Quantum Mechanics

Eighth Lecture

Hermitian Operator

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2022- 2023

1. What is Hermitian operator?

Operator $\hat{A}$ is said to be hermition when satisfying the relation:

$$\int_{-\infty}^{\infty} \varphi_n^* \hat{A} \varphi_m d\tau = \int_{-\infty}^{\infty} \varphi_m \left(\hat{A}\varphi_n\right)^* d\tau$$

2. Properties of Hermitian operator

1) the eigen value correspond to any Hermitian operator are real quantities.

2) eigen function correspond to different eigen value are always orthogonal i.e.

$$\int_{-\infty}^{\infty} \varphi_n^* \varphi_m d\tau = 0$$

Example: prove that P_x is hermitian operator?

$$\hat{A} = \widehat{p_x}$$

$$\int_{-\infty}^{\infty} \varphi_n^* \hat{A} \varphi_m d\tau = \int_{-\infty}^{\infty} \varphi_m \left(\hat{A}\varphi_n\right)^* d\tau$$

$$\int_{-\infty}^{\infty} \varphi_n^* (-i\hbar \frac{\partial}{\partial x}) \varphi_m dx$$

$$-i\hbar \int_{-\infty}^{\infty} \varphi_n^* \frac{\partial}{\partial x} \varphi_m dx - - - - - - - - 1$$

$$\int_{-\infty}^{\infty} u\,dv = uv_{-\infty}^{\infty}| - \int_{-\infty}^{\infty} v\,du$$

$$u = \varphi_n^* \qquad dv = \frac{\partial}{\partial x}\varphi_m dx$$

$$du = \frac{\partial \varphi_n^*}{\partial x}dx \qquad v = \varphi_m$$

$$= -i\hbar\varphi_n^*\varphi_m \Big|_{-\infty}^{\infty} + i\hbar \int_{-\infty}^{\infty} \varphi_m \frac{\partial \varphi_n^*}{\partial x}dx$$

$$= i\hbar \int_{-\infty}^{\infty} \varphi_m \frac{\partial \varphi_n^*}{\partial x}dx$$

$$= \int_{-\infty}^{\infty} \varphi_m \left(i\hbar\frac{\partial \varphi_n^*}{\partial x}\right)dx$$

$$= \int_{-\infty}^{\infty} \varphi_m \left(-i\hbar\frac{\partial \varphi_n}{\partial x}\right)^* dx$$

$$= \int_{-\infty}^{\infty} \varphi_m (p_x\varphi_n)^* dx$$

[P_x] is hermitian operator