Lecture No. 3

Thermal Stresses

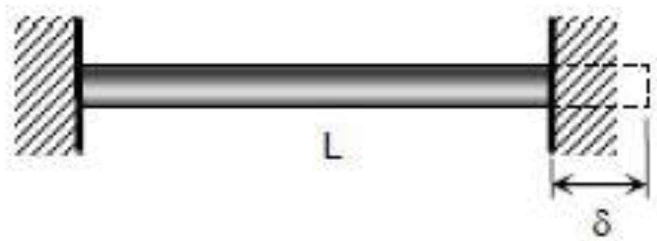
Temperature changes cause the body to expand or contract. The amount δT, is given by

$$\delta_T = \alpha L\,(T_f - T_i) = \alpha L\,\Delta T$$

where α is the coefficient of thermal expansion in m/m°C, L is the length in meter, *T* and 6f are the initial and final temperatures, respectively in °C. For steel, α = 11.25 × 10-6 m/m°C.

If temperature deformation is permitted to occur freely, no load or stress will be induced in the structure. In some cases where temperature deformation is not permitted, an internal stress is created. The internal stress created is termed as thermal stress.

For a homogeneous rod mounted between unyielding supports as shown, the thermal stress is computed as:



Deformation due to temperature changes;

$$\delta_T = \alpha L \, \Delta T$$

Deformation due to equivalent axial stress;

$$\delta_P = \frac{PL}{AE} = \frac{\sigma L}{E}$$

$$\delta_T = \delta_P$$

$$\alpha L \, \Delta T = \frac{\sigma L}{E}$$

$$\sigma = E \alpha \, \Delta T$$

Where σ is the thermal stress in MPa, E is the modulus of elasticity of the rod in MPa.

If the wall yields a distance of x as shown, the following calculations will be made:

$$\delta_T = x + \delta_P$$

$$\alpha L \Delta T = x + \frac{\sigma L}{E}$$

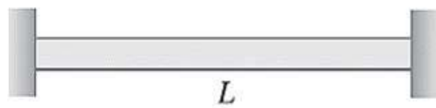
Where σ represents the thermal stress.

Take note that as the temperature rises above the normal, the rod will be in compression, and if the temperature drops below the normal, the rod is in tension

Example 1:

A steel rod of length *L* and uniform cross sectional area *A* is secured between two walls, as shown in the figure. Use *L*=1.5m, *E*=200 GPa, $\alpha = 11.7 \times 10^{-6}$ /°C and $\Delta T = 80$ °C. Calculate the stress for a temperature increase of *ΔT* for:

a) The walls are fixed.
b) The walls move apart a distance 0.5mm.



Solution:

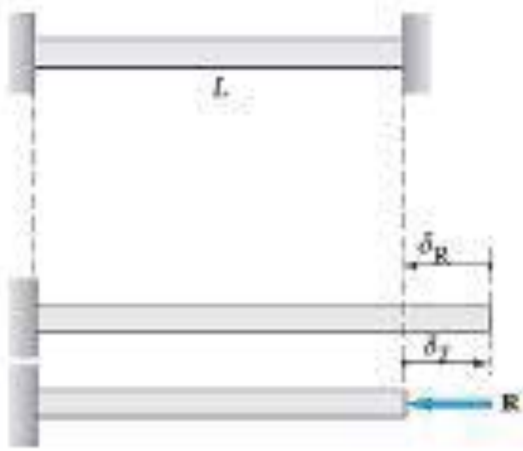
a) $\delta_{th} - \delta_R = 0$

$$\alpha(\Delta T)L - \frac{RL}{AE} = 0$$

$\therefore R = A\,E\,\alpha\,(\Delta T)$

$$\sigma = \frac{R}{A} = E\alpha(\Delta T)$$

$$= 200 \times 10^9 \times 11.7 \times 10^{-6} \times 80 = 187.2 \text{ MPa}$$ (Answer)



b) $\delta_{th} - \delta_R = \delta_w$

$$\alpha(\Delta T)L - \frac{RL}{AE} = \delta_w$$

$$R = AE\left(\alpha\,\Delta T - \frac{\delta_w}{L}\right)$$

The compressive stress is then,

$$\sigma = \frac{R}{A} = E\left(\alpha\Delta T - \frac{\delta_w}{L}\right)$$

$$= 200 \times 10^9\left(11.7 \times 10^{-6} \times 80 - \frac{0.5\times10^{-3}}{1.5}\right) = 120.52 \text{ MPa}$$ (Answer)

Example 1.

A steel rod is stretched between two rigid walls and carries a tensile load of 5000 N at 20°C. If the allowable stress is not to exceed 130 MPa at -20°C, what is the minimum diameter of the rod? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.

Solution

$$\delta = \delta_{th} + \delta_P$$

$$\frac{\sigma_{th} L}{E} = \alpha L \Delta T + \frac{PL}{AE} \Rightarrow \sigma_{th} = \alpha \cdot L \cdot \Delta T \cdot E + \frac{P}{A}$$

$$130 = 11.7 \times 10^{-6} \times (20 - (-20)) \times 200 \times 10^{3} + \frac{5000}{A}$$

$$\Rightarrow A = 137.36 \ mm^2$$

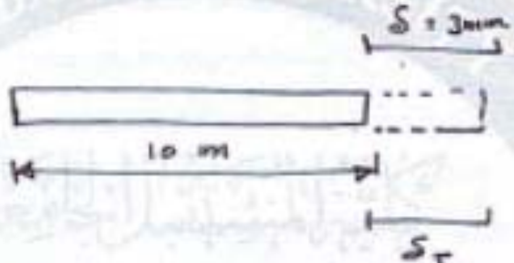
$$= \frac{\pi}{4} d^2$$

$$\Rightarrow d = 13.22 \ mm$$

→ Ans.

Example 2

Steel railroad reels 10 m long are laid with a clearance of 3 mm at a temperature of 15°C. At what temperature will the rails just touch? What stress would be induced in the rails at that temperature if there were no initial clearance? Assume α = 11.7 µm/(m·°C) and E = 200 GPa.



$$\delta_T = \alpha L \Delta T$$

$$3 = 11.7 \times 10^{-6} \times 10\,000\,(T_f - 15)$$

$$T_f = 40.64\,^\circ C \longrightarrow \text{Ans (1)}$$

Required stress

$$\delta = \delta_{th}$$

$$\frac{\sigma_{th} L}{E} = \alpha L \Delta T$$

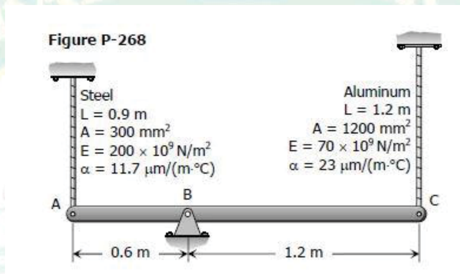
$$\sigma = \alpha E\,(T_f - T_i)$$

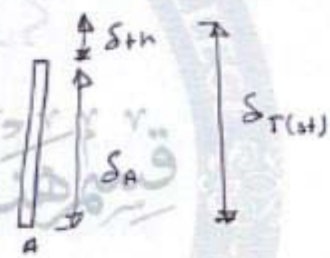
$$= 11.7 \times 10^{-6} \times 200\,000\,(40.64 - 15)$$

$$= 60 \text{ MPa} \longrightarrow \text{Ans (2)}$$

Example 3

The rigid bar ABC in Fig. P-268 is pinned at B and attached to the two vertical rods. Initially, the bar is horizontal and the vertical rods are stress-free. Determine the stress in the aluminum rod if the temperature of the steel rod is decreased by 40°C. Neglect the weight of bar ABC.





Solution/ For steel rod

$$\delta_{th} = \alpha L \Delta T$$

$$= 11.7 \times 10^{-6} \times 900 \times 40$$

$$= 0.4212 \text{ mm}$$

$$\delta_{T(st)} = \delta_{th} + \delta_A$$

$$\delta_A = \delta_{T(st)} - \delta_{th} \quad \text{—— (1)}$$

$$\frac{\delta_A}{0.6} = \frac{\delta_B}{1.2} \Rightarrow \delta_A = 0.5\,\delta_{B_{al}} \quad \text{—— (2)}$$

sub ② in ①

$$0.5\,\delta_{al} = \delta_{T(st)} - \delta_{st}$$

$$0.5\left(\frac{PL}{AE}\right)_{al} = \left(\frac{PL}{AE}\right)_{st} - 0.4212$$

$$0.5\left[\frac{P_{al} * 1200}{1200 * 70 * 10^3}\right] = \frac{P_{st} * 900}{300 * 200 * 10^3} - 0.4212$$

$$\Rightarrow 28080 - P_s = 0.4762\,P_{al} \quad \text{——} (*)$$

$$\Sigma M_B = 0$$

$$0.6\,P_{st} = 1.2\,P_{al}$$

$$P_s = 2P_{al} \quad \text{——} (**)$$

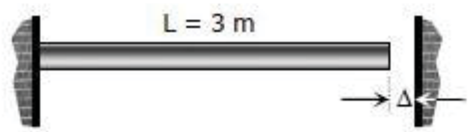
sub eq(**) in eq(*)

$$28080 - 2P_{al} = 0.4762\,P_{al}$$

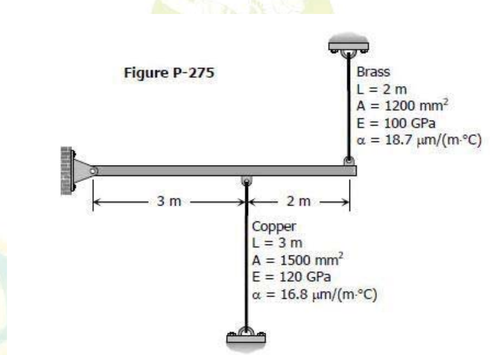
$$P_{al} = 9.45 \text{ MPa} \longrightarrow \text{Ans.}$$

HOMEWORK NO. 2

1. A bronze bar 3 m long with a cross sectional area of 320 mm2 is placed between two rigid walls as shown in Fig. below. At a temperature of -20°C, the gap Δ = 2.5 mm. Find the temperature at which the compressive stress in the bar will be 35 MPa. Use α = 18.0 × 10-6 m/(m·°C) and E = 80 GPa.



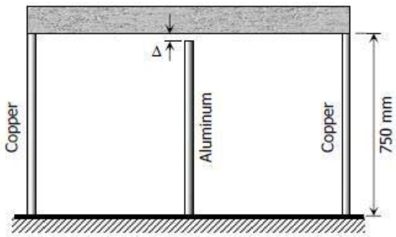
2. A rigid horizontal bar of negligible mass is connected to two rods as shown in Fig. If the system is initially stress-free. Calculate the temperature change that will cause a tensile stress of 90 MPa in the brass rod. Assume that both rods are subjected to the change in temperature.



Example 4

As shown in Fig, there is a gap between the aluminum bar and the rigid slab that is supported by two copper bars. At 10°C, Δ = 0.18 mm. Neglecting the mass of the slab, calculate the stress in each rod when the temperature in the assembly is increased to 95°C.

For each copper bar, A = 500 mm2, E = 120 GPa, and α = 16.8 μm/(m·°C). For the aluminum bar, A = 400 mm2, E = 70 GPa, and α = 23.1 μm/(m·°C).



Example 5

A rigid bar of negligible weight is supported as shown in Fig. P-271. If W = 80 kN, compute the temperature change that will cause the stress in the steel rod to be 55 MPa. Assume the coefficients of linear expansion are 11.7 μm/(m·°C) for steel and 18.9 μm/(m·°C) for bronze.

Figure P-271 and P-272