Work # 3. ECG spectral analysis

*Objective:* Practical realization of ECG spectral analysis.

- **The theory basics**

Spectral analysis is used to obtain the signal frequency content quantitative estimations. Most widely used method is spectral analysis based on the *discreet Fourie transform,* determined by the equation:

$$X(k) = \sum_{n=0}^{N-1} x(n)e^{-j\,2\pi/N\,nK},$$

where $x(n)$ – discreet signal samples, N – number of samples, $k = 0, 1, \ldots, N–1$ – numbers of spectral components. *Fast Fourie transform* (FFT) is a special algorithm used for reduction of the calculation complexity.

In this work, FFT is used for the spectral analysis of three types of ECG rhythm: normal rhythm (curve *"a"* at the figure below), ventricular tachycardia (curve *"b"*) and ventricular fibrillation (curve *"c"*).

a

b

c
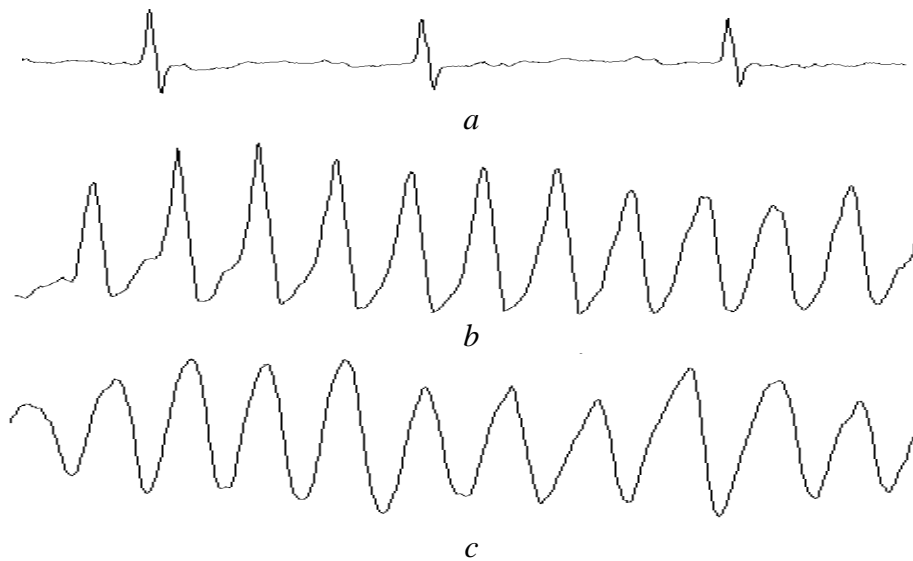
- **The main tasks of the work**

- Calculation of the test signal spectral estimations. Plotting the test signal and also its magnitude spectrum and *power spectral density* (PSD).
- Calculation of spectral estimations for the ECG fragments containing three different types of heart rate.

- **The procedure of the work**

Calculation of the test signal spectral estimations. The test signal represents sum of two sinusoidal signals and a constant. The values of sampling frequency (F_s), fragment duration (t_{max}), amplitudes (A_1, A_2) and frequencies (F_1, F_2) for the sinusoids and also the value of constant component (C) are given in the Table W.4.

1. Start MATLAB.
2. Set the path to your work folder.
3. Create a new m-file and save it using some name.
4. Create a graphical window that contains three rows and three columns of fields for plots using the **"subplot"** operator.
5. Calculate the test signal (sum of two sinusoids and constant component) and plot it inside the upper field of the left column.
6. Switch on the grid mode of the plot:

```
grid on
```

7. Calculate FFT for the test signals using MATLAB command "fft":

```
ft=fft(s);     % FFT for the signal "s"
```

8. Calculate the magnitude spectrum:

```
N=tmax*Fs;             % The number of test signal samples
for j=1:N              % Cycle for the magnitude spectrum calculation
    if (j==1)          % The case of zero frequency:
        ms(j)=sqrt(real(ft(j))^2+imag(ft(j))^2)/N;
    else               % All frequencies except zero:
        ms(j)=sqrt(real(ft(j))^2+imag(ft(j))^2)/N*2;
    end
end
```

9. Plot the calculated magnitude spectrum inside the upper field of the central column for the frequency range from 0 Hz to 0.5 F_s Hz:

```
df=Fs/N;               % Step along the frequency axis
for j=1:(N/2)
    f(j)=df*(j-1);  % Calculation of the argument vector
end
stem(f,ms(1:(N/2)),'.')  % Magnitude spectrum plot
```

10. Examine the magnitude spectrum: sizes of the spectral lines should be equivalent to the corresponding values of the sinusoids amplitudes. The line at the zero frequency corresponds to the constant component.

11. Calculate PSD of the test signal and plot it inside the upper field of the right column:

```
for j=1:N        % Cycle for the PSD calculation
    if (j==1)          % The case of zero frequency:
        psd(j)=1/N*(real(ft(j))^2+imag(ft(j))^2)/Fs;
    else               % All frequencies except zero:
        psd(j)=2/N*(real(ft(j))^2+imag(ft(j))^2)/Fs;
    end;
end;
axes(hAxes3)
plot (f,psd(1:(N/2)))% PSD plot
```

The spectral estimates calculated with the use of FFT have two significant disadvantages: 1) possibility of presence of very high component at zero frequency; 2) so called "spectral leakage" (false spectral components) due to the jumps at the ends of the analyzed signal fragment caused by the suggestion of the signal periodical nature. These two problems are resolved by the following measures: mean value has to be subtracted from the signal and the signal fragment has to be multiplied by a smoothing *window function* to suppress signal at the ends of the fragment.

12. Subtract mean value from the signal:

```
m=mean(s);       % Mean value for the signal "s"
s0=s-m;          % Mean value remove
```

13. In the fields of second row plot the obtained signal and also the plots of its magnitude spectrum and PSD. Set the grid for all plots on.

In case of correct mean value removing spectral componenst for the zero frequency should be equal to null.

14. Multiply the signal after the mean value removing by the "Hamming" window function:

```
w=hamming(N); % Window function array
for j=1:N      % Multiplication of signal by the window function:
    sw(j)=s0(j)*w(j);
end
```

15. In the left lower field plot simultaneously the following two curves: The signal multiplied by the window and the window function itself. Set the grid on.

16. In the other two fields of the lower row plot the magnitude spectrum and the PSD for the obtained signal.

17. Save the plots as "png" or "bmp" file using the File->Save As item of the figure menu. Save the program text.

18. Set some non-integer frequency value for one of the two sinusoids.

At the two upper spectra the *spectral leakage* effect should appear. At the lower spectrum this effect should be expressed not so evidently.

19. Save the plots as "png" or "bmp" file.

Calculation of spectral estimations for the real ECG fragments. Names of the files containing ECG recordings are given in the Table W.4. The sampling frequency $F_S = 250$ Hz.

Each file contains three ECG fragments from the same patient obtained during different stages of ventricular fibrillation attack: normal rhythm just before the attack, ventricular tachycardia and ventricular fibrillation.

20. Read this three ECG fragments into separate arrays. For instance:

```
ECG=load('W3_00.txt');   % Load all three signals into "ECG" array
Fs=250;                  % Sampling frequency, Hz
N=length(ECG(:,1));      % Fragment duration in samples
T=1/Fs;                  % Sampling interval, s
t=0:T:(N-1)*T;           % Argument for time axis
Norma=ECG(:,1);          % Normal rhythm array
VTach=ECG(:,2);          % Ventricular tachycardia array
VFibr=ECG(:,3);          % Ventricular fibrillation array
subplot(3,1,1)           % Plot these fragments to check it:
plot(t, Norma)
subplot(3,1,2)
plot(t, VTach)
subplot(3,1,3)
plot(t, VFibr)
```

21. Use the program prepared at the first part of thiswork as an example for the new program and save it using some name. The new program should plot two curves in each row of the left column: 1) Initial signal for the corresponding fragment; 2) The fragment after mean value removing and multiplication by smoothing window. Central and right column of each row should be used for the magnitude spectrum and PSD respectively. Set the following limits for the frequency axis: from 0 Hz to 20 Hz (that means first 80 values).

22. Save the plots as "png" or "bmp" file. Save the program text.

- **The report contents**

1. Title, goal and tasks of the work.
2. Texts of developed programs and all saved images.
3. Explanation of the results and conclusions.

1. How does the magnitude spectrum of the harmonic (sinusoidal) signal look?
2. Explain the appearance of the model signals spectra from the point of view of the signals properties.
3. How does the smoothing window influence a spectrum?
4. In what conditions the spectrum of harmonic (sinusoidal) signal has only one line?
5. What does "spectral leakage" mean? What are the causes of the spectral leakage?
6. What does "spectral resolution" mean? Does the use of smoothing windows improve spectral resolution or not?
7. What do spectral harmonics mean and that are the reasons of their presence in spectrum?
8. Explain the appearance of the power spectral density (PSD) for the three selected signal fragments.
9. What is the connection between the magnitude spectrum and the PSD?
10. What assumptions concerning the third ECG properties (when the ventricular fibrillation takes place) can be made considering this signal spectrum appearance and vice versa?

- **Supplement #3**

Table W.3

Variant	F_s, Hz	A_1	F_1, Hz	A_2	F_2, Hz	C	t_{max}, s	File
1	100	1.5	5.0	0.8	13.0	0.2	1.0	W3_01.txt
2	120	1.2	10.0	0.5	17.0	0.3	3.0	W3_02.txt
3	150	1.4	8.0	0.9	14.0	0.25	1.5	W3_03.txt
4	180	0.8	10.0	1.5	16.0	0.15	2.5	W3_04.txt
5	200	0.7	5.0	0.9	10.0	0.35	1.2	W3_05.txt
6	240	0.7	11.0	1.5	19.0	0.4	1.0	W3_06.txt
7	250	2.5	5.0	1.5	20.0	0.45	0.8	W3_07.txt
8	300	1.5	3.0	2.0	7.0	0.5	5.0	W3_08.txt
9	350	2.5	6.0	1.0	14.0	0.2	0.5	W3_09.txt
10	400	1.0	23.0	1.5	35.0	0.3	3.0	W3_10.txt