Al-Mustaqbal University College
Department of Medical Instrumentation Techniques Engineering
Class: second stage
Subject: Mathematics II
Lecturer: Dr. Diyar Hussain Habbeb
Lecture: Lec 2

unit vector

متجه الـ unit vector يعني ان المتجه طوله = 1 .

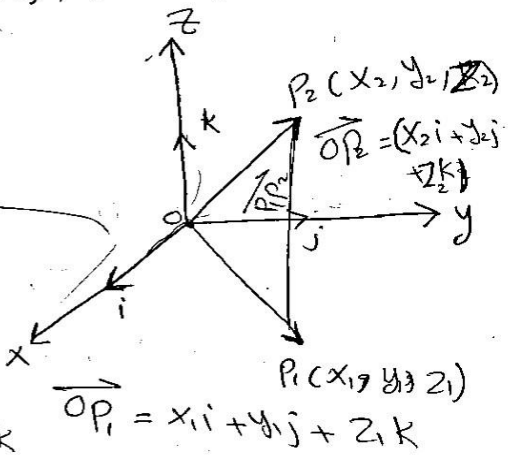
$i = (1,0,0)$

$j = (0,1,0)$

$K = (0,0,1)$

$V = (V_1, V_2, V_3) = (V_1,0,0) + (0,V_2,0) + (0,0,V_3)$

$\Rightarrow \boxed{V = V_1 i + V_2 j + V_3 K}$



$\overrightarrow{OP_1} = x_1 i + y_1 j + z_1 k$

$\overrightarrow{OP_2} = x_2 i + y_2 j + z_2 k$

$\overrightarrow{P_1P_2} = (x_2 - x_1) i + (y_2 - y_1) j + (z_2 - z_1) k$

ملاحظة / لو كان المتجه $V \neq 0$ اي ان طوله مساوي صفر $|V| \neq 0$

$$\therefore \left| \frac{1}{|V|} \cdot V \right| = \frac{1}{|V|} \cdot |V| = 1$$

Ex:- Find the unit vector u in the direction of the vector from $P_1(1,0,1)$ to $P_2(3,2,0)$?

Sol:-

$$\overrightarrow{P_1P_2} = (3-1) i + (2-0) j + (0-1) k$$

$$\overrightarrow{P_1P_2} = 2i + 2j - K$$

$$|P_1P_2| = \sqrt{2^2 + 2^2 + (-1)^2} = \sqrt{9} = 3$$

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Unit vector $(U) = \dfrac{\overrightarrow{P_1P_2}}{|\overrightarrow{P_1P_2}|} = \dfrac{2i+2j-k}{3}$ (7)

Unit vector $(U) = \frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k$

Ex2: A force of 6N is applied in the direction of the vector $V = 2i+2j-k$. Express the force (F) as a product of its' magnitude and direction?

Sol:-

Force vector has magnitude (length) = 6

the unit vector $= \dfrac{V}{|V|} = \dfrac{2i+2j-k}{|V|}$

$\therefore \dfrac{V}{|V|} = \dfrac{2i+2j-k}{\sqrt{(2)^2+(2)^2+(-1)^2}} = \dfrac{2i+2j-k}{3}$

$\therefore F = |V| \cdot \dfrac{V}{|V|} = 6\left(\frac{2}{3}i + \frac{2}{3}j - \frac{1}{3}k\right).$

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(9)

The dot product

الضرب النقطي

ملاحظة مهمة: الناتج من عملية الضرب النقطي هو عدد حقيقي (Scalar) أي ناتج عملية الضرب كمايلي:

$$\vec{u} = u_1 i + u_2 j + u_3 k$$
$$\vec{v} = v_1 i + v_2 j + v_3 k$$
$$\vec{u} \cdot \vec{v} = (u_1 \cdot v_1) + (u_2 \cdot v_2) + (u_3 \cdot v_3) = ?$$

Ex1:- $\vec{u} = i - 2j - k$, $\vec{v} = -6i + 2j - 3k$

$$\vec{u} \cdot \vec{v} = (-6) + (-4) + (3) = -7$$

Ex2: $\vec{u} = \frac{1}{2} i + 3j + k$

$$\vec{v} = 4i - j + 2k$$

$$\vec{u} \cdot \vec{v} = (2) + (-3) + (2) = 1$$

ملاحظة 1/2 حاصل ضرب نقطي لاي متجه مع نفسه يساوي 1

$$i \cdot i = j \cdot j = k \cdot k = 1$$

ملاحظة 3/

$$i \cdot j = j \cdot k = k \cdot i = k \cdot j = 0$$

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تطبيقات الضرب النقطي

① Angle Between vectors :-

$\vec{u}$, $\theta = ?$, $\vec{v}$

$$\vec{u}\cdot\vec{v} = |u|\cdot|v|\cos\theta$$

$$\cos\theta = \frac{\vec{u}\cdot\vec{v}}{|u|\cdot|v|}$$

$$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{|u|\cdot|v|}$$

① $0 \le \theta \le \pi$

② $\vec{u}$, $\vec{v}$ $\theta = 0$

③ $\vec{u}$, $\vec{v}$ $\theta = \pi$

EX:- Find the angle between $\vec{u} = i - 2j - 2k$, $\vec{v} = 6i + 3j + 2k$.

Sol:-

$$\vec{u}\cdot\vec{v} = (6) + (-6) + (-4) = -4$$

$$|u| = \sqrt{(1)^2 + (-2)^2 + (-2)^2} = \sqrt{9} = 3$$

$$|v| = \sqrt{(6)^2 + (3)^2 + (2)^2} = \sqrt{49} = 7$$

$$\theta = \cos^{-1}\frac{\vec{u}\cdot\vec{v}}{|u|\cdot|v|}$$

$$\theta = \cos^{-1}\left(\frac{-4}{21}\right) \Rightarrow \theta = 100.98^{\circ}$$

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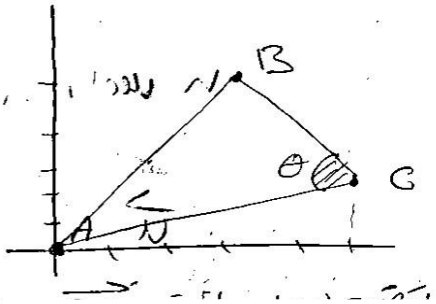
Ex:- Find the angle in triangle ABC determined by the vertices A=(0,0), B(3,5), and C=(5,2) ?

Sol:-



$\overrightarrow{CA} = (-5,-2) \rightarrow \overrightarrow{CA} = -5i - 2j$

$\overrightarrow{CB} = (-2,3) \rightarrow \overrightarrow{CB} = -2i + 3j$

$$\theta = \cos^{-1} \frac{\overrightarrow{CA} \cdot \overrightarrow{CB}}{|\overrightarrow{CA}| \cdot |\overrightarrow{CB}|}$$

$\overrightarrow{CA} \cdot \overrightarrow{CB} = (10) + (-6) = 4$

$|\overrightarrow{CA}| = \sqrt{25+4} = \sqrt{29}$

$|\overrightarrow{CB}| = \sqrt{4+9} = \sqrt{13}$

$$\theta = \cos^{-1}\left(\frac{4}{\sqrt{29}\cdot\sqrt{13}}\right)$$

$\therefore \theta = 78.11^{\circ}$