

# *Quantum Mechanics*

*Sixth Lecture*

## *Time Dependent Schrödinger Equation (T.D.S.E)*

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*Third Stage*

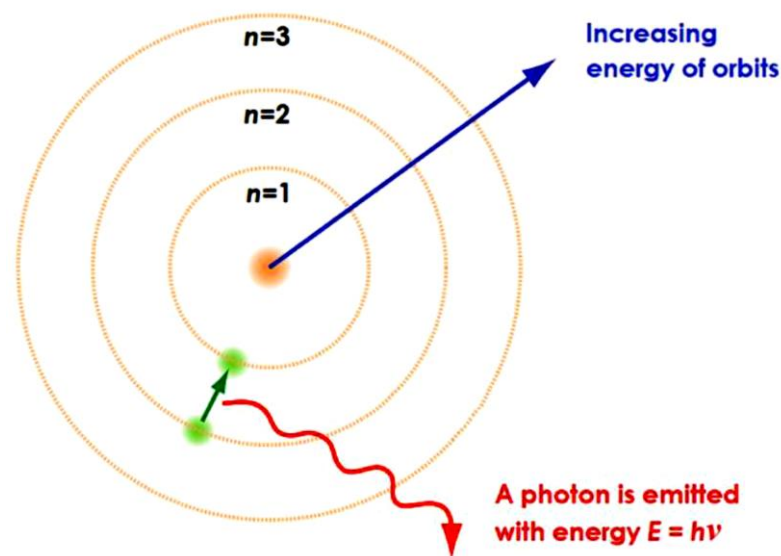
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## **1. Introduction**

- Through quantum experiments, Planck was able to prove that
  - ✓ energy is quantized (existence of electrons on specific energy levels)
  - ✓ The radiation has a dual behavior (wave-particle duality).
- Energy can only be absorbed and emitted with specific values.
- Quantum levels



### **✚ The Planck's postulate**

$$E = h\nu$$

This equation restates that “photon energy (E) is proportional to its frequency ( $\nu$ ) multiplied by Planck’s constant (h) which is equal to  $6.63 \times 10^{-34}$  Js”.

## **2. The Schrodinger Equation**

- The Schrödinger equation describes the quantum state of a particle.
- *It states that every particle has a wave function accompanying it, and that this function is spread to fill the entire cosmic void, and this function*

*is stronger in the place of the particle and weakens as we move away from it*

- Similar in classical mechanics, this formula is used to predict certain physical outcomes in a given system:  $F = ma$
- As well as another formula that is used to determine the energy for an object in a given system:  $E_{\text{total}} = E_{\text{kinetic}} + E_{\text{potential}}$
- The Schrodinger equation:

$$E\Psi(x) = \frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2} + V\Psi(x)$$

- The  $V$  in  $V\Psi(x)$  is sometimes written as the potential energy of a function.
- The  $\Psi$  is the Greek symbol psi and known as the wave function, “which describes the probability of the quantum state of a particle in a system”.

$\frac{-\hbar^2}{2m} \frac{d^2\Psi(x)}{dx^2}$  is the kinetic energy of the system.

- Deriving the formula for kinetic energy from the classical physics energy formula will give us this result:

$$E_t = E_k + E_p$$

$$E_t = \frac{1}{2}mv^2 + V$$

- ✚ For substituting momentum ( $p$ ) in the equation:

$$p = mv$$

$$E = \frac{p^2}{2m} + V \dots\dots\dots(1)$$

- ✚ The general form of the wave equation where  $k$  is the wave vector,  $\omega$  is the angular frequency

$$\Psi(x, t) = A e^{i(kx - \omega t)} \dots\dots\dots(2)$$

- ✚ From the de Broglie relation between wavelength and momentum

$$\lambda = \frac{h}{p}$$

$$p = \hbar k$$

where  $k = 2\pi/\lambda$  and  $\hbar = h/2\pi$ ,  $h$  being the Planck's constant.

$$k = \frac{p}{\hbar}$$

where  $k$  denotes the wave vector.

- from Einstein's explanation of the photoelectric effect, the energy  $E$  of the particle is related to the frequency through the relation:

$$E = \hbar \omega$$

$$\therefore \omega = \frac{E}{\hbar}$$

by using eq. of  $k$  and  $\omega$  in eq. (2) we take:

$$\Psi(x, t) = A \exp \left[ \frac{i}{\hbar} (px - Et) \right] \dots\dots\dots(3)$$

- Derive equation (3) with respect to time, substitute the value of the  $\Psi(x,t)$  and multiply by  $i\hbar$ , we have:

$$i \hbar \frac{\partial \Psi}{\partial t} = E \Psi(x, t) \dots\dots\dots(4)$$

- Derive equation (3) with respect to distance, substitute the value of the  $\Psi(x,t)$  and multiply by  $-i\hbar$ , we have:

$$-i \hbar \frac{\partial \Psi}{\partial x} = p \Psi(x, t) \dots\dots\dots(5)$$

- Derive equation (5) with respect to distance, substitute the value of the  $\Psi(x,t)$  and multiply by  $-i\hbar$ , we have:

$$-\hbar^2 \frac{\partial^2 \Psi}{\partial x^2} = p^2 \Psi(x,t) \quad \dots\dots\dots(6)$$

✚ Multiplying equation (1) by  $\Psi(x,t)$  we have:

$$E\Psi = \frac{p^2\Psi}{2m} + V\Psi \quad \dots\dots\dots(7)$$

By substituting the equations (4) and (6) in eq. (7) we have:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} + V\Psi \quad \dots\dots\dots(8)$$

✚ Equation (8) shows the time-dependent Schrödinger equation in one-dimension x.

### **3. Interpretation of the concept the Schrodinger Equation**

✚ By comparing the time-dependent Schrödinger equation with the classical energy equation, we note that this equation describes the total energy of a particle in a vacuum, which is the sum of the kinetic and potential energies.

✚ The wave function  $\Psi$  describes the probability of the quantum state of a particle in a system.

✚ For particle in one dimension the wave function of its state describe by  $\Psi(x,t)$ , where x is the position and t is the time.

✚ the wave function only describes the **probability** of the position of the particle.

✚ **The probability density** is another way of describing the relative likelihood that a value at any given sample in a space will be there.

✚ probability density function  $p(x,t)$  is given by:

$$|\Psi(x,t)|^2 = \Psi(x,t)^* \Psi(x,t) = p(x,t)$$

The probability that this particle will be at position  $x$  in the interval  $a \leq x \leq b$  is in the integral of the density:

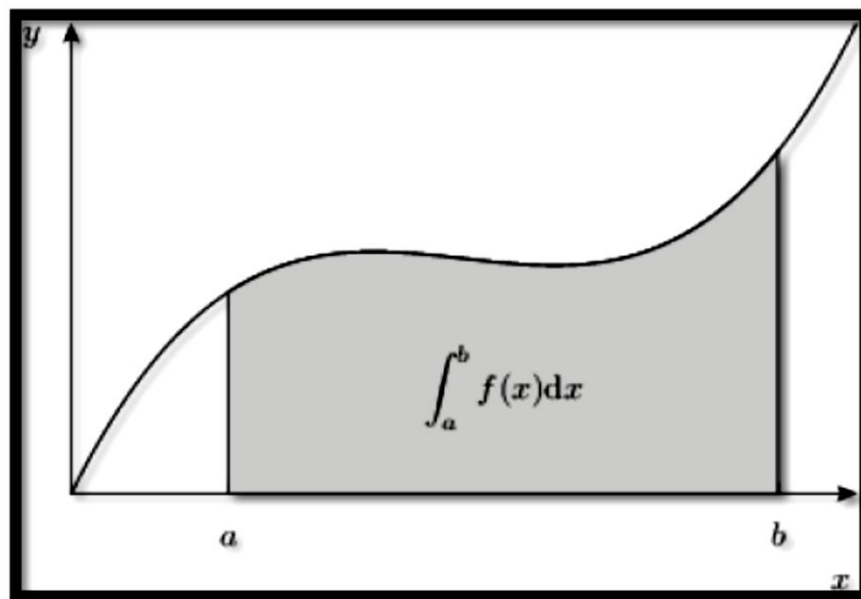
$$P_{a \leq x \leq b}(t) = \int_a^b dx |\Psi(x, t)|^2$$

A probability is a real number between 0 and 1, 0 meaning it has a 0% chance of happening and 1 meaning it has a 100% chance of happening. Since there is a 100% that if the particle is observed and measured, it will be always be somewhere.

This also means that a measurement of  $x$  must give a value between  $-\infty$  and  $+\infty$ , and

$P_{-\infty < x < \infty}$  must equal 1:

$$\int_{-\infty}^{\infty} dx |\Psi(x, t)|^2 = 1$$



Take this graph to be a 1-dimensional space for example. By taking the integral of the equation, we can therefore say that there is a 100% chance that a particle will be found in the area under the graph (space) from intervals  $a \leq x \leq b$ .