



## A\* search: Minimizing the total estimated solution cost

The most widely-known form of best-first search is called  $A^*$  search (pronounced "A-star search"). It evaluates nodes by combining g(n), the cost to reach the node, and h(n.), the cost to get from the node to the goal:

$$f(n) = g(n) + h(n)$$

Since g(n) gives the path cost from the start node to node *n*, and h(n) is the estimated cost of the *cheapest path* from n to the goal, we have

f(n) = estimated cost of the cheapest solution through n

Thus, if we are trying to find the cheapest solution, a reasonable thing to try first is the node with the lowest value of g(n) + h(n). It turns out that this strategy is more than just reasonable: provided that the heuristic function h(n) satisfies certain conditions. Note : This function is for review Only

```
function A*(start,goal)
    closedset := the empty set // T.
```

```
closedset := the empty set // The set of nodes already evaluated.
   openset := {start} // The set of tentative nodes to be evaluated,
   initially containing the start node
   came from := the empty map
                               // The map of navigated nodes.
   g score[start] := 0 // Cost from start along best known path.
    // Estimated total cost from start to goal through y.
    f score[start] := g score[start] + heuristic cost estimate(start,
    qoal)
    while openset is not empty
       current := the node in openset having the lowest f score[] value
        if current = goal
            return reconstruct path(came from, goal)
        remove current from openset
        add current to closedset
        for each neighbor in neighbor nodes (current)
            if neighbor in closedset
                continue
            tentative g score := g score[current] +
    dist between(current, neighbor)
            if neighbor not in openset or tentative g score <
    g score[neighbor]
                came from[neighbor] := current
                g score[neighbor] := tentative g score
                f score[neighbor] := g score[neighbor] +
   heuristic cost estimate (neighbor, goal)
                if neighbor not in openset
                    add neighbor to openset
    return failure
function reconstruct_path(came_from, current_node)
    if current node in came from
        p := reconstruct path(came from, came from[current node])
        return (p + current node)
    else
       return current node
```





### **Comperhensive Example for Function** A\*

- 1) Add the starting square (or node) to the open list.
- 2) Repeat the following:
  - a) Look for the lowest F cost square on the open list. We refer to this as the current square.
  - b) Switch it to the closed list.
  - c) For each of the 8 squares adjacent to this current square  $\dots$ 
    - If it is not walkable or if it is on the closed list, ignore it. Otherwise do the following.
    - If it isn't on the open list, add it to the open list. Make the current square the parent of this square. Record the F, G, and H costs of the square.
    - If it is on the open list already, check to see if this path to that square is better, using G cost as the measure. A lower G cost means that this is a better path. If so, change the parent of the square to the current square, and recalculate the G and F scores of the square. If you are keeping your open list sorted by F score, you may need to resort the list to account for the change.

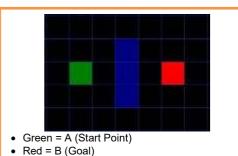
d) Stop when you:

• Blue= Obstacle Node

of 14 for a diagonal move

- Add the target square to the closed list, in which case the path has been found
- Fail to find the target square, and the open list is empty. In this case, there is no path.

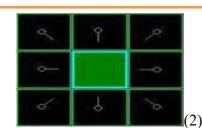
3) Save the path. Working backwards from the target square, go from each square to its parent square until you reach the starting square. That is your path.



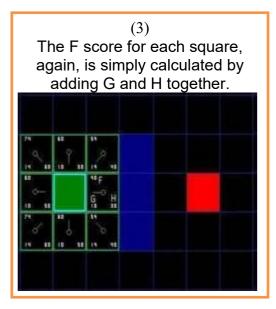
G cost, we will assign a cost of 10 to each horizontal or vertical square moved, and a cost

H Cost is the Manhattan Distance as an

example in vertical and horizontal only.



- Begin at the starting point A and add it to an "open list" of squares to be considered.
- Look at all the reachable or walkable squares adjacent to the starting point, ignoring squares with walls, water, or other illegal terrain. Add them to the open list, too. For each of these squares, save point A as its "parent square".
- Drop the starting square A from your open list, and add it to a "closed list" of squares that you don't need to look at again for now.



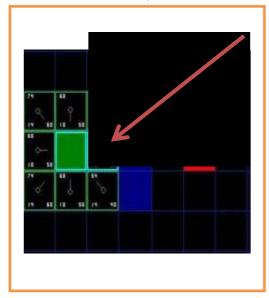
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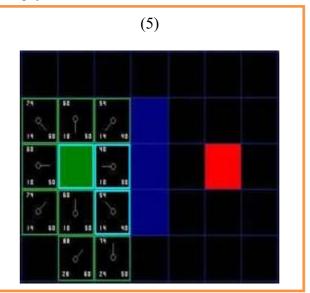
#### **Continuing the Search**

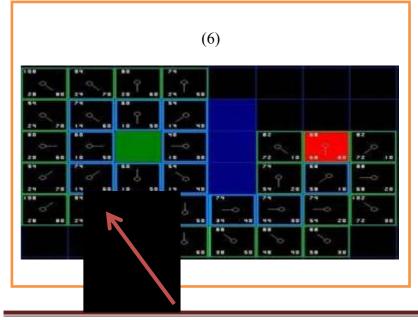
To continue the search, we simply choose the lowest F score square from all those that are on the open list (here we have 8 open nodes). We then do the following with the selected square:

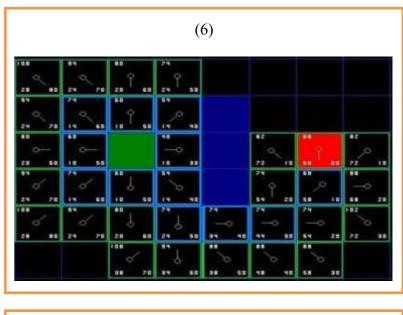
- 4) Drop it from the open list and add it to the closed list.
- 5) Check all of the adjacent squares. Ignoring those that are on the closed list or unwalkable (terrain with walls, water, or other illegal terrain), add squares to the open list if they are not on the open list already. Make the selected square the "parent" of the new squares.
- 6) If an adjacent square is already on the open list, check to see if this path to that square is a better one. In other words, check to see if the G score for that square is lower if we use the current square to get there. If not, don't do anything.

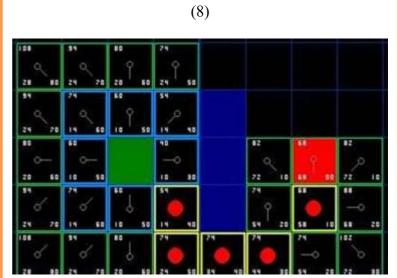
On the other hand, if the G cost of the new path is lower, change the parent of the adjacent square to the selected square (in the diagram above, change the direction of the pointer to point at the selected square). Finally, recalculate both the F and G scores of that square. If this seems confusing, you will see it illustrated below.











# <u>A\* Characteristics</u>

*1.* A\* search is both complete and optimal.

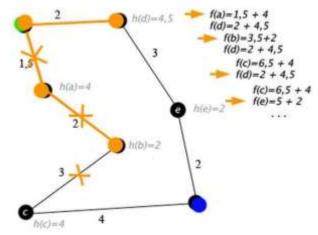
The optimality of  $A^*$  is straightforward to analyze if it is used with TREE-SEARCH. In this case,  $A^*$  is optimal if h(n) is an admissible heuristic-that is, provided that h(n) never overestimates the cost to reach the goal. Admissible heuristics (that is, it must not overestimate the distance to the goal) are by nature optimistic, because they think the cost of solving the problem is less than it actually is. Since g(n) is the exact cost to reach n, we have as immediate consequence that f(n) never overestimates the true cost of a solution through n. An obvious example of an admissible heuristic is the straight-line distance  $h_{SLD}$ that we used in getting to Bucharest example. Straight-line distance is admissible because the shortest path between any two points is a straight line, so the straight line cannot be an overestimate.

2. A\* uses a best-first search and finds a least-cost path from a given initial node to one goal node (out of one or more possible goals).

- 3. As A\* traverses the graph, it follows a path of the lowest expected total cost or distance, keeping a sorted priority queue of alternate path segments along the way.
- **4.** It uses a knowledge-plus-heuristic cost function of node x (usually denoted f(x)) to determine the order in which the search visits nodes in the tree. The cost function is a sum of two functions:
  - the past path-cost function, which is the known distance from the starting node to the current node x (usually denoted g(x))
  - a future path-cost function, which is an admissible "heuristic estimate" of the distance from x to the goal (usually denoted h(x)).

### Example

An example of an A star (A\*) algorithm in action where nodes are cities connected with roads and h(x) is the straight-line distance to target point:



### Example

Illustration of A\* search for finding path from a start node to a goal node in a robot motion planning problem. The empty circles represent the nodes in the open set, i.e., those that remain to be explored, and the filled ones are in the closed set. Color on each closed node indicates the distance from the start: the greener, the farther. One can first see the A\* moving in a straight line in the direction of the goal, then when hitting the obstacle, it explores alternative routes through the nodes from the open set.

