$\therefore \quad d^3 = 7676 \times 10^3 / 8.25 = 930 \times 10^3 \text{ or } d = 97.6 \text{ mm}$$

Taking the larger of the two values, we have

d = 98 say 100 mm **Ans.**

14.13 Shafts Subjected to Axial Load in addition to Combined Torsion and Bending Loads

When the shaft is subjected to an axial load (F) in addition to torsion and bending loads as in propeller shafts of ships and shafts for driving worm gears, then the stress due to axial load must be added to the bending stress (σ_b). We know that bending equation is

$$\frac{M}{I} = \frac{\sigma_b}{y} \quad \text{or} \quad \sigma_b = \frac{M.y}{I} = \frac{M \times d/2}{\frac{\pi}{64} \times d^4} = \frac{32\,M}{\pi\,d^3}$$

and stress due to axial load

$$= \frac{F}{\frac{\pi}{4} \times d^2} = \frac{4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

$$= \frac{F}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]} = \frac{4\,F}{\pi\left[(d_o)^2 - (d_i)^2\right]} \qquad \text{...(For hollow shaft)}$$

$$= \frac{F}{\pi\,(d_o)^2\,(1 - k^2)} \qquad \text{... } (\because\ k = d_i/d_o)$$

∴ Resultant stress (tensile or compressive) for solid shaft,

$$\sigma_1 = \frac{32\,M}{\pi\,d^3} + \frac{4\,F}{\pi\,d^2} = \frac{32}{\pi\,d^3}\left(M + \frac{F \times d}{8}\right) \qquad \text{...}(i)$$

$$= \frac{32\,M_1}{\pi\,d^3} \qquad \text{...}\left(\text{Substituting } M_1 = M + \frac{F \times d}{8}\right)$$

In case of a hollow shaft, the resultant stress,

$$\sigma_1 = \frac{32\,M}{\pi\,(d_o)^3\,(1 - k^4)} + \frac{4\,F}{\pi\,(d_o)^2\,(1 - k^2)}$$

$$= \frac{32}{\pi\,(d_o)^3\,(1 - k^4)}\left[M + \frac{F\,d_o\,(1 + k^2)}{8}\right] = \frac{32\,M_1}{\pi (d_o)^3\,(1 - k^4)}$$

$$\text{...}\left[\text{Substituting for hollow shaft, } M_1 = M + \frac{F\,d_o\,(1 + k^2)}{8}\right]$$

In case of long shafts (slender shafts) subjected to compressive loads, a factor known as ***column factor*** (α) must be introduced to take the column effect into account.

∴ Stress due to the compressive load,

$$\sigma_c = \frac{\alpha \times 4\,F}{\pi\,d^2} \qquad \text{...(For round solid shaft)}$$

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$$= \frac{\alpha \times 4F}{\pi (d_o)^2 \; (1 - k^2)} \qquad \text{...(For hollow shaft)}$$

The value of column factor (α) for compressive loads* may be obtained from the following relation :

Column factor, $\alpha = \dfrac{1}{1 - 0.0044 \; (L / K)}$

This expression is used when the slenderness ratio (L / K) is less than 115. When the slenderness ratio (L / K) is more than 115, then the value of column factor may be obtained from the following relation :

**Column factor, $\alpha = \dfrac{\sigma_y \, (L / K)^2}{C \pi^2 E}$

where L = Length of shaft between the bearings,

K = Least radius of gyration,

σ_y = Compressive yield point stress of shaft material, and

C = Coefficient in Euler's formula depending upon the end conditions.

The following are the different values of C depending upon the end conditions.

C =1, for hinged ends,

= 2.25, for fixed ends,

= 1.6, for ends that are partly restrained as in bearings.

Note: In general, for a hollow shaft subjected to fluctuating torsional and bending load, along with an axial load, the equations for equivalent twisting moment (T_e) and equivalent bending moment (M_e) may be written as

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha \, F \, d_o \, (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \frac{\pi}{16} \times \tau \, (d_o)^3 \, (1 - k^4)$$

and

$$M_e = \frac{1}{2}\left[K_m \times M + \frac{\alpha F d_o \, (1 + k^2)}{8} + \sqrt{\left\{K_m \times M + \frac{\alpha F d_o \, (1 + k^2)}{8}\right\}^2 + (K_t \times T)^2}\right]$$

$$= \frac{\pi}{32} \times \sigma_b \, (d_o)^3 \, (1 - k^4)$$

It may be noted that for a solid shaft, $k = 0$ and $d_0 = d$. When the shaft carries no axial load, then $F = 0$ and when the shaft carries axial tensile load, then $\alpha = 1$.

Example 14.18. *A hollow shaft is subjected to a maximum torque of 1.5 kN-m and a maximum bending moment of 3 kN-m. It is subjected, at the same time, to an axial load of 10 kN. Assume that the load is applied gradually and the ratio of the inner diameter to the outer diameter is 0.5. If the outer diameter of the shaft is 80 mm, find the shear stress induced in the shaft.*

Solution. Given : T = 1.5 kN-m = 1.5 × 10^3 N-m ; M = 3 kN-m = 3 × 10^3 N-m ; F = 10 kN = 10 × 10^3 N ; $k = d_i / d_o = 0.5$; d_o = 80 mm = 0.08 m

Let τ = Shear stress induced in the shaft.

Since the load is applied gradually, therefore from Table 14.2, we find that

K_m =1.5 ; and K_t = 1.0

* The value of column factor (α) for tensile load is unity.

** It is an Euler's formula for long columns.

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We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o (1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 3 \times 10^3 + \frac{1 \times 10 \times 10^3 \times 0.08 (1 + 0.5^2)}{8}\right]^2 + (1 \times 1.5 \times 10^3)^2}$$

... ($\because$ α = 1, for axial tensile loading)

$$= \sqrt{(4500 + 125)^2 + (1500)^2} = 4862 \text{ N-m} = 4862 \times 10^3 \text{ N-mm}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$4862 \times 10^3 = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) = \frac{\pi}{16} \times \tau\, (80)^3\, (1 - 0.5^4) = 94\,260\, \tau$$

$\therefore$ $\tau = 4862 \times 10^3 / 94\,260 = 51.6 \text{ N/mm}^2 = 51.6$ MPa **Ans.**

Crankshaft inside the crank-case

Example 14.19. *A hollow shaft of 0.5 m outside diameter and 0.3 m inside diameter is used to drive a propeller of a marine vessel. The shaft is mounted on bearings 6 metre apart and it transmits 5600 kW at 150 r.p.m. The maximum axial propeller thrust is 500 kN and the shaft weighs 70 kN. Determine :*

1. *The maximum shear stress developed in the shaft, and*
2. *The angular twist between the bearings.*

Solution. Given : d_o = 0.5 m ; d_i = 0.3 m ; P = 5600 kW = 5600 × 10^3 W ; L = 6 m ; N = 150 r.p.m. ; F = 500 kN = 500 × 10^3 N ; W = 70 kN = 70 × 10^3 N

1. Maximum shear stress developed in the shaft

Let τ = Maximum shear stress developed in the shaft.

We know that the torque transmitted by the shaft,

$$T = \frac{P \times 60}{2 \pi N} = \frac{5600 \times 10^3 \times 60}{2 \pi \times 150} = 356\,460 \text{ N-m}$$

and the maximum bending moment,

$$M = \frac{W \times L}{8} = \frac{70 \times 10^3 \times 6}{8} = 52\,500 \text{ N-m}$$

Now let us find out the column factor α. We know that least radius of gyration,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]}{\frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2][(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}}$$

$$= \frac{1}{4}\sqrt{(d_o)^2 + (d_i)^2} = \frac{1}{4}\sqrt{(0.5)^2 + (0.3)^2} = 0.1458 \text{ m}$$

∴ Slenderness ratio,

$$L / K = 6 / 0.1458 = 41.15$$

and column factor,

$$\alpha = \frac{1}{1 - 0.0044\left(\frac{L}{K}\right)} \qquad \ldots\left(\because \frac{L}{K} < 115\right)$$

$$= \frac{1}{1 - 0.0044 \times 41.15} = \frac{1}{1 - 0.18} = 1.22$$

Assuming that the load is applied gradually, therefore from Table 14.2, we find that

$$K_m = 1.5 \text{ and } K_t = 1.0$$

Also $\quad k = d_i / d_o = 0.3 / 0.5 = 0.6$

We know that the equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha F d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.5 \times 52\,500 + \frac{1.22 \times 500 \times 10^3 \times 0.5\,(1 + 0.6^2)}{8}\right]^2 + (1 \times 356\,460)^2}$$

$$= \sqrt{(78\,750 + 51\,850)^2 + (356\,460)^2} = 380 \times 10^3 \text{ N-m}$$

We also know that the equivalent twisting moment for a hollow shaft (T_e),

$$380 \times 10^3 = \frac{\pi}{16} \times \tau\,(d_o)^3\,(1 - k^4) = \frac{\pi}{16} \times \tau\,(0.5)^3\,[1 - (0.6)^4] = 0.02\,\tau$$

∴ $\quad \tau = 380 \times 10^3 / 0.02 = 19 \times 10^6 \text{ N/m}^2 = 19$ MPa **Ans.**

2. *Angular twist between the bearings*

Let $\quad \theta$ = Angular twist between the bearings in radians.

We know that the polar moment of inertia for a hollow shaft,

$$J = \frac{\pi}{32}[(d_o)^4 - (d_i)^4] = \frac{\pi}{32}[(0.5)^4 - (0.3)^4] = 0.005\,34 \text{ m}^4$$

From the torsion equation,

$$\frac{T}{J} = \frac{G \times \theta}{L}, \text{ we have}$$

$$\theta = \frac{T \times L}{G \times J} = \frac{356\,460 \times 6}{84 \times 10^9 \times 0.00\,534} = 0.0048 \text{ rad}$$

... (Taking G = 84 GPa = 84×10^9 N/m²)

$$= 0.0048 \times \frac{180}{\pi} = 0.275°$$ **Ans.**

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Example 14.20. *A hollow steel shaft is to transmit 20 kW at 300 r.p.m. The loading is such that the maximum bending moment is 1000 N-m, the maximum torsional moment is 500 N-m and axial compressive load is 15 kN. The shaft is supported on rigid bearings 1.5 m apart. The maximum permissible shear stress on the shaft is 40 MPa. The inside diameter is 0.8 times the outside diameter. The load is cyclic in nature and applied with shocks. The values for the shock factors are* $K_t = 1.5$ *and* $K_m = 1.6$.

Solution. Given : *P = 20 kW ; *N = 300 r.p.m. ; M = 1000 N-m = 1000 × 10^3 N-mm ; T = 500 N-m = 500 × 10^3 N-mm ; F = 15 kN = 15 000 N ; L = 1.5 m = 1500 mm ; τ = 40 MPa = 40 N/mm^2 ; $d_i = 0.8\, d_o$ or $k = d_i/d_o = 0.8$; $K_t = 1.5$; $K_m = 1.6$

Let d_o = Outside diameter of the shaft, and

d_i = Inside diameter of the shaft = 0.8 d_o ...(Given)

We know that moment of inertia of a hollow shaft,

$$I = \frac{\pi}{64}\left[(d_o)^4 - (d_i)^4\right]$$

and cross-sectional area of the hollow shaft,

$$A = \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right]$$

∴ Radius of gyration of the hollow shaft,

$$K = \sqrt{\frac{I}{A}} = \sqrt{\frac{\frac{\pi}{64}[(d_o)^4 - (d_i)^4]}{\frac{\pi}{4}[(d_o)^2 - (d_i)^2]}}$$

$$= \sqrt{\frac{[(d_o)^2 + (d_i)^2][(d_o)^2 - (d_i)^2]}{16\,[(d_o)^2 - (d_i)^2]}} = \sqrt{\frac{(d_o)^2 + (d_i)^2}{16}}$$

$$= \frac{d_o}{4}\sqrt{1 + \left(\frac{d_i}{d_o}\right)^2} = \frac{d_o}{4}\sqrt{1 + (0.8)^2} = 0.32\, d_o$$

and column factor for compressive loads,

$$\alpha = \frac{1}{1 - 0.0044\,(L/K)} = \frac{1}{1 - 0.0044\,(1500/0.32\,d_o)}$$

$$= \frac{1}{1 - 20.6/d_o} = \frac{d_o}{d_o - 20.6}$$

We know that equivalent twisting moment for a hollow shaft,

$$T_e = \sqrt{\left[K_m \times M + \frac{\alpha\, F\, d_o\,(1 + k^2)}{8}\right]^2 + (K_t \times T)^2}$$

$$= \sqrt{\left[1.6 \times 1000 \times 10^3 + \frac{\left(\frac{d_o}{d_o - 20.6}\right) 15000 \times d_o\,(1 + 0.8^2)}{8}\right]^2 + (1.5 \times 500 \times 10^3)^2}$$

$$= \sqrt{\left[1600 \times 10^3 + \frac{3075\,(d_o)^2}{d_o - 20.6}\right]^2 + (750 \times 10^3)^2} \quad \text{...}(i)$$

* Superfluous data.

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We also know that equivalent twisting moment for a hollow shaft,

$$T_e = \frac{\pi}{16} \times \tau (d_o)^3 (1 - k^4)$$

$$= \frac{\pi}{16} \times 40 (d_o)^3 (1 - 0.8^4) = 4.65 (d_o)^3 \quad \text{...(ii)}$$

Equating equations (*i*) and (*ii*), we have

$$4.65 (d_o)^3 = \sqrt{\left[1600 \times 10^3 + \frac{3075 (d_o)^2}{d_o - 20.6}\right]^2 + (750 \times 10^3)^2} \quad \text{...(iii)}$$

Solving this expression by hit and trial method, we find that

$$d_o = 76.32 \text{ say } 80 \text{ mm } \textbf{Ans.}$$

and $d_i = 0.8\, d_o = 0.8 \times 80 = 64$ mm **Ans.**

Note : In order to find the minimum value of d_o to be used for the hit and trial method, determine the equivalent twisting moment without considering the axial compressive load. We know that equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2} = \sqrt{(1.6 \times 1000 \times 10^3)^2 + (1.5 \times 500 \times 10^3)^2} \quad \text{... (iv)}$$

$$= 1767 \times 10^3 \text{ N-mm}$$

Equating equations (*ii*) and (*iv*),

$$4.65(d_o)^3 = 1767 \times 10^3 \quad \text{or} \quad (d_o)^3 = 1767 \times 10^3/4.65 = 380 \times 10^3$$

$$\therefore \quad d_o = 72.4 \text{ mm}$$

Thus the value of d_o to be substituted in equation (*iii*) must be greater than 72.4 mm.

14.14 Design of Shafts on the basis of Rigidity

Sometimes the shafts are to be designed on the basis of rigidity. We shall consider the following two types of rigidity.

1. *Torsional rigidity.* The torsional rigidity is important in the case of camshaft of an I.C. engine where the timing of the valves would be effected. The permissible amount of twist should not exceed 0.25° per metre length of such shafts. For line shafts or transmission shafts, deflections 2.5 to 3 degree per metre length may be used as limiting value. The widely used deflection for the shafts is limited to 1 degree in a length equal to twenty times the diameter of the shaft.

The torsional deflection may be obtained by using the torsion equation,

$$\frac{T}{J} = \frac{G \,.\, \theta}{L} \quad \text{or} \quad \theta = \frac{T \,.\, L}{J \,.\, G}$$

where θ = Torsional deflection or angle of twist in radians,

T = Twisting moment or torque on the shaft,

J = Polar moment of inertia of the cross-sectional area about the axis of rotation,

$$= \frac{\pi}{32} \times d^4 \quad \text{...(For solid shaft)}$$

$$= \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad \text{...(For hollow shaft)}$$

G = Modulus of rigidity for the shaft material, and

L = Length of the shaft.

2. *Lateral rigidity.* It is important in case of transmission shafting and shafts running at high speed, where small lateral deflection would cause huge out-of-balance forces. The lateral rigidity is also important for maintaining proper bearing clearances and for correct gear teeth alignment. If the shaft is of uniform cross-section, then the lateral deflection of a shaft may be obtained by using the deflection formulae as in Strength of Materials. But when the shaft is of variable cross-section, then

Air acclerating downwards, pushed by the rotating blades, produced an upwards reaction that lifts the helicopter.

Note : This picture is given as additional information and is not a direct example of the current chapter.

the lateral deflection may be determined from the fundamental equation for the elastic curve of a beam, *i.e.*

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

Example 14.21. *A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25° per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.*

Solution. Given : $P = 4$ kW = 4000 W ; $N = 800$ r.p.m. ; $\theta = 0.25° = 0.25 \times \frac{\pi}{180} = 0.0044$ rad ; $L = 1$ m = 1000 mm ; $G = 84$ GPa $= 84 \times 10^9$ N/m^2 $= 84 \times 10^3$ N/mm^2

Diameter of the spindle

Let d = Diameter of the spindle in mm.

We know that the torque transmitted by the spindle,

$$T = \frac{P \times 60}{2\pi N} = \frac{4000 \times 60}{2\pi \times 800} = 47.74 \text{ N-m} = 47\ 740 \text{ N-mm}$$

We also know that $\frac{T}{J} = \frac{G \times \theta}{L}$ or $J = \frac{T \times l}{G \times \theta}$

or

$$\frac{\pi}{32} \times d^4 = \frac{47\ 740 \times 1000}{84 \times 10^3 \times 0.0044} = 129\ 167$$

$\therefore$ $d^4 = 129\ 167 \times 32 / \pi = 1.3 \times 10^6$ or $d = 33.87$ say 35 mm **Ans.**

Shear stress induced in the spindle

Let τ = Shear stress induced in the spindle.

We know that the torque transmitted by the spindle (T),

$$47\ 740 = \frac{\pi}{16} \times \tau \times d^3 = \frac{\pi}{16} \times \tau\,(35)^3 = 8420\,\tau$$

$\therefore$ $\tau = 47\ 740 / 8420 = 5.67$ N/mm^2 $= 5.67$ MPa **Ans.**

Example 14.22. *Compare the weight, strength and stiffness of a hollow shaft of the same external diameter as that of solid shaft. The inside diameter of the hollow shaft being half the external diameter. Both the shafts have the same material and length.*

Solution. Given : $d_o = d$; $d_i = d_o / 2$ or $k = d_i / d_o = 1 / 2 = 0.5$

Comparison of weight

We know that weight of a hollow shaft,

$$W_H = \text{Cross-sectional area} \times \text{Length} \times \text{Density}$$

$$= \frac{\pi}{4}\left[(d_o)^2 - (d_i)^2\right] \times \text{Length} \times \text{Density} \quad ...(i)$$

and weight of the solid shaft,

$$W_S = \frac{\pi}{4} \times d^2 \times \text{Length} \times \text{Density} \quad ...(ii)$$

Since both the shafts have the same material and length, therefore by dividing equation **(*i*)** by equation **(*ii*)**, we get

$$\frac{W_H}{W_S} = \frac{(d_o)^2 - (d_i)^2}{d^2} = \frac{(d_o)^2 - (d_i)^2}{(d_o)^2} \quad ...(\because d = d_o)$$

$$= 1 - \frac{(d_i)^2}{(d_o)_2} = 1 - k^2 = 1 - (0.5)^2 = 0.75 \textbf{ Ans.}$$

Comparison of strength

We know that strength of the hollow shaft,

$$T_H = \frac{\pi}{16} \times \tau\, (d_o)^3\, (1 - k^4) \quad ...(iii)$$

and strength of the solid shaft,

$$T_S = \frac{\pi}{16} \times \tau \times d^3 \quad ...(iv)$$

Dividing equation **(*iii*)** by equation **(*iv*)**, we get

$$\frac{T_H}{T_S} = \frac{(d_o)^3\,(1 - k^4)}{d^3} = \frac{(d_o)^3\,(1 - k^4)}{(d_o)^3} = 1 - k^4 \quad ...(\because d = d_o)$$

$$= 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

Comparison of stiffness

We know that stiffness

$$= \frac{T}{\theta} = \frac{G \times J}{L}$$

The propeller shaft of this heavy duty helicopter is subjected to very high torsion.

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∴ Stiffness of a hollow shaft,

$$S_H = \frac{G}{L} \times \frac{\pi}{32}\left[(d_o)^4 - (d_i)^4\right] \quad ...(v)$$

and stiffness of a solid shaft,

$$S_S = \frac{G}{L} \times \frac{\pi}{32} \times d^4 \quad ...(vi)$$

Dividing equation (*v*) by equation (*vi*), we get

$$\frac{S_H}{S_S} = \frac{(d_o)^4 - (d_i)^4}{d^4} = \frac{(d_o)^4 - (d_i)^4}{(d_o)^4} = 1 - \frac{(d_i)^4}{(d_o)^4} \quad ...(\because d = d_o)$$

$$= 1 - k^4 = 1 - (0.5)^4 = 0.9375 \textbf{ Ans.}$$

EXERCISES

1. A shaft running at 400 r.p.m. transmits 10 kW. Assuming allowable shear stress in shaft as 40 MPa, find the diameter of the shaft. **[Ans. 35 mm]**

2. A hollow steel shaft transmits 600 kW at 500 r.p.m. The maximum shear stress is 62.4 MPa. Find the outside and inside diameter of the shaft, if the outer diameter is twice of inside diameter, assuming that the maximum torque is 20% greater than the mean torque. **[Ans. 100 mm ; 50 mm]**

3. A hollow shaft for a rotary compressor is to be designed to transmit a maximum torque of 4750 N-m. The shear stress in the shaft is limited to 50 MPa. Determine the inside and outside diameters of the shaft, if the ratio of the inside to the outside diameter is 0.4. **[Ans. 35 mm ; 90 mm]**

4. A motor car shaft consists of a steel tube 30 mm internal diameter and 4 mm thick. The engine develops 10 kW at 2000 r.p.m. Find the maximum shear stress in the tube when the power is transmitted through a 4 : 1 gearing. **[Ans. 30 MPa]**

5. A cylindrical shaft made of steel of yield strength 700 MPa is subjected to static loads consisting of a bending moment of 10 kN-m and a torsional moment of 30 kN-m. Determine the diameter of the shaft using two different theories of failure and assuming a factor of safety of 2. **[Ans. 100 mm]**

6. A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The allowable shear stress for the material of the shaft is 42 MPa. If the shaft carries a central load of 900 N and is simply supported between bearing 3 metre apart, determine the diameter of the shaft. The maximum tensile or compressive stress is not to exceed 56 MPa. **[Ans. 50 mm]**

7. Two 400 mm diameter pulleys are keyed to a simply supported shaft 500 mm apart. Each pulley is 100 mm from its support and has horizontal belts, tension ratio being 2.5. If the shear stress is to be limited to 80 MPa while transmitting 45 kW at 900 r.p.m., find the shaft diameter if it is to be used for the input-output belts being on the same or opposite sides. **[Ans. 40 mm]**

8. A cast gear wheel is driven by a pinion and transmits 100 kW at 375 r.p.m. The gear has 200 machine cut teeth having 20° pressure angle and is mounted at the centre of a 0.4 m long shaft. The gear weighs 2000 N and its pitch circle diameter is 1.2 m. Design the gear shaft. Assume that the axes of the gear and pinion lie in the same horizontal plane. **[Ans. 80 mm]**

9. Fig. 14.17 shows a shaft from a hand-operated machine. The frictional torque in the journal bearings at *A* and *B* is 15 N-m each. Find the diameter (*d*) of the shaft (on which the pulley is mounted) using maximum distortion energy criterion. The shaft material is 40 C 8 steel for which the yield stress in tension is 380 MPa and the factor of safety is 1.5. **[Ans. 20 mm]**

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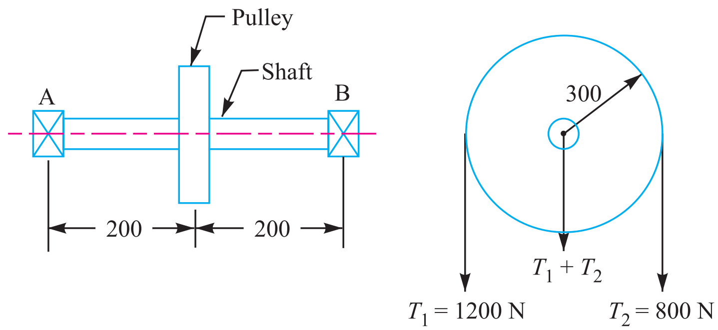


All dimensions in mm.

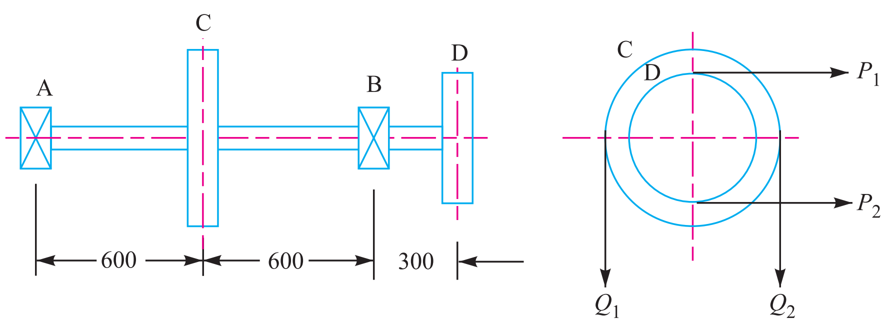
Fig. 14.17

10. A line shaft is to transmit 30 kW at 160 r.p.m. It is driven by a motor placed directly under it by means of a belt running on a 1 m diameter pulley keyed to the end of the shaft. The tension in the tight side of the belt is 2.5 times that in the slack side and the centre of the pulley over-hangs 150 mm beyond the centre line of the end bearing. Determine the diameter of the shaft, if the allowable shear stress is 56 MPa and the pulley weighs 1600 N. **[Ans. 60 mm]**

11. Determine the diameter of hollow shaft having inside diameter 0.5 times the outside diameter. The permissible shear stress is limited to 200 MPa. The shaft carries a 900 mm diameter cast iron pulley. This pulley is driven by another pulley mounted on the shaft placed below it. The belt ends are parallel and vertical. The ratio of tensions in the belt is 3. The pulley on the hollow shaft weighs 800 N and overhangs the nearest bearing by 250 mm. The pulley is to transmit 35 kW at 400 r.p.m.

[Ans. d_o = 40 mm, d_i = 20 mm]

12. A horizontal shaft *AD* supported in bearings at *A* and *B* and carrying pulleys at *C* and *D* is to transmit 75 kW at 500 r.p.m. from drive pulley *D* to off-take pulley *C*, as shown in Fig. 14.18.



All dimensions in mm.

Fig. 14.18

Calculate the diameter of shaft. The data given is : $P_1 = 2\ P_2$ (both horizontal), $Q_1 = 2\ Q_2$ (both vertical), radius of pulley *C* = 220 mm, radius of pulley *D* = 160 mm, allowable shear stress = 45 MPa. **[Ans. 100 mm]**

13. A line shaft *ABCD*, 9 metres long, has four pulleys *A*, *B*, *C* and *D* at equal distance apart. Power of 45 kW is being supplied to the shaft through the pulley *C* while the power is being taken off equally from the pulleys *A*, *B* and *D*. The shaft runs at 630 r.p.m.

Calculate the most economical diameters for the various portions of the shaft so that the shear stress does not exceed 55 MPa. If the shear modulus is 85 GPa, determine the twist of the pulley *D* with respect to the pulley *A*. **[Ans. 28 mm, 36 mm, 28 mm ; 0.0985°]**

14. A shaft made of steel receives 7.5 kW power at 1500 r.p.m. A pulley mounted on the shaft as shown in Fig. 14.19 has ratio of belt tensions 4.

The gear forces are as follows :

$$F_t = 1590 \text{ N}; F_r = 580 \text{ N}$$

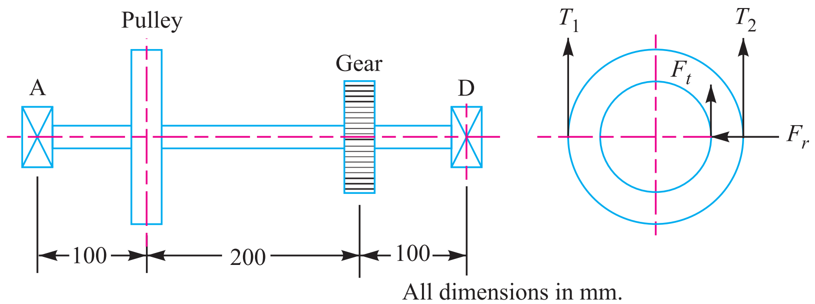


Fig. 14.19

Design the shaft diameter by maximum shear stress theory. The shaft material has the following properties :

Ultimate tensile strength = 720 MPa; Yield strength = 380 MPa; Factor of safety = 1.5.

[Ans. 20 mm]

15. An overhang hollow shaft carries a 900 mm diameter pulley, whose centre is 250 mm from the centre of the nearest bearing. The weight of the pulley is 600 N and the angle of lap is 180°. The pulley is driven by a motor vertically below it. If permissible tension in the belt is 2650 N and if coefficient of friction between the belt and pulley surface is 0.3, estimate, diameters of shaft, when the internal diameter is 0.6 of the external.

Neglect centrifugal tension and assume permissible tensile and shear stresses in the shaft as 84 MPa and 68 MPa respectively. **[Ans. 65 mm]**

16. The shaft, as shown in Fig. 14.20, is driven by pulley *B* from an electric motor. Another belt drive from pulley *A* is running a compressor. The belt tensions for pulley *A* are 1500 N and 600 N. The ratio of belt tensions for pulley *B* is 3.5.

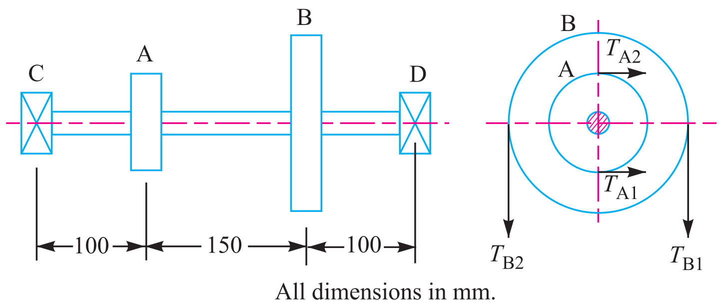


Fig. 14.20

The diameter of pulley *A* is 150 mm and the diameter of pulley *B* is 480 mm. The allowable tensile stress for the shaft material is 170 MPa and the allowable shear stress is 85 MPa. Taking torsion and bending factors as 1.25 and 1.75 respectively, find the shaft diameter.

Also find out the dimensions for a hollow shaft with outside diameter limited to 30 mm. Compare the weights of the two shafts. **[Ans. 30 mm ; 24 mm ; 1.82]**

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17. A mild steel shaft transmits 15 kW at 210 r.p.m. It is supported on two bearings 750 mm apart and has two gears keyed to it. The pinion having 24 teeth of 6 mm module is located 100 mm to the left of the right hand bearing and delivers the power horizontally to the right. The gear having 80 teeth of 6 mm module is located 15 mm to the right of the left hand bearing and receives power in a vertical direction from below. Assuming an allowable working shear stress as 53 MPa, and a combined shock and fatigue factor of 1.5 in bending as well as in torsion, determine the diameter of the shaft.

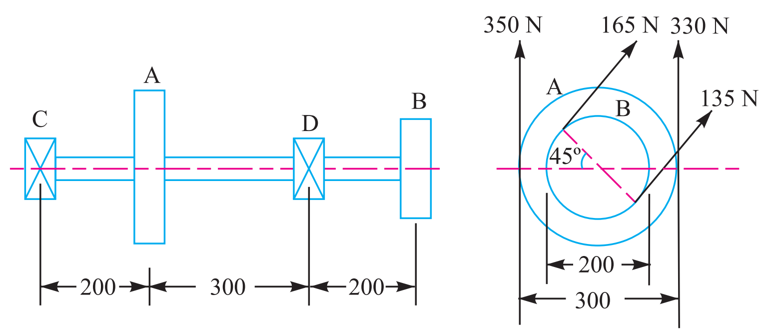
[Ans. 60 mm]

18. A steel shaft 800 mm long transmitting 15 kW at 400 r.p.m. is supported at two bearings at the two ends. A gear wheel having 80 teeth and 500 mm pitch circle diameter is mounted at 200 mm from the left hand side bearing and receives power from a pinion meshing with it. The axis of pinion and gear lie in the horizontal plane. A pulley of 300 mm diameter is mounted at 200 mm from right hand side bearing and is used for transmitting power by a belt. The belt drive is inclined at 30° to the vertical in the forward direction. The belt lap angle is 180 degrees. The coefficient of friction between belt and pulley is 0.3. Design and sketch the arrangement of the shaft assuming the values of safe stresses as : τ = 55 MPa; σ_t = 80 MPa. Take torsion and bending factor 1.5 and 2 respectively. **[Ans. 120 mm]**

19. A machine shaft, supported on bearings having their centres 750 mm apart, transmitted 185 kW at 600 r.p.m. A gear of 200 mm and 20° tooth profile is located 250 mm to the right of left hand bearing and a 450 mm diameter pulley is mounted at 200 mm to right of right hand bearing. The gear is driven by a pinion with a downward tangential force while the pulley drives a horizontal belt having 180° angle of contact. The pulley weighs 1000 N and tension ratio is 3. Find the diameter of the shaft, if the allowable shear stress of the material is 63 MPa. **[Ans. 80 mm]**

20. If in the above Exercise 19, the belt drive is at an angle of 60° to the horizontal and a combined shock and fatigue factor is 1.5 for bending and 1.0 for torque, find the diameter of the shaft. **[Ans. 90 mm]**

21. A shaft made of 40 C 8 steel is used to drive a machine. It rotates at 1500 r.p.m. The pulleys *A*, *B* and the bearings *C*, *D* are located as shown in Fig. 14.21. The belt tensions are also shown in the figure.



All dimensions in mm.

Fig. 14.21

Determine the diameter of the shaft. The permissible shear stress for the shaft material is 100 MPa. The combined shock and fatigue factor applied to bending and torsion are 1.5 and 1.2 respectively.

[Ans. 25 mm]

22. The engine of a ship develops 440 kW and transmits the power by a horizontal propeller shaft which runs at 120 r.p.m. It is proposed to design a hollow propeller shaft with inner diameter as 0.6 of the outer diameter. Considering torsion alone, calculate the diameter of the propeller shaft if stress in the material is not to exceed 63 MPa and also the angular twist over a length of 2.5 m is not to be more than 1°. The modulus of rigidity of the shaft material is 80 GPa. **[Ans. 30 mm ; 18 mm]**

23. A shaft is required to transmit 1 MW power at 240 r.p.m. The shaft must not twist more than 1 degree on a length of 15 diameters. If the modulus of rigidity for material of the shaft is 80 GPa, find the diameter of the shaft and shear stress induced. **[Ans. 165 mm ; 46.5 MPa]**

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24. The internal diameter of a hollow shaft is 2/3 rd of its external diameter. Compare the strength and stiffness of the shaft with that of a solid shaft of the same material. **[Ans. 1.93 ; 2.6]**

25. The shaft of an axial flow rotary compressor is subjected to a maximum torque of 2000 N-m and a maximum bending moment of 4000 N-m. The combined shock and fatigue factor in torsion is 1.5 and that in bending is 2. Design the diameter of the shaft, if the shear stress in the shaft is 50 MPa. Design a hollow shaft for the above compressor taking the ratio of outer diameter to the inner diameter as 2. What is the percentage saving in material ? Also compare the stiffness.

[Ans. 96 mm ; 98 mm, 49 mm ; 21.84%; 1.018]

QUESTIONS

1. Distinguish clearly, giving examples between pin, axle and shaft.
2. How the shafts are formed ?
3. Discuss the various types of shafts and the standard sizes of transmissions shafts.
4. What type of stresses are induced in shafts ?
5. How the shaft is designed when it is subjected to twisting moment only ?
6. Define equivalent twisting moment and equivalent bending moment. State when these two terms are used in design of shafts.
7. When the shaft is subjected to fluctuating loads, what will be the equivalent twisting moment and equivalent bending moment ?
8. What do you understand by torsional rigidity and lateral rigidity.
9. A hollow shaft has greater strength and stiffness than solid shaft of equal weight. Explain.
10. Under what circumstances are hollow shafts preferred over solid shafts ? Give any two examples where hollow shafts are used. How are they generally manufactured ?

OBJECTIVE TYPE QUESTIONS

1. The standard length of the shaft is

(*a*) 5 m (*b*) 6 m

(*c*) 7 m (*d*) all of these

2. Two shafts *A* and *B* are made of the same material. The diameter of the shaft *A* is twice as that of shaft *B*. The power transmitted by the shaft *A* will be .... of shaft *B*.

(*a*) twice (*b*) four times

(*c*) eight times (*d*) sixteen times

3. Two shafts *A* and *B* of solid circular cross-section are identical except for their diameters d_A and d_B. The ratio of power transmitted by the shaft *A* to that of shaft *B* is

(*a*) $\frac{d_A}{d_B}$ (*b*) $\frac{(d_A)^2}{(d_B)^2}$

(*c*) $\frac{(d_A)^3}{(d_B)^3}$ (*d*) $\frac{(d_A)^4}{(d_B)^4}$

4. Two shafts will have equal strength, if

(*a*) diameter of both the shafts is same

(*b*) angle of twist of both the shafts is same

(*c*) material of both the shafts is same

(*d*) twisting moment of both the shafts is same

Contents

5. A transmission shaft subjected to bending loads must be designed on the basis of
   (*a*) maximum normal stress theory
   (*b*) maximum shear stress theory
   (*c*) maximum normal stress and maximum shear stress theories
   (*d*) fatigue strength
6. Which of the following loading is considered for the design of axles ?
   (*a*) Bending moment only
   (*b*) Twisting moment only
   (*c*) Combined bending moment and torsion
   (*d*) Combined action of bending moment, twisting moment and axial thrust
7. When a shaft is subjected to a bending moment M and a twisting moment T, then the equivalent twisting moment is equal to
   (*a*) $M + T$ (*b*) $M^2 + T^2$
   (*c*) $\sqrt{M^2 + T^2}$ (*d*) $\sqrt{M^2 - T^2}$
8. The maximum shear stress theory is used for
   (*a*) brittle materials (*b*) ductile materials
   (*c*) plastic materials (*d*) non-ferrous materials
9. The maximum normal stress theory is used for
   (*a*) brittle materials (*b*) ductile materials
   (*c*) plastic materials (*d*) non-ferrous materials
10. The design of shafts made of brittle materials is based on
   (*a*) Guest's theory (*b*) Rankine's theory
   (*c*) St. Venant's theory (*d*) Von Mises Theory

ANSWERS

1. (*d*)	**2.** (*c*)	**3.** (*c*)	**4.** (*d*)	**5.** (*a*)
6. (*a*)	**7.** (*c*)	**8.** (*b*)	**9.** (*a*)	**10.** (*b*)

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