Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: 3rd
Subject: Digital Signal Processing
Lecturer: Dr. Rami Qays Malik
Lecture: 3- system properties

1. Static and Dynamic System:-
The system is static when the output of the system depends only upon present input sample it is called also memoryless system. for example

$$y(n) = 10 . x(n)$$

or $$y(n) = 15 . x^2(n) + 10x(n)$$

The system is Dynamic if the output depends upon the past values of input for example

$$y(n) = x(n) + x(n-1)$$

this system is Dynamic because the output value depends upon the previous input sample (past) $(x(n-1))$.

2. Shift invariant and shift variant System:-
If the input output characteristics of the system do not change with shift of time origin, for example

ex. $y(n) = x(n) - x(n-1)$

Consider the system discribe by

$$y(n) = T[x(n)] = x(n) - x(n-1)$$

now Let us delay the input by "K" constant

$$y(n,K) = T[x(n-K)]$$
$$= x(n-K) - x(n-K-1)$$

* now Let us delay the output $y(n)$ by "K" sample

$$y(n-K) = x(n-K) - x(n-K-1)$$

Here observe that

$$y(n,K) = y(n-K)$$

Hence The system is shift invariant.

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ex.. $y(n) = n\,x(n)$; are shift invariant or not?

$$y(n) = T[x(n)] = n\,x(n)$$

when input $x(n)$ is delayed by "K" sample the response is

$$y(n,K) = T[x(n-K)]$$
$$= n\,x(n-K)$$

* Here observe that only input $x(n)$ is delayed. the multiplier "n" is not part of the input.

Now Let us delay or shift the out put $y(n)$ by "K" sample

$$y(n-K) = (n-K)\,x(n-K)$$

* Here both "n" & $x(n)$ in the equation $y(n) = n\,x(n)$ will be shifted by "K" samples because they are part of output sequence; then $\Rightarrow y(n,K) \neq y(n-K)$
* Hence the system is shift variant.

3. Causal and Noncausal System :-

In this system the output depends upon past and present input only. that is the output function of $x(n), x(n-1), x(n-2), x(n-3)$ ... and so on.

* the system is Noncausal if the output depends upon future input ; $x(n+1), x(n+2), \ldots$ and so on.

Ex.. check the following systems are causal or noncausal ?

ⓐ $y(n) = x(n) + x(n-1)$

ⓑ $y(n) = x(n) + x(n+1)$

ⓒ $y(n) = x(2n)$

Sol :- ⓐ $y(n) = x(n) + x(n-1)$

* here $y(n)$ depends upon $x(n)$ and $x(n-1)$; $x(n)$ is the present input and $x(n-1)$ is the previous input

Hence the system is causal.

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Sol: (b) $y(n) = x(n) + x(n+1)$

* Here $y(n)$ depends upon present input $x(n)$ and the next or future input $x(n+1)$
* Hence the System is noncausal.

Sol: (c) $y(n) = x(2n)$

$n = 1 \Rightarrow y(1) = x(2)$

Here when $n = 2 \Rightarrow y(2) = x(4)$

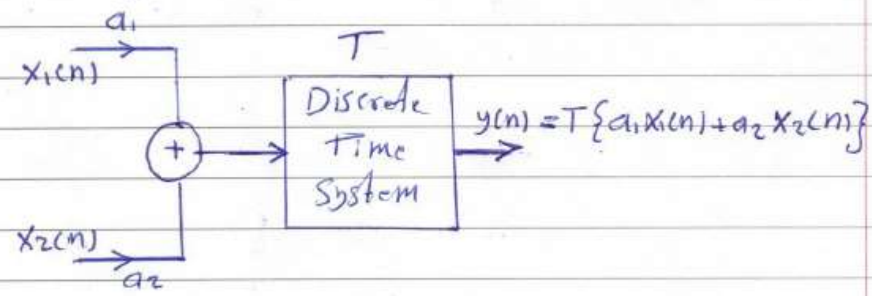
thus the output $y(n)$ depends upon the future inputs Hence the system is noncausal.

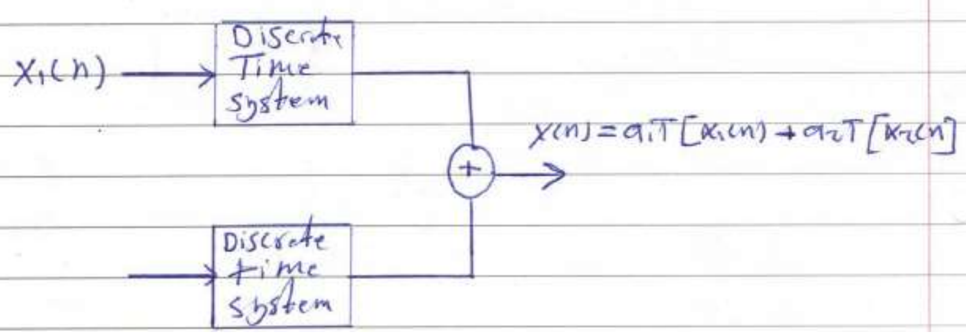
4. Linear and non Linear Systems. (Linearity property)

A system is said to be Linear if it satisfies the superposition principle. Let $x_1(n)$ and $x_2(n)$ be two input sequence. then the system is Linear if only

$$T\{a_1 x_1(n) + a_2 x_2(n)\} = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$

Here a_1 and a_2 are constant. the above condition state that the system is linear if the combined response due to $x_1(n)$ and $x_2(n)$ together is same as the sum of individual response.





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Ex: Determine the following systems
ⓐ $y(n) = x(n^2)$
ⓑ $y(n) = x^2(n)$ are Linear or non Linear ?

Sol: ⓐ $y(n) = x(n^2)$
→ for two separate input $x_1(n)$ & $x_2(n)$ the system produce response of

$$y_1(n) = x_1(n^2)$$
$$y_2(n) = x_2(n^2)$$

→ and the system response to the Linear combination of $x_1(n)$ & $x_2(n)$ will be

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$

Since the Linear systems satisfy additive property, the above equation will be

$$y_3(n) = a_1 T[x_1(n)] + a_2 T[x_2(n)]$$
$$= a_1 x_1(n^2) + a_2 x_2(n^2)$$

this is the response of the system to Linear combination of two input

→ Now the response of the system due to Linear combination of two output will be

$$y_3'(n) = a_1 y_1(n) + a_2 y_2(n)$$
$$= a_1 x_1(n^2) + a_2 x_2(n^2)$$

* we observe that

$$y_3(n) = y_3'(n)$$

Hence the system is Linear..

Sol: ⓑ $y(n) = x^2(n)$
when the input $x_1(n)$ & $x_2(n)$ applied sepparately the response $y_1(n)$ & $y_2(n)$ will be

$$y_1(n) = x_1^2(n) \quad \text{--- ①}$$

$$y_2(n) = x_2^2(n) \quad \text{--- ②}$$

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the response of the system to the Linear combination of $x_1(n)$ & $x_2(n)$ will be:

$$y_3(n) = T[a_1 x_1(n) + a_2 x_2(n)]$$
$$= [a_1 x_1(n) + a_2 x_2(n)]^2$$
$$= a_1^2 x_1^2(n) + 2a_1 a_2 x_1(n) x_2(n) + a_1^2 x_2^2(n)$$

* the Linear combination of two output will be

$$y_3'(n) = a_1 x_1^2(n) + a_2 x_2^2(n)$$

We observe that :- $y_3(n) \neq y_3'(n)$
Hence the system is non Linear.

5. Stable and Unstable System (Stability property).
when every bounded input produce abounded output, then the system is called Bounded input Bounded output (BIBO) stable.

* the input $x(n)$ is bounded if there exists some finite number "M_x" such that

$$|x(n)| \leq M_x < \infty$$

Similarly output $y(n)$ is bounded if there exists some finite number "M_y" such that

$$|y(n)| \leq M_y < \infty$$

* If the output is unbounded for any bounded input, then the system is unstable,
the unstable system produce erritic output.

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