Analytic Mechanics

First lecture

Fundamental Concepts

Vectors

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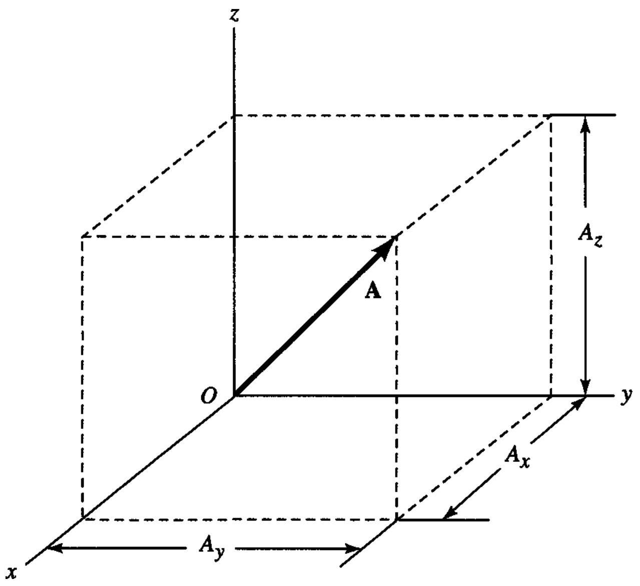
1. *Introduction*

The motion of dynamical systems is typically described in terms of two basic quantities: scalars and vectors.

1. A scalar is a physical quantity that has magnitude only, such as the mass of an object.

2. Vector has both magnitude and direction, such as the displacement, velocity, acceleration, and force.

The scalar quantity is represented by the symbol (A), we denote vector quantities simply by $(\vec{A})$, a given vector $(\vec{A})$, is specified by stating its magnitude and its direction relative to some arbitrarily chosen coordinate system. It is represented diagrammatically as a directed line segment, as shown in three-dimensional space in Figure 1.



The Vector $\vec{A} = iA_x + jA_y + kA_z$ means that there vector (A) is expressed on the right in terms of its components in a particular Coordinate system.

2. The Definitions and Rules

1. Equality of vectors تساوي المتجهات

$$\vec{A} = \vec{B}$$
$$[A_x, A_y, A_z] = [B_x, B_y, B_z]$$
$$A_x = B_x, A_y = B_y, A_z = B_z$$

Two vectors are equal if, and if their respective components are equal.

2. Vector Addition جمع المتجهات

$$\vec{A} + \vec{B} = (A_x, A_y, A_z) + (B_x, B_y, B_z) = (A_x + B_x, A_y + B_y, A_z + B_z)$$

3. Multiplication by a scalar الضرب بكمية عددية

If c is a scalar and A vector

$$c\vec{A} = c(A_x, A_y, A_z) = (cA_x, cA_y, cA_z) = \vec{A}c$$

The product cA is a vector whose components are c times those of A.

4.Vector subtraction طرح المتجهات

$$\vec{A} - \vec{B} = \vec{A} + (-1)\vec{B} = (A_x - B_x, A_y - B_y, A_z - B_z)$$

5.The null vector متجه الصفري

The vector 0 = (0, 0, 0) is called the null vector. The direction of the null vector is undefined. From (4) it follows that A - A =0. Because there can be no confusion when the null vector is denoted by a zero, we shall hereafter use the notation 0=0.

6. The Commutative Law of Addition قانون تبادل الحدود في الجمع

$$\vec{A} + \vec{B} = \vec{B} + \vec{A}$$

$A_x + B_x = B_x + A_x$ and similarly for the y and z components.

7. The Associative Law قانون ترتيب الحدود

$$\vec{A} + (\vec{B} + \vec{C}) = [A_x + (B_x + C_x),\ A_y + (B_y + C_y),\ A_z + (B_z + C_z)]$$

$$= (A_x + B_x) + C_x,\ (A_y + B_y) + C_y,\ (A_z + B_z) + C_z) = (\vec{A} + \vec{B}) + \vec{C}$$

8. The Distributive Law قانون توزيع الحدود

$$c(\vec{A} + \vec{B}) = c(A_x + B_x,\ A_y + B_y,\ A_z + B_z)$$

$$= c(A_x + B_x),\ c(A_y + B_y),\ c(A_z + B_z)$$

$$= (cA_x + cB_x,\ cA_y + cB_y,\ cA_z + cB_z)$$

$$= c\vec{A} + c\vec{B}$$

9. Magnitude of a vector مقدار المتجه

The magnitude of a vector A, denoted by $|A|$ or by A, is defined as the square root of the sum of the squares of the components, namely,

$$A = |\mathbf{A}| = \left(A_x^2 + A_y^2 + A_z^2\right)^{1/2}$$

The magnitude of a vector is its length.

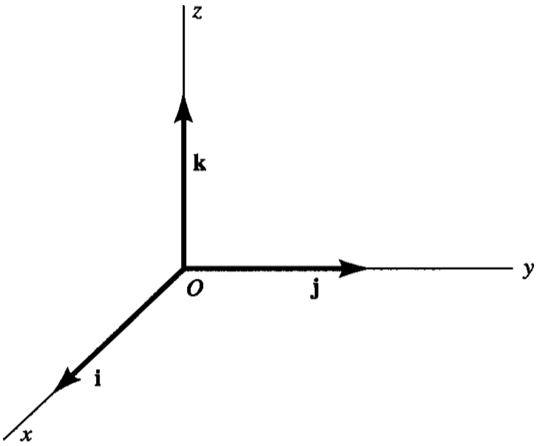
10. Unite Coordinate vector الوحدات المتجه للمحاور

A unit vector is a vector whose magnitude is unity. Unit vectors are often designated by the symbol

$$\hat{i} = [1,0,0], \quad \hat{j} = [0,1,0], \quad \hat{k} = [0,0,1]$$

The three unit vectors are called unit coordinate vectors or basis vectors. In terms of basis vectors, any vector can be expressed as a vector sum of components as follows:

$$\vec{A} = (A_x, A_y, A_z) = (A_x, 0, 0) + (0, A_y, 0 + (0, 0, A_z)$$
$$= A_x(1,0,0) + A_y(0,1,0) + A_z(0,0,1)$$
$$= \hat{i}A_x + \hat{j}A_y + \hat{k}A_z$$



The unit vectors ijk وحدات المتجه

11. The scalar Product الضرب العددي

Given two vectors A and B, the scalar product or "dot" product, A. B, is the scalar defined by the equation:

$$\mathbf{A} \cdot \mathbf{B} = A_x B_x + A_y B_y + A_z B_z$$

*scalar multiplication is commutative

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$$

* because $A_x B_x = B_x A_x$, and so on. It is also *distributive*,

$$\vec{A} \cdot (\vec{B} + \vec{C}) = \vec{A} \cdot \vec{B} + \vec{A} \cdot \vec{C}$$

$$\vec{A}.(\vec{B} + \vec{C}) = A_x(B_x + C_x) + A_y(B_y + C_y) + A_z(B_z + C_z)$$

$$= A_xB_x + A_yB_y + A_zB_z + A_xC_x + A_yC_y + A_zC_z$$

$$= \vec{A} . \vec{B} + \vec{A} . \vec{C}$$

$$\cos\theta = \frac{A_xB_x + A_yB_y + A_zB_z}{(A_x^2 + A_y^2 + A_z^2)^{\frac{1}{2}}\,(B_x^2 + B_y^2 + B_z^2)^{\frac{1}{2}}}$$

$$\cos\theta = \frac{\vec{A}.\vec{B}}{AB}$$

$$\vec{A}.\vec{B} = AB\cos\theta$$

From the definitions of the unit coordinate vectors i, j, and k, it is clear that the following relations hold:

$$\hat{i}.\hat{i} = \hat{j}.\hat{j} = \hat{k}.\hat{k} = 1$$

$$\hat{i}.\hat{j} = \hat{i}.\hat{k} = \hat{j}.\hat{k} = 0$$

12. The vector Product الضرب الاتجاهي

$* \ \bar{A} \times \bar{B} = [(A_yB_z - A_zB_y), (A_zB_x - A_xB_z), (A_xB_y - A_yB_x)]$

$[(A_xB_y - A_yB_x)].$

$* \ A \times B = Ab\sin\theta$

$* \ \bar{A} \times \bar{B} = -\bar{B} \times \bar{A}$

$* \ A \times (B + c) = (A \times B) + (A \times c)$

$* \ i \times i = j \times j = k \times k = 0$

$* \ j \times k = i = -k \times j$

Note: The first equation states that the cross product is anti-commutative.

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}$$

Let us calculate the magnitude of the cross product.

$$|\mathbf{A}\times\mathbf{B}|^2=(A_yB_z-A_zB_y)^2+(A_zB_x-A_xB_z)^2+(A_xB_y-A_yB_x)^2$$

$$|\mathbf{A}\times\mathbf{B}|^2=\left(A_x^2+A_y^2+A_z^2\right)\left(B_x^2+B_y^2+B_z^2\right)-(A_xB_x+A_yB_y+A_zB_z)^2$$

$$|\mathbf{A}\times\mathbf{B}|^2=A^2B^2-(\mathbf{A}\cdot\mathbf{B})^2$$

$$|\mathbf{A}\times\mathbf{B}|=AB(1-\cos^2\theta)^{1/2}=AB\sin\theta$$

$$\vec{A}\times\vec{B}=(AB\sin\theta)\vec{n}$$

Where n is a unit vector normal to the plane of the two vectors A and B

Example {1}

Given the two vectors $\mathbf{A}=2\mathbf{i}+\mathbf{j}-\mathbf{k}$, $\mathbf{B}=\mathbf{i}-\mathbf{j}+2\mathbf{k}$, find $\mathbf{A}\times\mathbf{B}$.

Solution:

In this case it is convenient to use the determinant form

$$\mathbf{A}\times\mathbf{B}=\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ 2 & 1 & -1\\ 1 & -1 & 2\end{vmatrix}=\mathbf{i}(2-1)+\mathbf{j}(-1-4)+\mathbf{k}(-2-1)$$

$$=\mathbf{i}-5\mathbf{j}-3\mathbf{k}$$

Example {2}

Find a unit vector norms to the plane containing the two vectors A and B ahove.

Solution:

$$\mathbf{n} = \frac{\mathbf{A}\times\mathbf{B}}{|\mathbf{A}\times\mathbf{B}|} = \frac{\mathbf{i}-5\mathbf{j}-3\mathbf{k}}{[1^2+5^2+3^2]^{1/2}}$$

$$= \frac{\mathbf{i}}{\sqrt{35}} - \frac{5\mathbf{j}}{\sqrt{35}} - \frac{3\mathbf{k}}{\sqrt{35}}$$