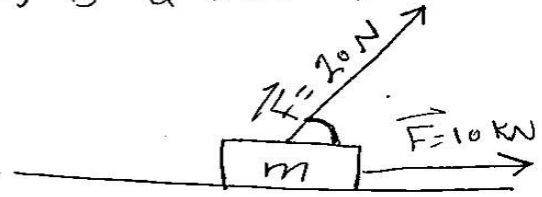
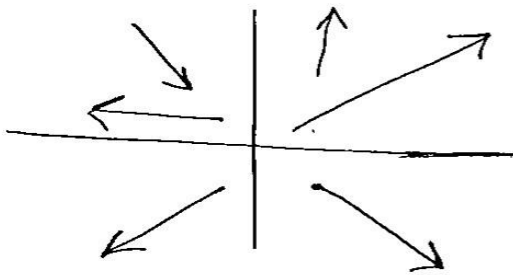




# linear Algebra

①

vectors: any thing has (magnitude) & (direction)  
 For example the force (F) is a factor.

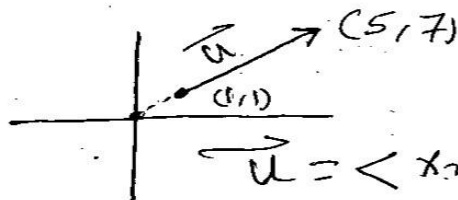


• اختلاف اطوال المتجهات يعني ان طول متجه  
 له قوى مختلفة عن الاخر.  
 • حول المتجه = القيمة.

• بالاضافة جميع المتجهات يجب ان تبدأ من نقطة الاصل (0,0) لذلك عندما  
 يكون المتجه ليس من نقطة الاصل وله نقطتان، نقطة بداية  $(x_1, y_1)$   
 ونقطة نهاية  $(x_2, y_2)$  نستخدم عملية (التصغير) للنتيجة -

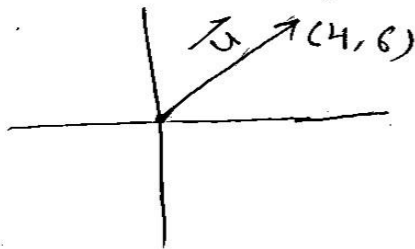
$$\vec{u} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

EX:-



$$\vec{u} = \langle x_2 - x_1, y_2 - y_1 \rangle$$

sol:-  $\vec{u} = \langle 5-1, 7-1 \rangle \Rightarrow \vec{u} = \langle 4, 6 \rangle$





②

Determination the length and direction of vectors:

$$① \quad |\vec{u}| = \sqrt{u_1^2 + u_2^2}$$

$$② \quad \cos \theta = \frac{u_1}{|\vec{u}|}$$

$$③ \quad \sin \theta = \frac{u_2}{|\vec{u}|}$$

Ex<sub>o</sub> - Find the length and direction of  $\vec{u}$ ?

$$\vec{u} = \langle 1, \sqrt{3} \rangle$$

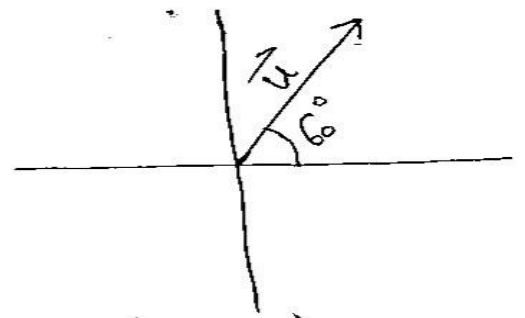
Sol:

$$|\vec{u}| = \sqrt{u_1^2 + u_2^2} = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2$$

$$\cos \theta = \frac{u_1}{|\vec{u}|} = \frac{1}{2}$$

$$\sin \theta = \frac{u_2}{|\vec{u}|} = \frac{\sqrt{3}}{2}$$

$$\theta = \frac{\pi}{3} = 60^\circ$$





Operations on Vector

① Addition operation:

عملية الجمع

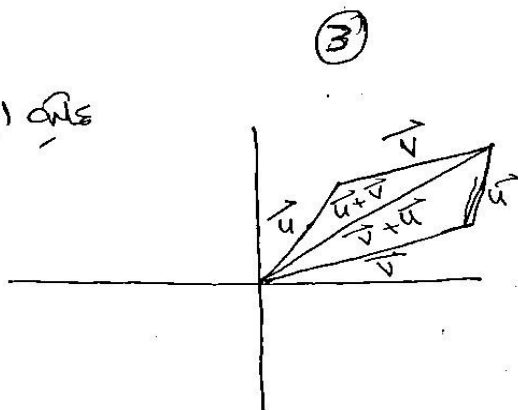
let  $u = (u_1, u_2, u_3)$

$v = (v_1, v_2, v_3)$

$\vec{u} + \vec{v} = (u_1 + v_1, u_2 + v_2, u_3 + v_3)$

$\vec{u} + \vec{v} = (v_1 + u_1, v_2 + u_2, v_3 + u_3)$

$\vec{u} + \vec{v} = \vec{v} + \vec{u} \Rightarrow$  الجمع ابراهيم



② multiply by scalar (عملية الضرب بـ ثابت)

$\vec{u} = (u_1, u_2, u_3)$

$k \cdot \vec{u} = (ku_1, ku_2, ku_3)$

الضرب بـ ثابت  
المتجه

Notes :-

① if  $k > +1 \Rightarrow$  change the magnitude only, and remain have the same direction.

② if  $k > -1 \Rightarrow$  change (magnitude + direction).

③ if  $k = -1 \Rightarrow$  change the direction only.

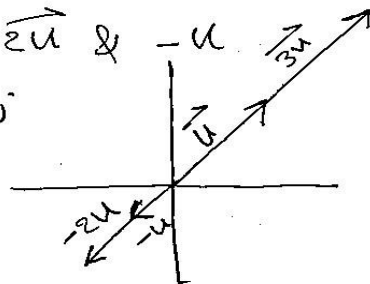
EX :- if  $\vec{u} = \langle 3, 5 \rangle$ , find  $3\vec{u}$ ,  $-2\vec{u}$  &  $-\vec{u}$

$\vec{u} = 3i + 5j$

$-\vec{u} = -3i - 5j$

$3\vec{u} = (9i + 15j)$

$-2\vec{u} = -6i - 10j$





Ex:- Prove that the length of  $k \cdot u$  equal to (4) Prop

$$|k| \cdot |u|, \quad \vec{u} = \langle u_1, u_2, u_3 \rangle$$

Sol:-

$$k \cdot u = (ku_1, ku_2, ku_3)$$

$$|k \cdot u| = \sqrt{(ku_1)^2 + (ku_2)^2 + (ku_3)^2}$$

$$|k \cdot u| = \sqrt{k^2 (u_1 + u_2 + u_3)^2}$$

$$|k \cdot u| = \sqrt{k^2} \cdot \sqrt{u_1^2 + u_2^2 + u_3^2}$$

$$|k \cdot u| = |k| \cdot |u|$$

③ The difference between vectors

$$u - v = u + (-v)$$

$$u - v = (u_1 - v_1), (u_2 - v_2), (u_3 - v_3)$$

Ex:- let  $u = \langle -1, 3, 1 \rangle$

$$v = \langle 4, 7, 0 \rangle$$

$$\text{③ } \left| \frac{1}{3} \cdot v \right|$$

Sol:-

$$2u = \langle -2, 6, 2 \rangle, \quad 3v = \langle 12, 21, 0 \rangle$$

$$\text{① } 2u + 3v = \langle 10, 27, 2 \rangle$$

$$\text{② } u - v = (-1, 3) - (4, 7, 0)$$

$$u - v = (-5, -4, 1)$$

$$\text{③ } \left| \frac{1}{3} \cdot u \right| = \left| \frac{1}{3} \cdot |u| \right|$$

$$= \frac{1}{3} \cdot \sqrt{(-1)^2 + (3)^2 + (1)^2} \Rightarrow \frac{1}{3} \cdot \sqrt{11} = \frac{\sqrt{11}}{3}$$