

Work # 2. ECG digital filtering

Objective: Practical realization of digital filtering.

- **The theory basics**

Digital filter — is a signal processing mathematical algorithm that can be determined by the following *difference equation*:

$$y(n) = \sum_{i=0}^{M-1} b_i x(n-i) + \sum_{j=1}^N a_j y(n-j), \quad (2.1)$$

where $x(n)$ and $y(n)$ – input and output signals samples, b_i and a_j – the filter *coefficients*, M and N are the coefficient numbers for the first and second sums.

Digital filter is called *finite impulse response* (FIR) filter, if all a_j coefficients are equal to zero, and *infinite impulse response* (IIR) filter at the opposite case. This work deals only with FIR filters.

The variants table (Table W.2) contains two sets of b_i coefficients that determine two different types of filters (called further “1” and “2” types). The filter “1” coefficients values are symmetrical relative to the central coefficient of the corresponding filter while the filter “2” coefficients values are anti-symmetrical (i.e. they are symmetrical with the sign minus). The *magnitude responses* for these two types of filters are determined correspondingly by the following two equations:

$$H(f) = b_{(M-1)/2} + 2 \sum_{k=1}^{(M-1)/2} \frac{b_{M-1-k}}{2} \cos(2\pi f k T); \quad (2.2)$$

$$H(f) = b_{(M-1)/2} + 2 \sum_{k=1}^{(M-1)/2} \frac{b_{M-1-k}}{2} \sin(2\pi f k T), \quad (2.3)$$

where f – frequency (Hz), and $T = 1/F_s$ – sampling interval for the sampling frequency F_s .

Impulse response of a digital filter is the output signal of the filter in case when its input signal represents a single impulse with the magnitude equal to one. *Step response* of a digital filter is the output signal of the filter in case when its input signal represents sequence of impulses with the magnitudes equal to one, preceded by zeros sequence.

- **The main tasks of the work**

- For the two given filters (“1” and “2” types) the following main characteristics must be calculated and plotted: coefficients set, magnitude response, impulse response and step response.

- The real ECG fragment must be read from the file and filtered by the two given filters. The curves of the ECG signals before and after the filtration must be plotted.

- **The procedure of the work**

Filters characteristics calculation. The filters coefficients sets for the filters of “1” and “2” types for each variant are given at the Table W.2.

1. Start MATLAB.
2. Set the path to your work folder.
3. Write the difference equation and magnitude response formulas for the filters of “1” and “2”, using the equations (2.1), (2.2) and (2.3) according to your variant from the Table W.2.

Consider as an example the type “1” filter determined by the following set of the coefficients: $b_0 = 1/10$, $b_1 = 2/10$, $b_2 = 4/10$, $b_3 = 2/10$, $b_4 = 1/10$.

Using (1.1) and (1.2) we can obtain:

$$y(n) = 0.1x(n) + 0.2x(n-1) + 0.4x(n-2) + 0.2x(n-3) + 0.1x(n-4)$$

$$\text{and } H(f) = 0.4 + 0.4\cos(2\pi fT) + 0.2\cos(4\pi fT).$$

4. Create a new m-file (MATLAB program) with the use of the New->Script item at the upper panel of the main MATLAB window and save it using some name.

5. Begin your program with creation of coefficients vectors for the two given filters. I.e. for the suggested above filter:

```
B1=[1/10 2/10 4/10 2/10 1/10]; % The filter coefficients vector
M1=length(B1); % Number of coefficients
```

6. Create a graphical window that contains eight fields for plots. These fields should form four rows and two columns:

```
subplot(4,2,1) % Create 4 by 2 window and activate the field #1
```

Left column is for the plots for the filter “1” and second column – for the filter “2”. These plots should have the following order:

- the filter coefficients;
- the magnitude response;
- the impulse response;
- the step response.

7. Plot in the upper fields the diagrams of “1” and “2” filters coefficients using the following example:

```
stem(B1)           % Stem diagram of the filter coefficients
title('Filter 1') % Title above the plot
```

In order to plot the magnitude response, the sampling frequency F_s should be determined. Here $F_s = 250$ Hz. Then the magnitude response can be calculated and plotted as follows:

```
Fs=250;           % Sampling frequency (Hz)
T=1/Fs;          % Sampling interval (s)
f=0:Fs/2;        % Frequency range for the magnitude response
                  % (from 0 Hz to Fs/2 Hz with the step 1 Hz)
H1=abs(2/5+2/5*cos(2*pi*f*T)+1/5*cos(4*pi*f*T)); % Magn. resp.
subplot(4,2,3)   % Activation of the field number "3"
plot(f,H1)       % Magnitude response plot
```

8. Run the program with the use of the Run command (green symbol ► at the upper panel of the editor window).

9. Calculate and plot the impulse responses for the filters “1” and “2”.

To calculate impulse response the special sequence should be formed that contains all values equal to zero except one value that is equal to one. This sequence should be used further as an input for the digital filter (for the difference equation of the filter). For the filter used as an example in can be done as follows:

```
N=20;           % Number of input sequence values
for n=1:N;      % The program cycle creating the input array
    X(n)=0;     % that contains "N" zero values
end
X(10)=1;       % Set 10-th element equal to "1"
Y=zeros(1,N);  % Create an output array containing all zeros
for n=M1:N      % Filtering of the input array
    Y(n)=0.1*X(n)+0.2*X(n-1)+0.4*X(n-2)+0.2*X(n-3)+0.1*X(n-4);
end
subplot(4,2,5) % Activation of the field number "5"
stem(X, '.')   % Input sequence plot
hold on
stem(Y, 'r')   % Output sequence plot; red color
```

Note: The output values can be calculated only for the output array elements $Y(M1)$ and further.

10. Using the above example calculate and plot the step responses for the filters “1” and “2”. The input sequence should contain zeros from the beginning and ones further.

11. Expand the graphical window to all screen, create a title for the window and save the plots as “png” or “bmp” file using the File->Save As item of the figure menu. Save the program text.

Filtration of the real ECG fragment. The names of the files containing ECG fragments for all variants are given at the Table W.2.

12. Put the needed ECG file into your work folder.

13. Create a new m-file and save it using some name.

14. Using as an example the first program, prepare the program that implements the following operations:

- reads the ECG fragment from the file (use the “load” command);
- creates graphical window containing three fields for plots;
- implements filtration of the ECG fragments using the both given filters;
- plots in different fields the curves of the input signal and of the output signals after the filters “1” and “2”.

15. Expand the window to all screen. If the “Y” scales of the plots are different, set them to the same ranges using the menu item Edit→Axes Properties.

16. Create a title of the window and save the plots as “png” or “bmp” file. Save the program text.

• **The report contents**

1. Title, goal and tasks of the work.
2. Texts of developed programs and all saved images.
3. Explanation of the results and conclusions.

• **Questions for the self-checking**

1. Write down the difference equation for a FIR filter. What is the difference from IIR filter?
2. What do impulse response and step response of digital filter mean?
3. What is the connection between impulse response of a FIR filter and the coefficients of its difference equation?
4. How to get the formulas for the magnitude and the phase responses using a set of a filter coefficients?
5. What does the magnitude response of a filter show?
6. How can be estimated the influence of a filter on a harmonic (sinusoidal) signal having a certain frequency with the use of the filter magnitude response?

7. What is the frequency range of the ECG signal?
8. Explain the changes in the ECG signal caused by the both filters in this work, using their magnitude responses.
9. What kind of properties has the second of the used filters within the lower part of the frequency range?
10. Why are the filtered signals slightly shifted to the right relative to the input ECG signal?

• **Supplement #2**

Table W.2

Variant	Filter	Coefficients b_i							File
		b_0	b_1	b_2	b_3	b_4	b_5	b_6	
1	“1”	1/8	1/4	1/4	1/4	1/8	–	–	W2_01.txt
	“2”	1/3	1/6	0	–1/6	–1/3	–	–	
2	“1”	1/9	2/9	1/3	2/9	1/9	–	–	W2_02.txt
	“2”	1/4	1/6	1/12	0	–1/12	–1/6	–1/4	
3	“1”	1/16	1/8	3/16	1/4	3/16	1/8	1/16	W2_03.txt
	“2”	1/12	1/6	1/4	0	–1/4	–1/6	–1/12	
4	“1”	1/5	1/5	1/5	1/5	1/5	–	–	W2_04.txt
	“2”	1/8	1/4	1/8	0	–1/8	–1/4	–1/8	
5	“1”	–1/3	1/3	1	1/3	–1/3	–	–	W2_05.txt
	“2”	1/6	1/6	1/6	0	–1/6	–1/6	–1/6	
6	“1”	–1/7	–1/7	3/7	5/7	3/7	–1/7	–1/7	W2_06.txt
	“2”	1/4	1/4	0	–1/4	–1/4	–	–	
7	“1”	–1/8	0	3/8	1/2	3/8	0	–1/8	W2_07.txt
	“2”	1/6	1/3	0	–1/3	–1/6	–	–	
8	“1”	–1/5	–1/5	2/5	1	2/5	–1/5	–1/5	W2_08.txt
	“2”	–1/10	1/8	3/8	0	–3/8	–1/8	1/10	
9	“1”	–3/10	0	1/2	3/5	1/2	0	–3/10	W2_09.txt
	“2”	1/5	1/3	1/9	0	–1/9	–1/3	–1/5	
10	“1”	1/12	1/9	1/6	1/4	1/6	1/9	1/12	W2_10.txt
	“2”	1/9	5/9	1/3	0	–1/3	–5/9	–1/9	

