Lecture No. 5

Shear Force and Bending Moment Diagram

5.1 Introduction

The term *beam* refers to a slender bar that carries transverse loading; that is, the applied forces are perpendicular to the bar. In a beam, the internal force system consists of a shear force and a bending moment acting on the cross section of the bar. The study of beams, however, is complicated by the fact that the shear force and the bending moment usually vary continuously along the length of the beam.

Supports, Types and Loads

Beams are classified according to their supports and may be summarized as:

- **A simply supported beam**, Figure 6-1, has a pin support at one end and a roller support at the other end. The pin support prevents displacement of the end of the beam, but not its rotation. The term *roller support* refers to a pin connection that is free to move parallel to the axis of the beam; hence, this type of support suppresses only the transverse displacement.

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- **A cantilever beam** is built into a rigid support at one end, with the other end being free, Figure 6-1 b. The built-in support prevents displacements as well as rotations of the end of the beam.
- **An overhanging beam**, illustrated in Figure 6-1 c is supported by a pin and a roller support, with one or both ends of the beam extending beyond the supports.

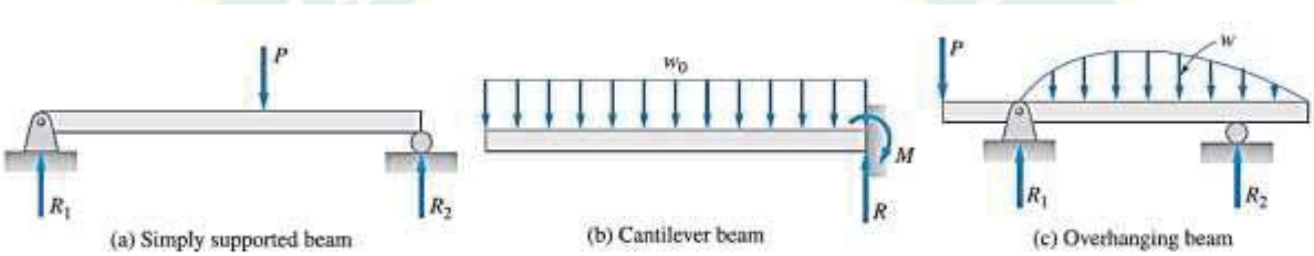


Figure 5-1 types of beams.

A *concentrated load*, such as *P in* Figure 1-*a*, is an approximation of a force that acts over a very small area. In contrast, a *distributed load* is applied over a finite area. If the distributed load acts on a very narrow area, the load may be approximated by a line load. The intensity *w* of this loading is expressed as force per unit length (N/m). The load distribution may be uniform, as shown in Figure 1-*b*, or it may vary with distance along the beam, as in Figure 1-*c*.

Shear-Moment Equations and Shear-Moment Diagrams

Consider the cantilever beam shown in Figure 6-2 a, which is subjected to a concentrated load *P* at the free end. If a cutting plane at *C* is drawn, a free body diagram through this section (Figure 6-2 b) shows a shear forces *V* and bending moment *M* at the cutting section. It is the objective in this section to determine the shear force *V* and the bending moment *M* at every cross section of the beam. To accomplish this task, we must derive the expressions for V and M in terms of the distance x measured along the beam. By plotting these expressions to scale, we obtain the *shear force and bending moment diagrams* for the beam.

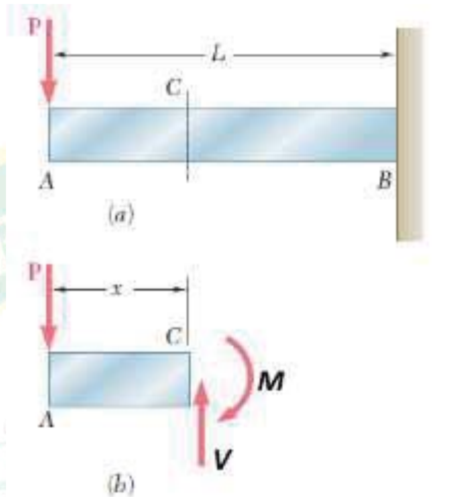


Figure 5-2 (*a*) Cantilever beam subjected to a concentrated load, (*b*) Section through *C*

Sign Conventions

It is necessary to adopt sign conventions for applied loading, shear forces, and bending moments. We will use the conventions shown in Figure 4, which assume the following to be *positive*.

Shear forces that tend to rotate a beam element *clockwise*.

Bending moments that tend to bend a beam element *concave upward*.

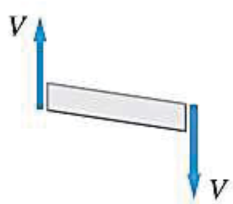
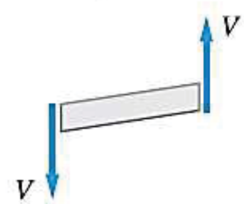
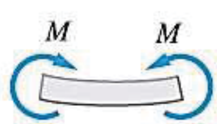
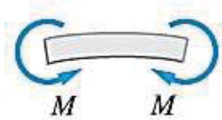
	Positive	Negative
Shear force		
Bending moment		

Figure 5-3: Sign Conventions.

Procedure for Determining Shear force and Bending Moment Diagrams

The following is a general procedure for obtaining shear force and bending moment diagrams of a statically determinate beam:

- Compute the support reactions from the *FBD* of the entire beam.
- Divide the beam into segments so that the loading within each segment is continuous. Thus, the end-points of the segments are discontinuities of loading, including concentrated loads and couples.

Perform the following steps for each segment of the beam:

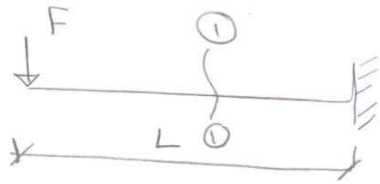
- Introduce an imaginary cutting plane within the segment, located at a distance *x* from the left end of the beam, that cuts the beam into two parts.
- Draw a *FBD* for the part of the beam lying either to the left or to the right of the cutting plane, whichever is more convenient. At the cut section, show *V* and *M* acting in their positive directions.
- Determine the expressions for *V* and *M* from the equilibrium equations obtainable from the FBD. These expressions, which are usually functions of *x*, are the shear force and bending moment equations for the segment. These equation can be obtained using the following:

$$V = (\Sigma F_y)_R = (\Sigma F_y)_L \qquad \text{(For Shear Force)}$$

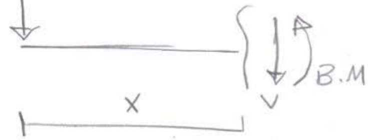
$$M = (\Sigma \mathrm{M})_L = (\Sigma \mathrm{M})_R \qquad \text{(For Bending Moment)}$$

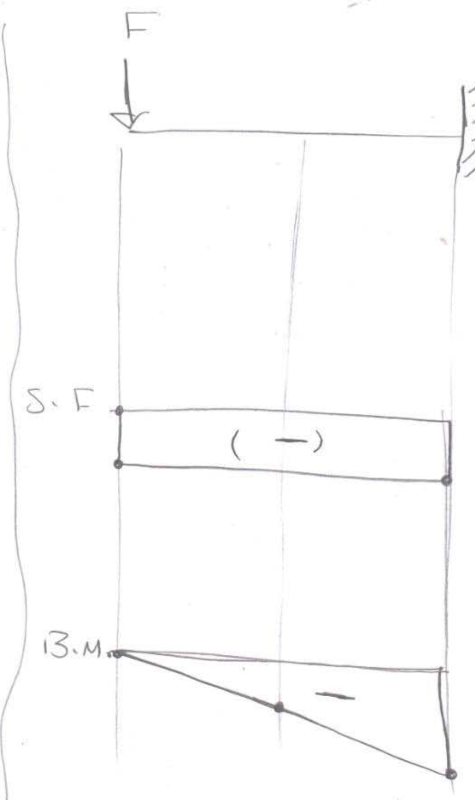
- Plot the expressions for *V* and *M* for the segment. It is visually desirable to draw the *V*-diagram (SFD) below the FBD of the entire beam, and then draw the *M*-diagram (BMD) below the *V*-diagram.

Cantilever with a *concentrated Load* at its Free End



S1 — 1 $\quad 0 \le x \le L$





S.F

$$\sum F_y = 0$$

$$-F - V = 0$$

$$\boxed{V = -F \ N}$$

B.M

$$\sum M_{1-1} = 0$$

$$-F(x) - B.M = 0$$

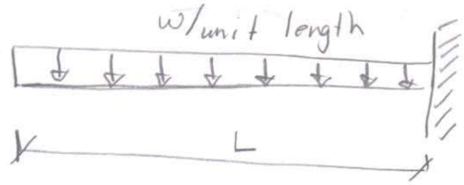
$$B.M = -F(x)$$

at $x = 0 \Rightarrow B.M = 0$

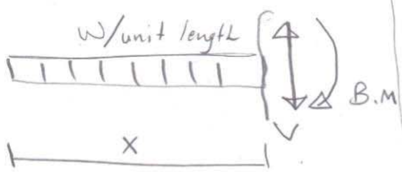
$x = \frac{L}{2} \Rightarrow B.M = -\frac{FL}{2}$

$x = L \Rightarrow B.M = -FL$

Cantilever with a *Uniformly Distributed* Load



Section ① — ①



$0 \leq x \leq L$

S.F

$\Sigma F_y = 0$

$-V - w(x) = 0$

$\boxed{V = -w(x)}$

at $x = 0 \rightarrow V = 0$

$x = \frac{L}{2} \rightarrow V = -\frac{wL}{2}$

$x = L \rightarrow V = -wL$

B.M

$\Sigma M = 0$

$B.M = w(x)\left(\frac{x}{2}\right)$

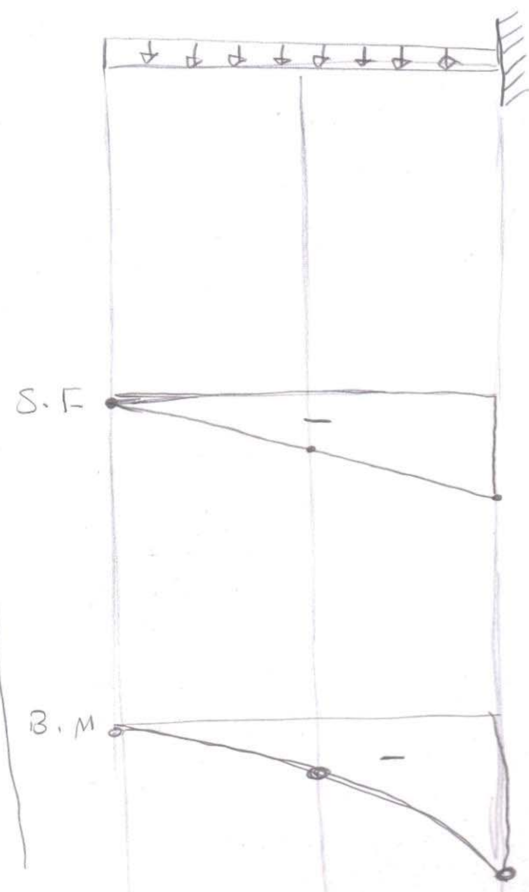
$\boxed{B.M = w\frac{x^2}{2}}$

B.M∘E

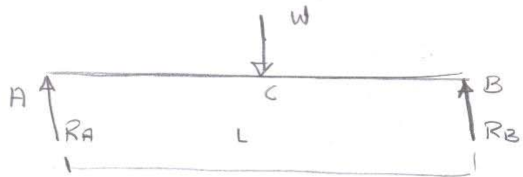
at $x = 0 \Rightarrow B.M = 0$

at $x = \frac{L}{2} \Rightarrow B.M = \frac{wL^2}{8}$

at $x = L \Rightarrow B.M = \frac{wL^2}{4}$



Simply Supported Beam with a *Concentrated Load* at its Mid-point



① Reactions

$\sum M_B = 0$

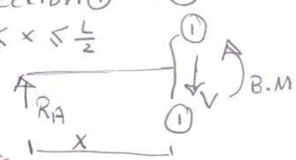
$\Rightarrow R_A(L) - W(\frac{L}{2}) = 0$

$\boxed{R_A = \frac{W}{2} \ N}$

$\sum F_y = 0 \Rightarrow \boxed{R_B = \frac{W}{2}}$

② Section ① — ①

$0 \leq x \leq \frac{L}{2}$



S.F ⌐

$\sum F_y = 0 \Rightarrow \boxed{V = \frac{W}{2}}$

S.F.E

B.M ⌐

$\sum M_{①-①} = 0$

$\boxed{B.M = \frac{W}{2}(x)}$

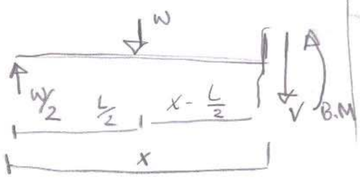
B.M.E

at $x = 0 \rightarrow B.M = 0$ N.m

$x = \frac{L}{4} \rightarrow B.M = \frac{WL}{8}$ N.m

$x = \frac{L}{2} \rightarrow B.M = \frac{WL}{4}$ N.m

③ Section ② — ②



S.F ⌐

$\sum F_y = 0$

$\frac{W}{2} - W - V = 0$

$\Rightarrow \boxed{V = -\frac{W}{2}}$

B.M ⌐

$\sum M = 0$

$B.M = \frac{W}{2}(x) - W(x - \frac{L}{2})$

at $x = \frac{L}{2}$

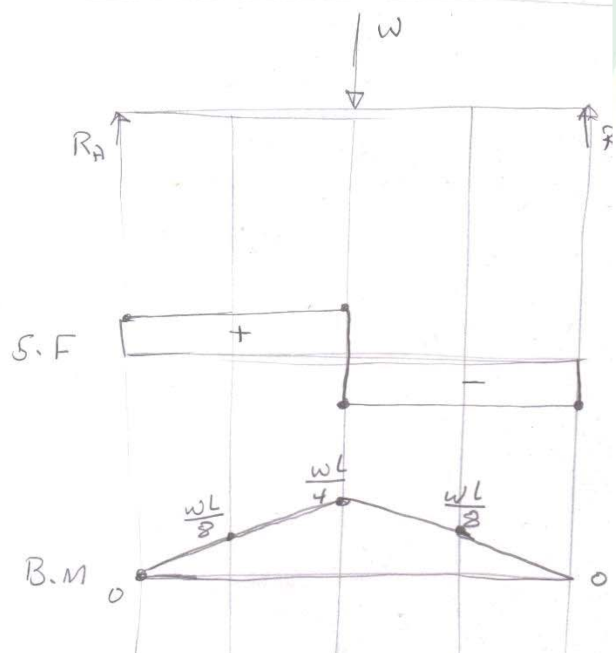
$B.M = \frac{WL}{4}$

at $x = \frac{3L}{4}$

$\Rightarrow B.M = \frac{WL}{8}$

at $x = L$

$\Rightarrow B.M = 0$



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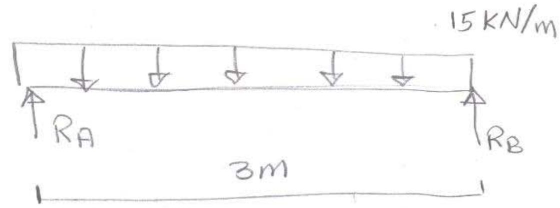
Simply Supported Beam with a *Uniformly Distributed Load*



① Reactions

$\Sigma M_B = 0$

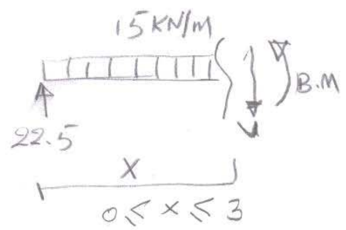
$R_A(3) - 15(3)(1.5) = 0$

$\boxed{R_A = 22.5\ KN}$

$\Sigma F_y = 0$

$\boxed{R_B = 22.5\ KN}$

② Sections



S-F

$\Sigma F_y = 0$

$22.5 - 15(x) - V = 0$

$\boxed{V = 22.5 - 15x}$

S.F.E

at $x = 0 \Rightarrow V = 22.5\ KN$

$x = 1 \Rightarrow V = 7.5\ KN$

$x = 2 \Rightarrow V = -7.5\ KN$

$x = 3 \Rightarrow V = -22.5$

B.M

$\Sigma M = 0$

$B.M = 22.5(x) - 15(x)(\frac{x}{2})$

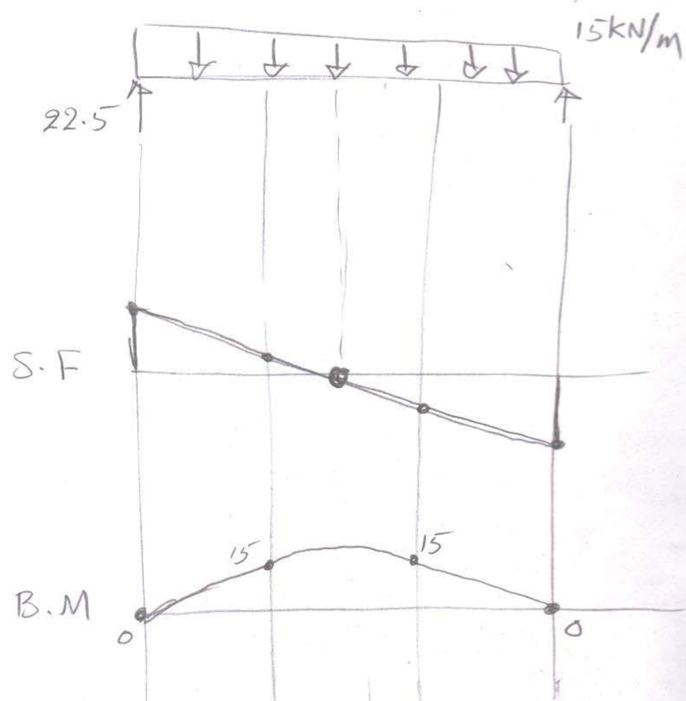
$\boxed{B.M = 22.5x - 15\frac{x^2}{2}}$

at $x = 0 \Rightarrow B.M = 0$

$x = 1 \Rightarrow B.M = 15\ KN.m$

$x = 2 \Rightarrow B.M = 15\ KN.m$

$x = 3 \Rightarrow B.M = 0$

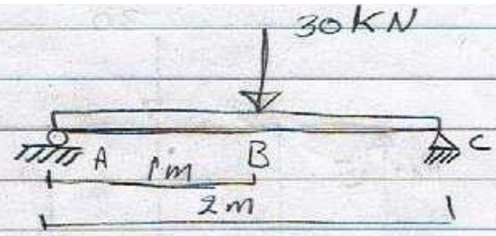


H.W

S.F at $x = 1.5\ m$

B.M at $x = 1.5\ m$

Example 1



Reactions/

$\sum M_B = 0$

$R_A(2) - 30(1) = 0$

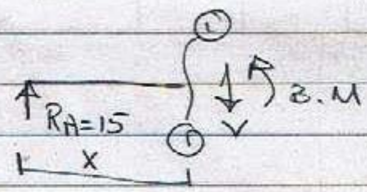
$R_A = 15\ KN$

$\sum F_y = 0$

$15 - 30 + R_B = 0$

$R_B = 15\ KN$

Section A-B ($0 \le x \le 1$)



$\sum F_y = 0$

$15 - V = 0$

$V = 15\ KN$

$\sum M = 0$

$15(x) - B.M = 0$

$B.M = 15x$

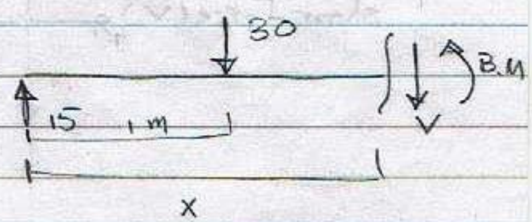
at $x = 0 \Rightarrow B.M = 0$

$x = 0.5 \Rightarrow B.M = 7.5\ KN.m$

$x = 1 \Rightarrow B.M = 15\ KN.m$

Section B-C

($1 \le x \le 2$)



$\sum F_y = 0$

$15 - 30 - V = 0 \Rightarrow \boxed{V = -15\ KN.m}$

$\sum F_y = 0$

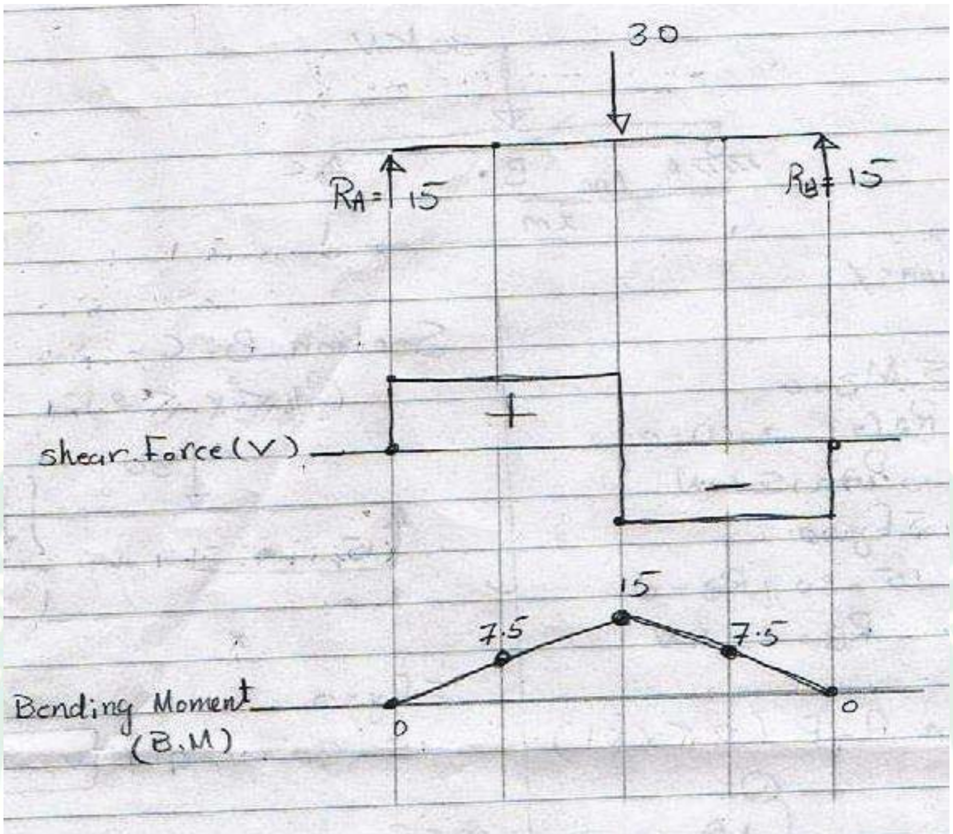
$15(x) - 30(x-1) - B.M = 0$

$B.M = 15(x) - 30(x-1)$

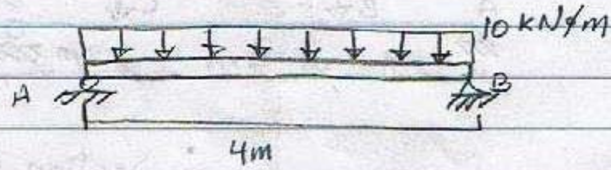
at $x = 1 \Rightarrow B.M = 15\ KN.m$

$x = 1.5 \Rightarrow B.M = 7.5\ KN.m$

$x = 2 \Rightarrow B.M = 0$


30
$R_A = 15$
$R_B = 15$
+
shear Force (V)
—
15
7.5
7.5
Bending Moment (B.M)
0
0

Example 2



Reactions

$\Sigma M_B = 0$

$R_A(4) - 10(4) = 0$

$R_A = 20\ kN$

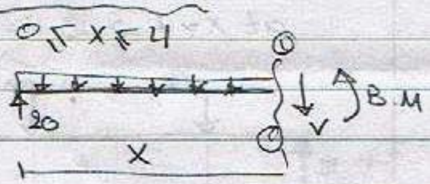
$\Sigma F_y = 0$

$20 - 10(4) + R_B = 0$

$R_B = 20\ kN.$

Section A-B

$0 \le x \le 4$



$\Sigma F_y = 0 \Rightarrow 20 - 40(x) - V = 0$

$V = 20 - 10(x)$

at $x = 0 \Rightarrow V = 20\ kN$

$x = 1 \Rightarrow V = 10\ kN$

$x = 2 \Rightarrow V = 0\ kN$

$x = 3 \Rightarrow V = -10\ kN$

$x = 4 \Rightarrow V = -20\ kN$

$\Sigma M = 0$

$20(x) - 10(x)\frac{x}{2} - B.M$

$B.M = 20x - 5x^2$

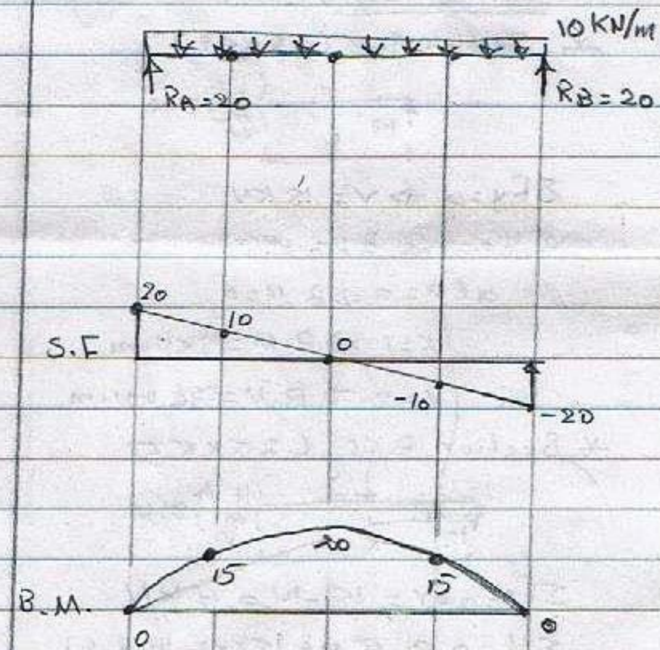
at $x = 0 \Rightarrow B.M = 0$

$x = 1 \Rightarrow B.M = 15\ kN \cdot m$

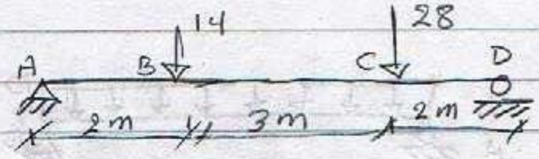
$x = 2 \Rightarrow B.M = 20\ kN \cdot m$

$x = 3 \Rightarrow B.M = 15\ kN \cdot m$

$x = 4 \Rightarrow B.M = 0\ kN \cdot m$



Example 3



Reactions/

$\Sigma M_D = 0$

$R_A(7) - 14(5) - 28(2) = 0$

$R_A = 18\ kN$

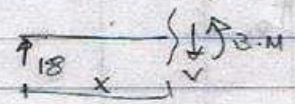
$\Sigma F_y = 0$

$18 - 14 - 28 + R_B = 0$

$R_B = 24\ kN$

S.F & B.M/

* Section A-B $(0 \leq x \leq 2)$



$\Sigma F_y = 0 \Rightarrow V = 18\ kN$

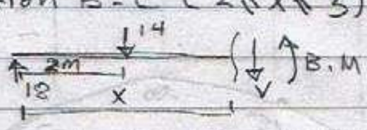
$\Sigma M = 0 \Rightarrow B.M = 18(x)$

at $x = 0 \Rightarrow B.M = 0$

$x = 1 \Rightarrow B.M = 18\ kN.m$

$x = 2 \Rightarrow B.M = 36\ kN.m$

* Section B-C $(2 \leq x \leq 5)$



$\Sigma F_y = 0 \Rightarrow V = 18 - 14 = 4\ kN$

$\Sigma M = 0 \Rightarrow B.M = 18(x) - 14(x-2)$

at $x = 2 \Rightarrow B.M = 36\ kN.m$

$x = 3 \Rightarrow B.M = 40\ kN.m$

$x = 4 \Rightarrow B.M = 44\ kN.m$

$x = 5 \Rightarrow B.M = 48\ kN.m$

* Section C-D $(5 \leq x \leq 7)$

Section C-D $(5 \leq x \leq 7)$

$\Sigma F_y = 0$

$18 - 14 - 28 - V = 0$

$V = -24\ kN$

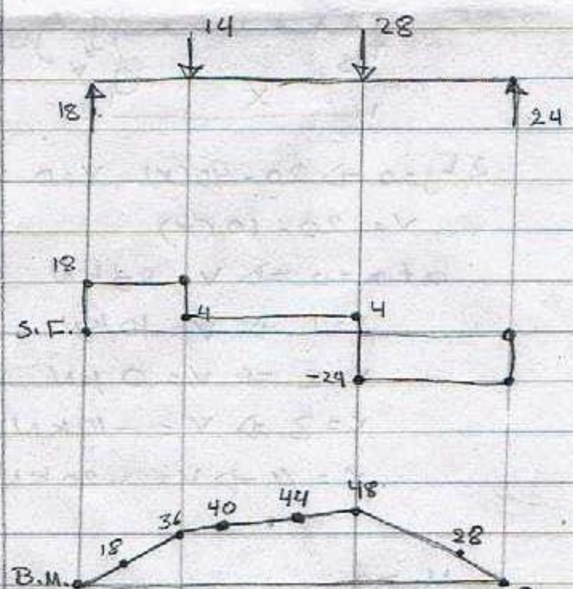
$\Sigma M = 0$

$B.M = 18(x) - 14(x-2) - 28(x-5)$

at $x = 5 \Rightarrow B.M = 48\ kN.m$

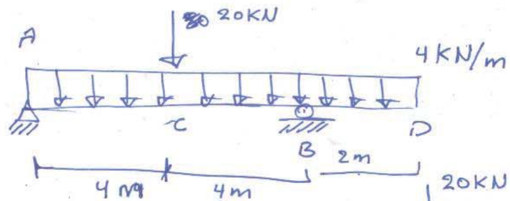
at $x = 6 \Rightarrow B.M = 28\ kN.m$

at $x = 7 \Rightarrow B.M = 0\ kN.m$



Example 4

example / Derive the shear force and bending moment equations for the beam shown in the Figure, and draw the shear force and bending moment diagrams.



Reactions

$\Sigma M_B = 0$

$R_A(8) - 4(8)(4) - 20(4) + 4(2)(1) = 0$

$\boxed{R_A = 25\ KN}$

$\Sigma F_y = 0$

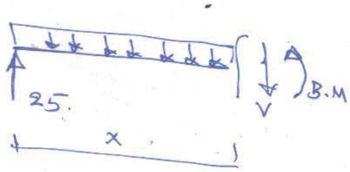
$25 - 4(10) - 20 + R_B = 0$

$\boxed{R_B = 35\ KN}$

S.F & B.M

$0 \leq x \leq 4$

Section A-C



$\Sigma F_y = 0$

$25 - 4(x) - V = 0$

$\boxed{V = 25 - 4x}$

at $x = 0 \Rightarrow V = 25\ KN$

at $x = 1 \Rightarrow V = 21\ KN$

at $x = 2 \Rightarrow V = 17\ KN$

at $x = 3 \Rightarrow V = 13\ KN$

at $x = 4 \Rightarrow V = 9\ KN$

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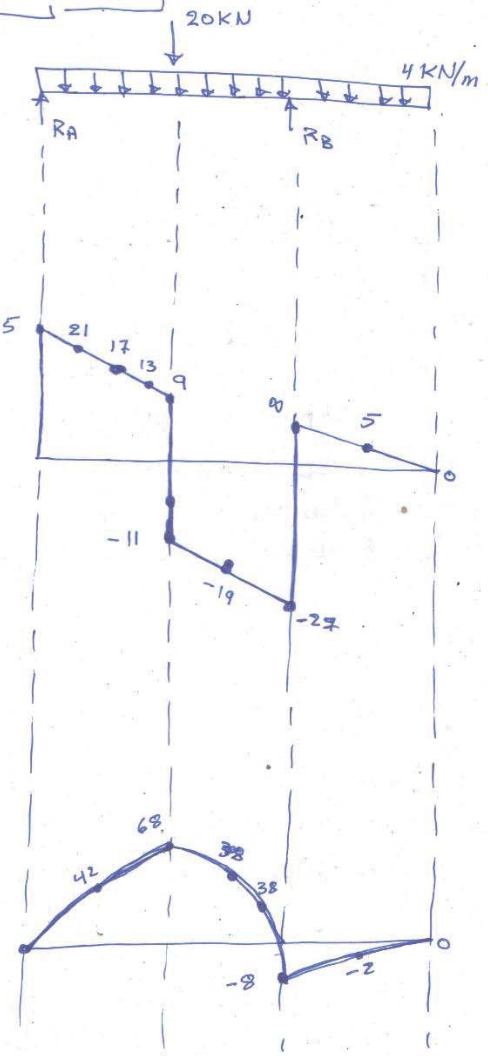


$\Sigma M = 0$

$25(x) - 4 \times \frac{x^2}{2} - B.M = 0$

$\boxed{B.M = 25x - 4\frac{x^2}{2}}$

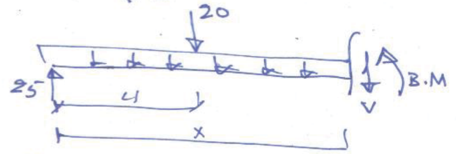
at $x = 0 \Rightarrow B.M = 0$

at $x = 2 \Rightarrow B.M = 42\,kN.m$

at $x = 4 \Rightarrow B.M = 68\,kN.m$

Section C-B

$4 \leq x \leq 8$



$\Sigma F_y = 0$

$25 - 20(x) - 20 - V = 0$

$\boxed{V = 25 - 20x - 20} \Rightarrow \boxed{V = 5 - 20x}$

at $x = 4 \Rightarrow V = -11\,kN$

at $x = 6 \Rightarrow V = -19\,kN$

at $x = 8 \Rightarrow V = -27\,kN$

$\Sigma M = 0$

$25(x) - 4\frac{x^2}{2} - 20(x-4) - B.M = 0$

$B.M = 25x - 2x^2 - 20(x-4)$

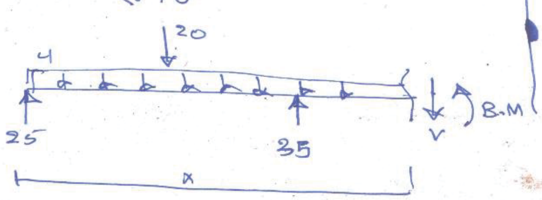
at $x = 4 \Rightarrow M = 68\,kN.m$

at $x = 6 \Rightarrow M = 38\,kN.m$

at $x = 8 \Rightarrow M = -8\,kN.m$

Section B-C

$8 \leq x \leq 10$



$\Sigma F_y = 0$

$25 - 4(x) - 20 + 35 - V = 0$

$\boxed{V = 40 - 4(x)}$

at $x = 8 \Rightarrow V = +8\,kN$

at $x = 9 \Rightarrow V = +5$

at $x = 10 \Rightarrow V = 0$

$\Sigma M = 0$

$25(x) - 4\frac{x^2}{2} - 20(x-4) + 35(x-8) - B.M = 0$

$\boxed{B.M = 25(x) - 2x^2 - 20(x-4) + 35(x-8)}$

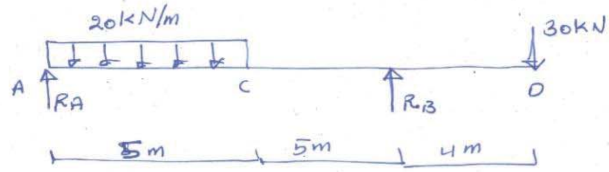
at $x = 8 \Rightarrow B.M = -8\,kN.m$

at $x = 9 \Rightarrow B.M = -2\,kN.m$

at $x = 10 \Rightarrow B.M = 0\,kN.m$

Example 5

example / Derive the shear force and bending moment equations For the beam shown in the Figure, and draw the shear force and bending moment diagrams.



Reactions

$\Sigma M_B = 0$

$R_A(10) - 20(5)(7.5) + 30(4) = 0$

$\boxed{R_A = 63\,kN}$

$\Sigma F_y = 0$

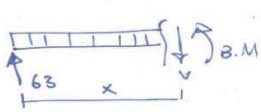
$63 - 20(5) + R_B - 30 = 0$

$\boxed{R_B = 67\,KN}$

S.F. & B.M

Section A-C

$0 \leq x \leq 5$



$\Sigma F_y = 0$

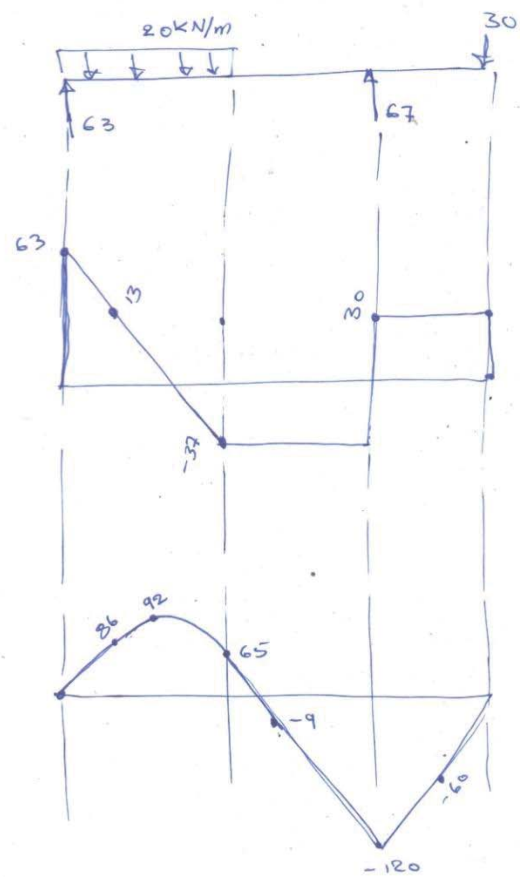
$63 - 20(x) - V = 0$

$\boxed{V = 63 - 20(x)}$

at $x = 0 \rightarrow V = 63\,KN$

at $x = 2 \rightarrow V = 13\,KN$

at $x = 5 \rightarrow V = -37\,KN$



$\Sigma M = 0$

$63(x) - 20\frac{x^2}{2} - B.M = 0$

$\boxed{B.M = 63x - 10x^2}$

at $x = 0 \Rightarrow B.M = 0$

at $x = 2 \Rightarrow B.M = 86\,kN.m$

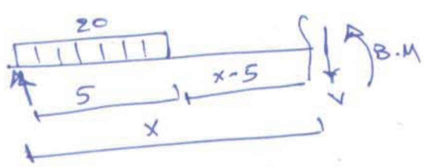
~~at x = 5 ⇒ B.M = 65 kN.m~~

at $x = 4 \Rightarrow B.M = 92\,kN.m$

at $x = 5 \Rightarrow B.M = 65\,kN.m$

Section C-B

$5 \le x \le 10$



$\Sigma F_y = 0$

$63 - 20(5) - V = 0$

$\boxed{V = 63 - 20(5)}$

at $x = 5 \rightarrow V = -37\,kN$

at $x = 7 \rightarrow V = -37\,kN$

at $x = 10 \rightarrow V = -37\,kN$

$\Sigma M = 0$

$25(x) - 20(5)(x - 2.5) - B.M = 0$

$B.M = 63(x) - 100(x - 2.5)$

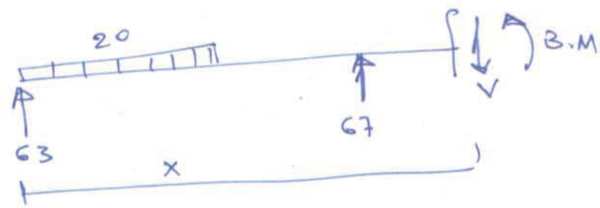
at $x = 5 \rightarrow B.M = 65\,kN.m$

at $x = 7 \rightarrow B.M = -9\,kN.m$

at $x = 10 \rightarrow B.M = -120$

Section B-D

$10 \le x \le 14$



$\Sigma F_y = 0$

$63 - 20(5) + 67 - V = 0$

$\boxed{V = 30\,kN}$ at $x = 10$ ⋮ $x = 14$

$\Sigma M = 0$

$63(x) - 20(5)(x - 2.5) + 67(x - 10) - B.M = 0$

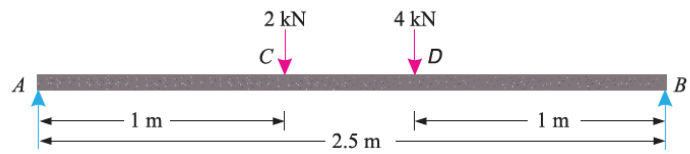
$\boxed{B.M = 63x - 100(x - 2.5) + 67(x - 10)}$

at $x = 10 \rightarrow B.M = -120\,kN.m$

at $x = 12 \rightarrow B.M = -60\,kN.m$

at $x = 14 \rightarrow B.M = 0$

Example : For the simply supported beam shown in the figure, write equations for the shearing force and bending moment at any point in the beam and plot shear and moment diagrams.



Solution)

Reactions/

$\sum M_B = 0$

$R_A(2.5) - 2(1.5) - 4(1) = 0$

$\boxed{R_A = 2.8\ KN}$

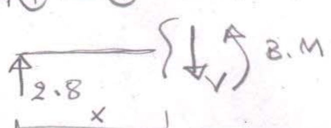
$\sum F_y = 0$

$2.8 - 2 - 4 + R_B = 0$

$\boxed{R_B = 3.2\ KN}$

Sections/

section ① — ① $\quad 0 \leq x \leq 1$



$\sum F_y = 0 \Rightarrow \boxed{V = 2.8\ KN}$

$\sum M = 0 \Rightarrow 2.8(x) - B.M = 0$

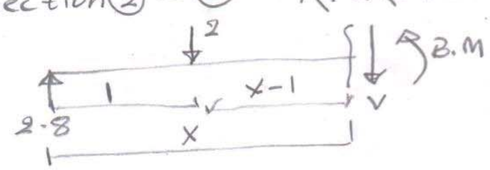
$\boxed{B.M = 2.8(x)}$

at $x = 0 \Rightarrow B.M = 0$

$x = 0.5 \Rightarrow B.M = 1.4\ KN.m$

$x = 1 \Rightarrow B.M = 2.8\ KN.m$

Section ② — ② $\quad 1 \leq x \leq 1.5$



$\sum F_y = 0$

$2.8 - 2 - V = 0 \Rightarrow \boxed{V = 0.8\ KN}$

$\sum M = 0$

$2.8(x) - 2(x-1) - B.M = 0$

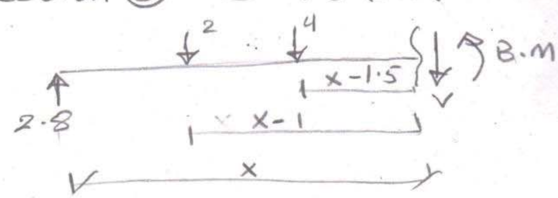
$\boxed{B.M = 2.8(x) - 2(x-1)}$

at $x = 1 \Rightarrow B.M = 2.8\ KN.m$

$x = 1.25 \Rightarrow B.M = 3\ KN.m$

$x = 1.5 \Rightarrow B.M = 3.2\ KN.m$

Section ③ — ③ $\quad 1.5 \leq x \leq 2.5$



$\sum F_y = 0 \Rightarrow 2.8 - 2 - 4 - V = 0$

$\boxed{V = -3.2\ KN}$

$\sum M = 0$

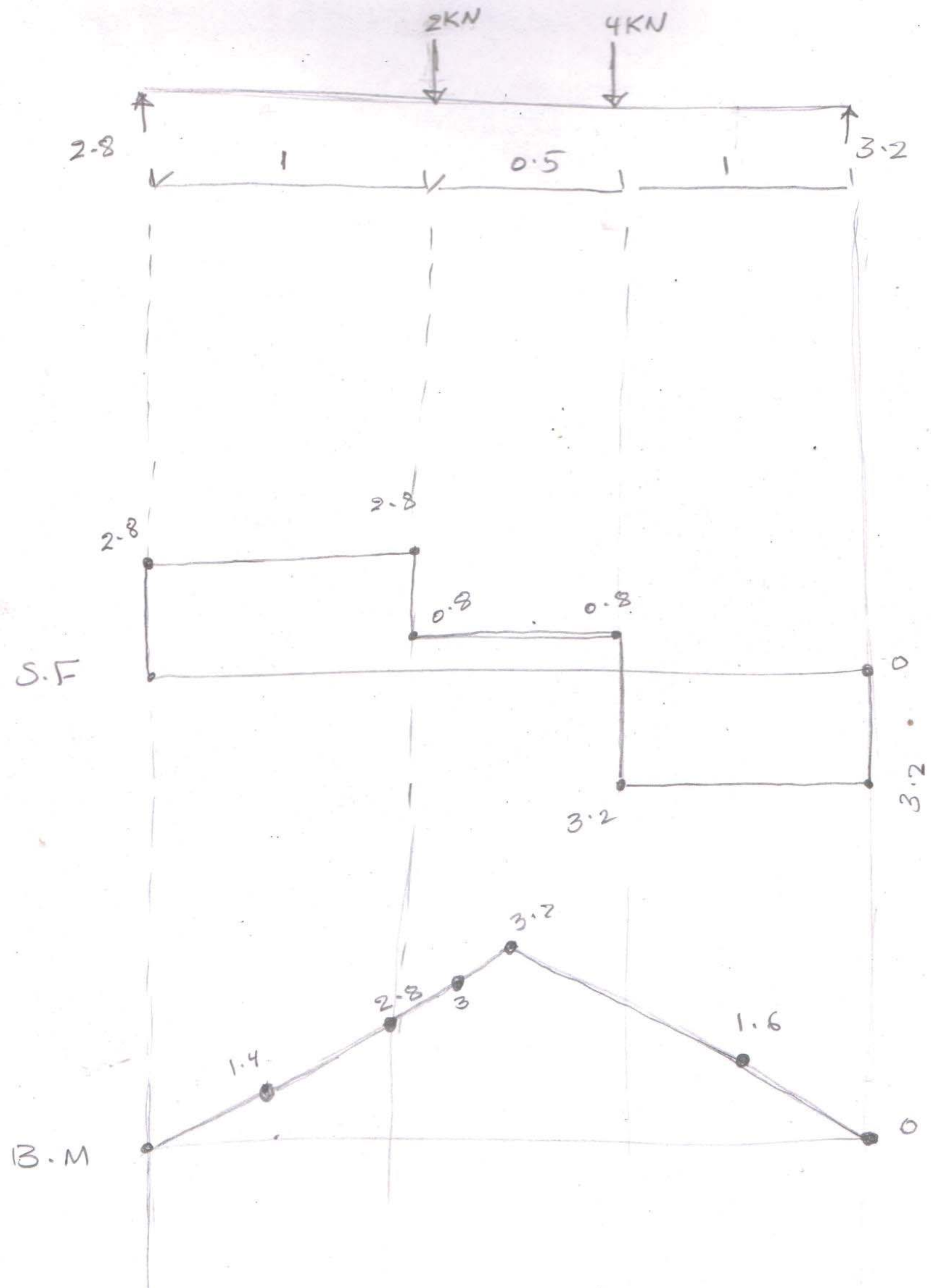
$2.8(x) - 2(x-1) - 4(x-1.5) - B.M = 0$

$\boxed{B.M = 2.8(x) - 2(x-1) - 4(x-1.5)}$

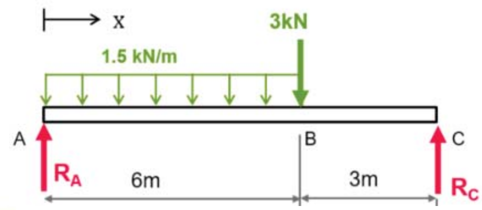
at $x = 1.5 \Rightarrow B.M = 3.2\ KN.m$

$x = 2 \Rightarrow B.M = 1.6\ KN.m$

$x = 2.5 \Rightarrow B.M = 0\ KN.m$


2KN
4KN
2.8
3.2
1
0.5
1
2.8
2.8
0.8
0.8
S.F
0
3.2
3.2
3.2
2.8
3
1.4
1.6
B.M
0

Example : For the simply supported beam shown in the figure, write equations for the shearing force and bending moment at any point in the beam and plot shear and moment diagrams.



Solution

Reactions

$R_A(9) - 1.5(6)(6) - 3(3) = 0$

$\boxed{R_A = 7\ KN}$

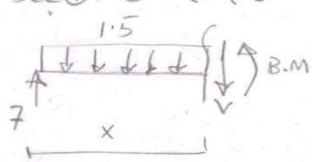
$\Sigma F_y = 0$

$-1.5(6) - 3 + R_C = 0$

$\boxed{R_C = \quad KN}$

Sections

sec ① — ① $0 \leq x \leq 6$



$\Sigma F_y = 0 \Rightarrow 7 - 1.5(x) - V = 0$

$\boxed{V = 7 - 1.5(x)}$

at $x = 0 \Rightarrow V = 7\ KN$
$x = 3 \Rightarrow V = 2.5\ KN$
$x = 6 \Rightarrow V = -2\ KN$

$\Sigma M = 0$

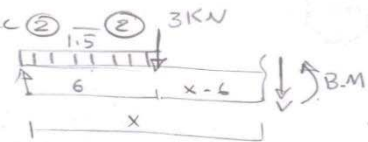
$7(x) - 1.5(x)(\frac{x}{2}) - B.M = 0$

$\boxed{B.M = 7(x) - 0.75x^2}$

$x = 0 \Rightarrow B.M = 0$
$x = 3 \Rightarrow B.M = 14.25$
$x = 6 \Rightarrow B.M = 15$

$x = 4 \Rightarrow B.M = 16$
$x = 5 \Rightarrow B.M = 16.25$

Sec ② — ②



$\Sigma F_y = 0$

$7 - 1.5(6) - 3 - V = 0 \Rightarrow \boxed{V = -5\ KN}$

$\Sigma M = 0$

$7(x) - 1.5(6)(x-3) - 3(x-6) - B.M = 0$

$\boxed{B.M = 7(x) - 9(x-3) - 3(x-6)}$

at $x = 6 \Rightarrow B.M = 15\ KN.m$
$x = 7.5 \Rightarrow B.M = 7.5\ KN.m$
$x = 9 \Rightarrow B.M = 9\ KN.m$

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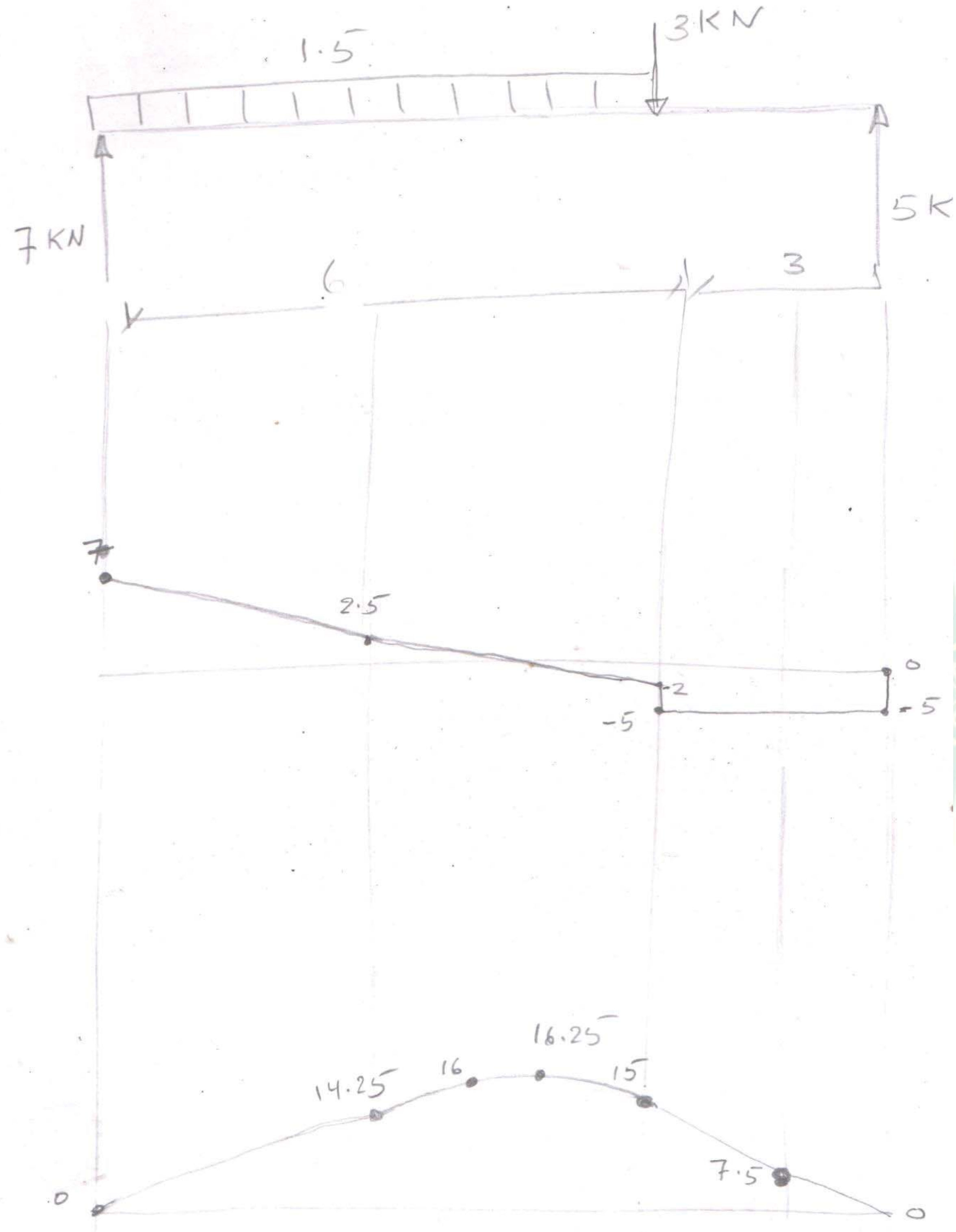
1.5
3KN
7KN
5K
6
3
7
2.5
0
-2
-5
-5
14.25
16
16.25
15
7.5
0
0