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Laser Basics
Third Stage

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قسم الفيزياء الطبية
اساسيات الليزر
المرحلة الثالثة

اساسيات الليزر

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Boltzmann Distribution

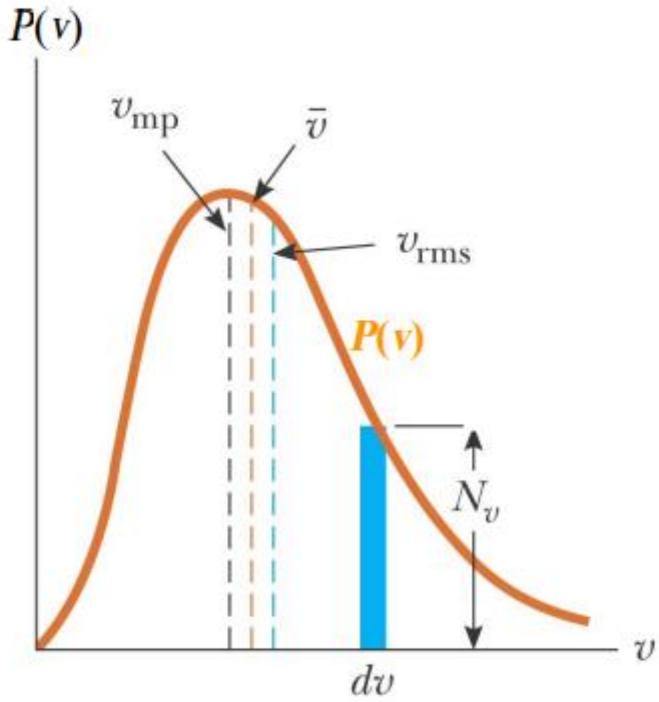
Boltzmann Distribution Law

- The motion of molecules is extremely chaotic
- Any individual molecule is colliding with others at an enormous rate
  - Typically at a rate of a billion times per second
- We introduce the **number density** $n_V(E)$
  - This is called a distribution function
  - It is defined so that $n_V(E)\,dE$ is the number of molecules per unit volume with energy between E and $E + dE$
- From statistical mechanics, the number density is
  $n_V(E) = n_0 e^{-E/k_BT}$ **Boltzmann distribution law**

■ The Boltzmann distribution law states that the probability of finding the molecule in a particular energy state varies exponentially as the energy divided by $k_B T$

■ The observed speed distribution of gas molecules in thermal equilibrium is shown at right

■P (v) is called the Maxwell- Boltzmann speed distribution function



■ The fundamental expression that describes the distribution of speeds in N gas molecules is

$$P(v) = 4\pi \left(\frac{m}{2\pi k_B T} \right)^{3/2} v^2 e^{-\frac{mv^2}{2k_B T}}$$

■ m is the mass of a gas molecule, k_B is Boltzmann's constant and T is the absolute temperature

■ The average speed is somewhat lower than the rms speed

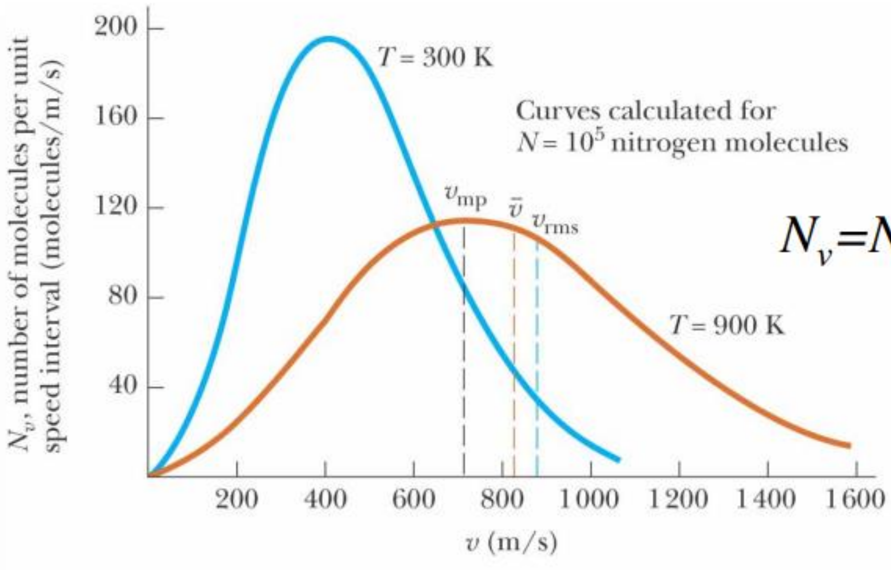
■ The most probable speed, v_{mp} is the speed at which the distribution curve reaches a peak

$$v_{mp} = \sqrt{\frac{2k_B T}{m}} = 1.41\sqrt{\frac{k_B T}{m}}$$

■ The peak shifts to the right as T increases

■ This shows that the average speed increases with increasing temperature

■ The asymmetric shape occurs because the lowest possible speed is 0 and the highest is infinity



$$N_v = NP(v)$$

■ P (v) is a probability distribution function, it gives the fraction of molecules whose speeds lie in the interval dv centered on the speed v

$$\int_0^\infty P(v)dv = 1$$

■The distribution of molecular speeds depends both on the mass and on temperature

■ The speed distribution for liquids is similar to that of gases even though the speeds are smaller in liquids than in gases

■ In solids, atoms do not have translational energy anymore, they vibrate. The only exception is solid helium, which is known to be a “quantum solid” where atoms can still move around.

Evaporation

■ Some molecules in the liquid are more energetic than others

■ Some of the faster moving molecules penetrate the surface and leave the liquid

■This occurs even before the boiling point is reached

■ The molecules that escape are those that have enough energy to overcome the attractive forces of the molecules in the liquid phase

■The molecules left behind have lower kinetic energies

■ Therefore, evaporation is a cooling process

<u>Example:</u> What is the *rms* speed of hydrogen at T=300 K? How much slower are O_2 molecules compared to H_2?

$$v_{rms} = \sqrt{\frac{3RT}{M}}$$

$M(H_2) = 2.016$ g/mole ➡ $v_{rms}^{H_2} = 1930$ m/s at $T = 300$ K

$$\frac{v_{rms}(O_2)}{v_{rms}(H_2)} = \sqrt{\frac{M_{H_2}}{M_{O_2}}}$$

$$\frac{v_{rms}(O_2)}{v_{rms}(H_2)} = \sqrt{\frac{2}{32}} = \frac{1}{4}$$ ➡ O_2 is 4 times slower than H_2