



Polar System

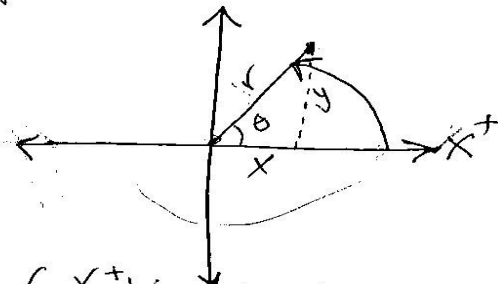
lec-8

* The system in which the coordinates represented by (r, θ) .

* It possible to convert the Cartesian system to Polar system as following:

$$(x, y) \rightarrow (r, \theta)$$

$$\begin{aligned} ① \quad r &= \sqrt{x^2 + y^2} \\ ② \quad \theta &= \tan^{-1} \left(\frac{y}{x} \right) \end{aligned}$$



θ : The angle between (r) and (x^+) in direction \circ

r : The distance between the origin and any point.

* It possible to convert the polar system to Cartesian system $[(r, \theta) \rightarrow (x, y)]$.

$$① \quad \cos \theta = \frac{x}{r}$$

$$\therefore \boxed{x = r \cos \theta}$$

$$② \quad \sin \theta = \frac{y}{r}$$

$$\therefore \boxed{y = r \sin \theta}$$

$$③ \quad r = \sqrt{x^2 + y^2}$$

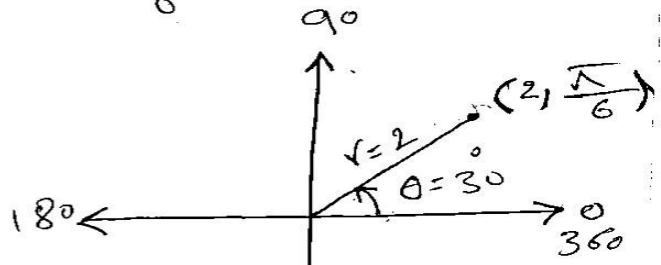
ملاحظة / دائماً الزاوية تحسب بين x^+ وإحداثيات y مع اتجاه \circ .

①

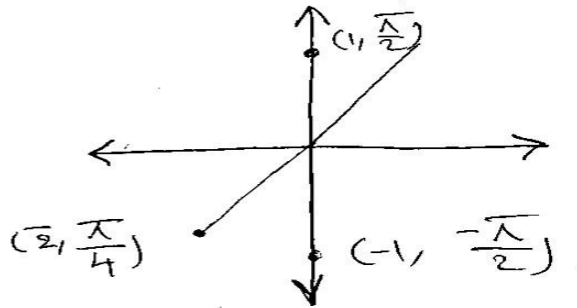


Ex 1

Ex 1:- Express the point $(2, \frac{\pi}{6})$?



- ② Express $(1, \frac{\pi}{2})$
 $(2, \frac{\pi}{4})$
 $(-1, -\frac{\pi}{2})$



③ $(r, \theta, z) = (1, \frac{\pi}{2}, 1)$, find Cartesian coordinates?

Sol:- $r=1$, $\theta = \frac{\pi}{2}$, $z=1$

$$x = r \cos \theta$$

$$x = 1 \times \cos \frac{\pi}{2} = 0$$

$$y = r \sin \theta$$

$$y = 1 \times \sin \frac{\pi}{2} = 1$$

$$z = 1$$

$$\therefore (x, y, z) = (0, 1, 1)$$

②



Exo- if the polar coordinates are $(5, 53^\circ)$, find the cartesian coordinates?

Sol-

$$x = r \cos \theta$$

$$x = 5 * \cos 53 = 3$$

$$\therefore x = 3$$

$$y = r \sin \theta$$

$$y = 5 * \sin 53 = 4$$

$$\therefore y = 4$$

$$\therefore (x, y) = (3, 4)$$

Ex- convert the cartesian system $(8, 6)$ to polar system if the point in first, second, third and fourth quarter?
حول انظما كارتيزي الى انظما قطبي في نقطة ربع اول، ربع ثانيا، ربع ثالث وربع رابعا

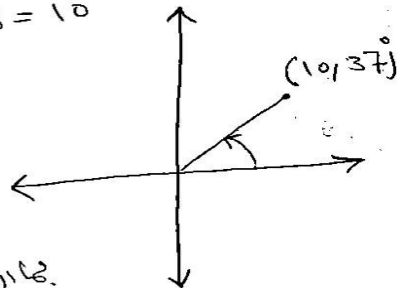
$$\text{Sol: } r^2 = x^2 + y^2 \Rightarrow r = \sqrt{64 + 36} = 10$$

$$\tan \theta = \frac{y}{x} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \theta = \tan^{-1}\left(\frac{3}{4}\right) = 37^\circ$$

$$(r, \theta) = (10, 37^\circ)$$

بما ان النقطه بالربع الاول فصيغتها θ نفسها المقترنه مع القانون
③





② إذا كانت النقطة بالربيع الثاني تقع $(-8, 6)$

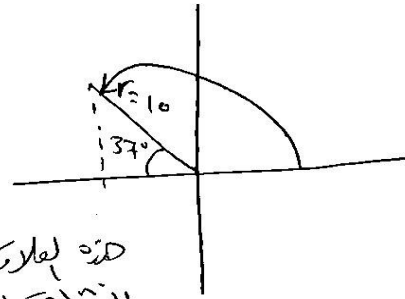
$$r = 10$$

$$\theta = \tan^{-1} \left(\frac{6}{-8} \right) = \tan^{-1} \left(-\frac{3}{4} \right)$$

$$\therefore \theta = \tan^{-1} \left(\frac{3}{4} \right) = 37^\circ$$

$$\theta_{\text{قطبي}} = 180 - \theta_{\text{من القانون}}$$

هذه العلامة تكون لإيجاد الزاوية القطبية إذا كانت
النقطة بالربيع الثاني



$$\therefore \theta_2 = 180 - 37$$

$$\therefore \theta = 143^\circ$$

$$(r, \theta) = (10, 143^\circ)$$

③ إذا كانت النقطة $(8, 6)$ بالربيع الثاني تقع $(-8, 6)$

$$r = \sqrt{x^2 + y^2} = 10$$

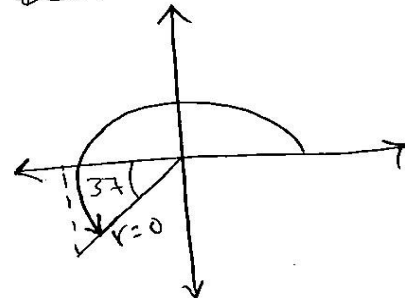
$$\theta = \tan^{-1} \left(\frac{-6}{8} \right) = 37^\circ$$

$$\theta_{\text{قطبي}} = 180 + \theta_{\text{من القانون}} \rightarrow \text{عنا تكون النقطة بالربيع الثالث}$$

$$\therefore \theta_2 = 180 + 37 = 217^\circ$$

$$\therefore (r, \theta) = (10, 217^\circ)$$

④





④ اذا كانت النقطة $(8, 6)$ بالربع الرابع تبصر $(8, -6)$:

$r = 10$

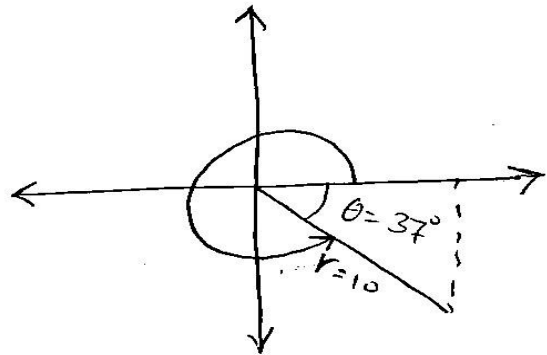
$\theta = 37^\circ$

$\theta = 360 - \theta$
 من القارة

$\theta = 360 - 37 =$

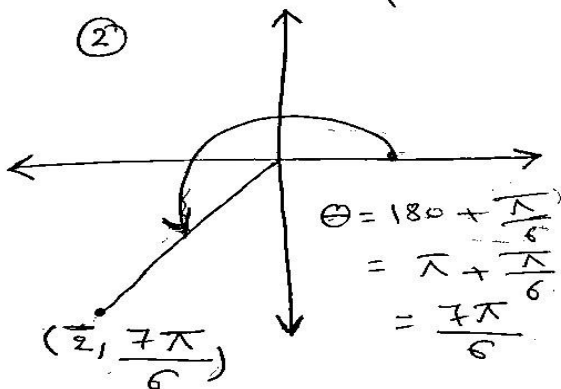
$\theta = 323^\circ$

$(r, \theta) = (10, 323^\circ)$

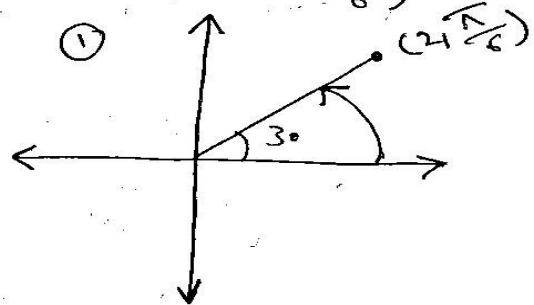


Ex: Draw all possibilities of the system $(2, \frac{\pi}{6})$

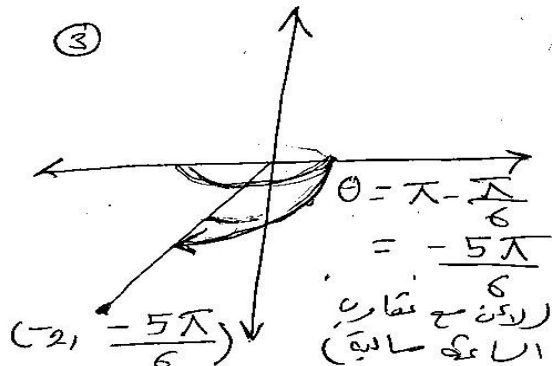
②



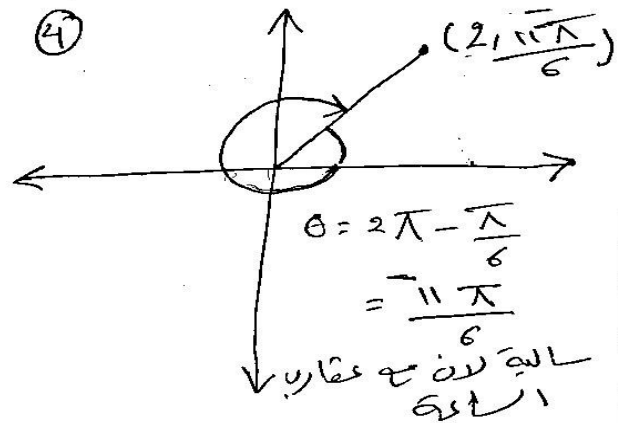
①



③



④



⑤