Lecture No. 1

Stress and Strain

1.1 Introduction

The study of the strength of materials is the study of the behavior of solid bodies under load. The way in which they react to applied forces, the deflections resulting and the stresses and strains set up within the bodies, are all considered in an attempt to provide sufficient knowledge to enable any component to be designed such that it will not fail within its service life. Typical components considered in this course include beams, shafts, cylinders and struts.

1.2 Analysis of Internal Forces

Consider a body of any shape acted upon by the forces shown in **Figure 1-1**, the forces $\overrightarrow{F_1}, \overrightarrow{F_2}, \overrightarrow{F_3}$, and $\overrightarrow{F_4}$ are the external forces acted on the body. This body is considered in static equilibrium (i.e. remains at rest). To study the internal forces, a section a-a through the body will cut the body into two pieces, Figure 1-b shows one piece of section a-a of the body balanced by components of internal forces. If the X axis is normal to the section, Y and Z axes are chosen parallel to the section.

Section a-a

(a) (b)

Figure 1-1 (a) Body of any shape subjected to external forces; (b) Balance of forces through section a-a

Each component reflects a different effect of the applied loads on the member and is given a special name, as follows:

P_{XX}	Axial forces, if the forces try to pull the body, it is called tensile forces and called compressive if it tends to shorten the body.
P_{XY} and P_{XZ}	Shear forces, usually designated by V, which acts parallel to the plane of section
M_{XX}	Torque T, this component measure the resistance to twisting the member
M_{XY} and P_{Xz}	Bending moments, these components measure the resistance to bending the member about Y or Z axis.

1.3 Stress

Stress is the measure of an external force acting over the cross sectional area of an object. Stress has units of force per area: N/m2 (SI) or lb/in2 (US). The SI units are commonly referred to as Pascals, abbreviated Pa.

1.3.1 Simple Stress:

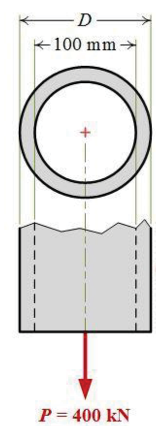
When a force acts perpendicular (or "normal") to the surface of an object

$$\sigma = \frac{Force}{Area} = \frac{F}{A}$$

Where σ (Greek lower case letter sigma) is the intensity of forces per unit area, or stress (N/m^2), F is the applied load (N), and A is the cross sectional area (m^2).

Example 1

A hollow steel tube with an inside diameter of 100 mm must carry a tensile load of 400 kN. Determine the outside diameter of the tube if the stress is limited to 120 MN/m^2.



Solution

$d_i = 100\ mm$

$F = 400\ kN$

$\sigma = 120\ MPa$

$d_o = ?$

$\sigma = \frac{F}{A}$

$* \ A = \frac{\pi}{4}(d_o^2 - d_i^2)$

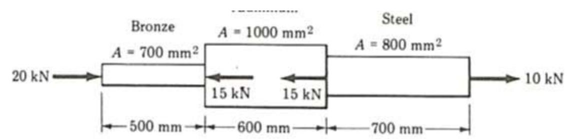
$120 = \frac{400 * 10^3}{\frac{\pi}{4}[d_o^2 - 100^2]}$

$d_o^2 = 14244.13$

$\boxed{d_o = 119.34\ mm} \rightarrow$ Ans.

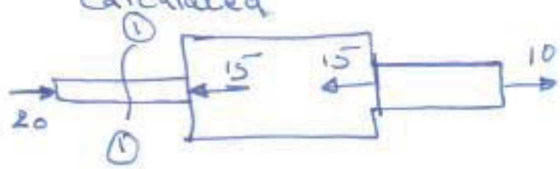
Example 2

An aluminum tube is rigidly fastened between a bronze rod and a steel rod as shown in the figure. Axial loads are applied at positions indicated. Determine the stress in each material.

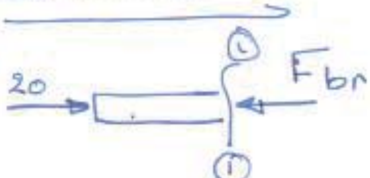


Solution

By superposition method, the force in each section calculated



Section 1—1



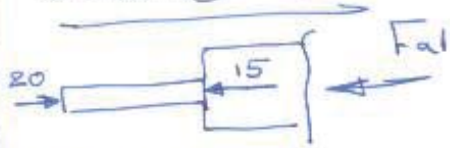
$\Sigma Fx = 0$

$\boxed{F_{br} = 20\,kN}$

$$\sigma_{br} = \frac{F_{br}}{A_{br}} = \frac{20 * 10^3}{700}$$

$\boxed{\sigma_{br} = 28.57\,MPa}$ → Ans

section 2—2



$\Sigma Fx = 0$

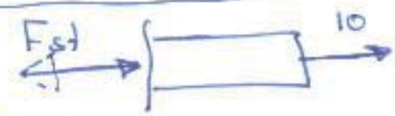
$-F_{al} - 15 + 20 = 0$

$\boxed{F_{al} = 5\,KN}$

$$\sigma_{al} = \frac{F_{al}}{A} = \frac{5 * 10^3}{1000}$$

$\boxed{\sigma_{al} = 5\,MPa}$ → Ans.

section 3—3



$\Sigma Fx = 0$

$-F_{st} - 10 = 0 \Rightarrow F_{st} = -10$

$\boxed{F_{st} = 10\,KN \leftarrow}$

$$\sigma_{st} = \frac{F_{st}}{A_s} = \frac{10 * 10^3}{700} = 14.28\,MPa$$ → Ans.

1.3.2 Shearing Stress

Shearing stress is the stress caused by force acting along or parallel to area resisting the forces and can be defined as:

$$\tau = \frac{V}{A}$$

Where τ (Greek lowercase letter tau) is the shearing stress (N/m^2), V is the shearing force (N), and A is the area (m^2).

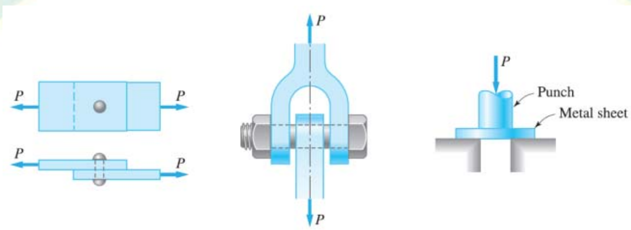
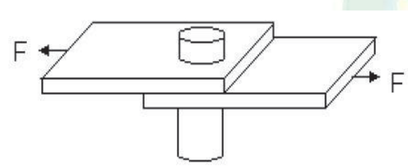


Figure 1-2 example of shear stress

Example 3

Two strip of metal are pinned to gather as shown in fig below with a rod of 10 mm diameter. The ultimate shear stress of the rod is 60 MPa. Determine the maximum force required to break the pin.



Solution

$d = 10\ mm$

$\tau = 60 MPa$, $V = ?$

$\tau = \frac{V}{A}$

$* A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (10)^2 = 78.5 mm^2$

$\tau = \frac{V}{A} \Rightarrow V = \tau * A$

$V = 60 * 78.5$

$\boxed{V = 4712.389\ N}$ → Ans.

1.3.3 Bearing Stress

If two bodies are pressed against each other, compressive forces are developed on the area of contact. The pressure caused by these surface loads is called bearing stress. Examples of bearing stress are the soil pressure beneath a pier and the contact pressure between a rivet and the side of its hole. If the bearing stress is large enough, it can locally crush the material, which in turn can lead to more serious problems.

As an illustration of bearing stress, consider the lap joint formed by the two plates that are riveted together as shown in Figure 1-3 a The bearing stress caused by the rivet is not constant; it actually varies from zero at the sides of the hole to a maximum behind the rivet as illustrated in Figure 1-3 b The difficulty inherent in such a complicated stress distribution is avoided by the common practice of assuming that the bearing stress σ_b is uniformly distributed over a reduced area. The reduced area A_b is taken to be the projected area of the rivet:

$$A_b = t\ d$$

Where *t* is the thickness of the plate and *d* represents the diameter of the rivet, as shown in the *free body diagram* (FBD) of the upper plate in Figure 1-3 c. From this FBD we see that the bearing force Fb equals the applied load P, so that the bearing stress becomes

$$\sigma_b = \frac{F_b}{A_b} = \frac{P}{t * d}$$

Where A_b is the projected area of the rivet hole

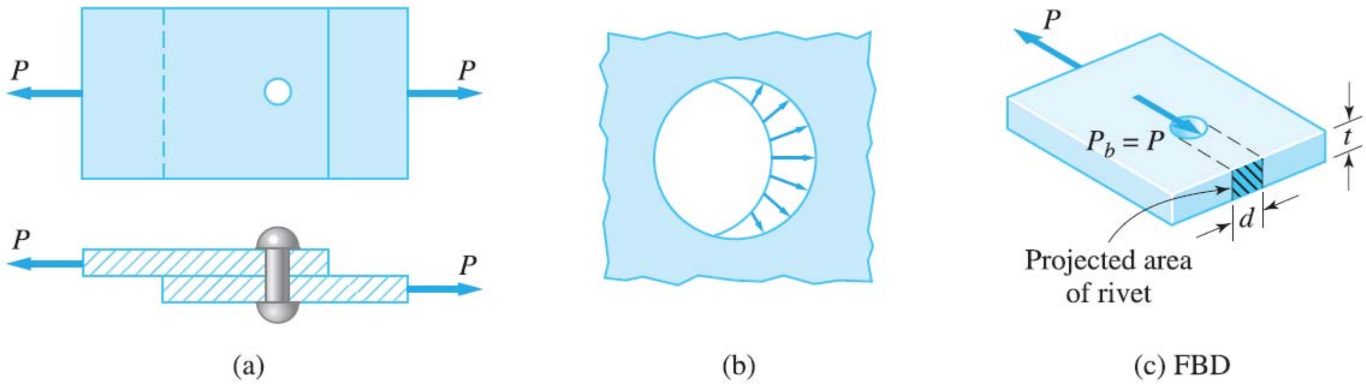
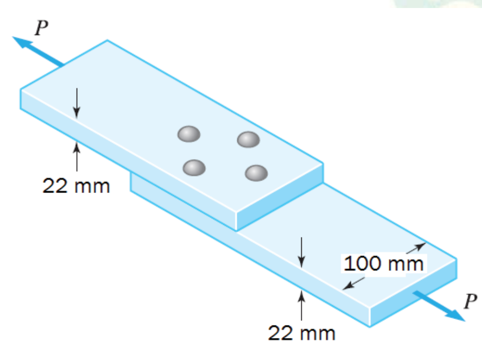


Figure 1-3 Bearing stress: (a) a rivet in a lap joint; (b) bearing stress is not constant; (c) bearing stress caused by the bearing force Pb is assumed to be uniform on projected area $t * d$.

Example 4

The lap joint shown in the figure is fastened by four rivets of 19 mm diameter. Find the maximum load P that can be applied if the working stresses are 96 MPa for shear in the rivet and 124 MPa for bearing in the plate.



Solution

$d = 19$ mm
$t = 22$ mm
$\tau = 96$ MPa
$\sigma_b = 124$ MPa

ملاحظة / في مثل هذا النوع من
المسائل تكون هنالك
نوعين من الاجهادات :-
① اجهادات قصية (τ)
على البرشام
② اجهادات تحميلية
(σ_b) على اللوح المعدني
والقوة التي تسبب الاجهادين
اعلاه هي نفس القوة (P)

For shear stress

$$\tau = \frac{V}{A} = \frac{P}{A} \quad ; \quad A = 4 * \frac{\pi}{4} d^2$$

$$= 4 * \frac{\pi}{4} (19)^2 = 1134.11 \ mm^2$$

$$\Rightarrow 96 = \frac{P_{shear}}{1134.11}$$

$$\boxed{P_{sh} = 108875.035 \ N}$$

For bearing stress

$$\sigma_{bearing} = \frac{P_{bearing}}{A} \quad ; \quad A_{bearing} = 4 * d * t$$

$$= 4 * 19 * 22$$

$$= 1672 \ mm$$

$$\sigma_b = \frac{P_b}{A} \Rightarrow 124 = \frac{P_b}{1672}$$

$$\boxed{P_b = 207328 \ N}$$

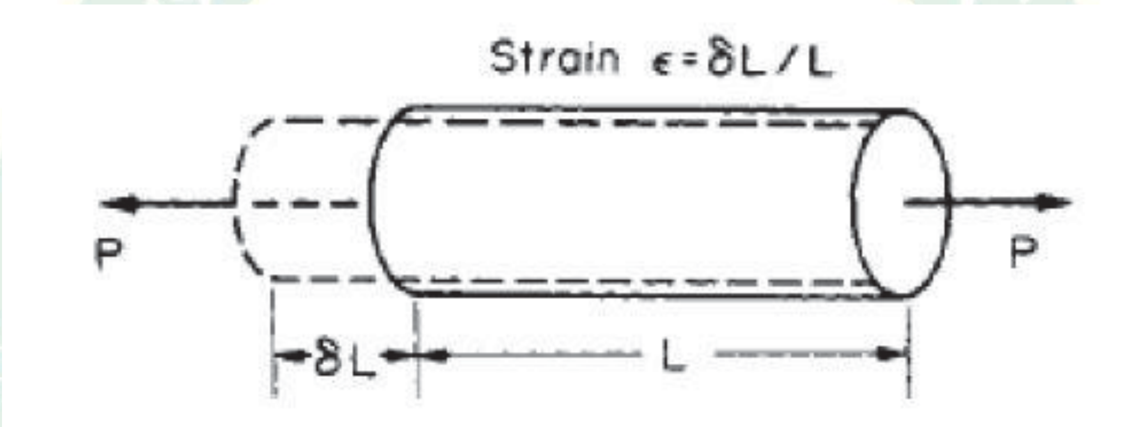
The safe load must be subjected to the joint is the small one $\Rightarrow \boxed{P = 108875.035 \ N}$ $\longrightarrow$ Ans.

1.4 Strains

If a bar is subjected to a direct load, the bar will change in length. If the bar has a length L and changes in length by ΔL, the strain produced is defined as:

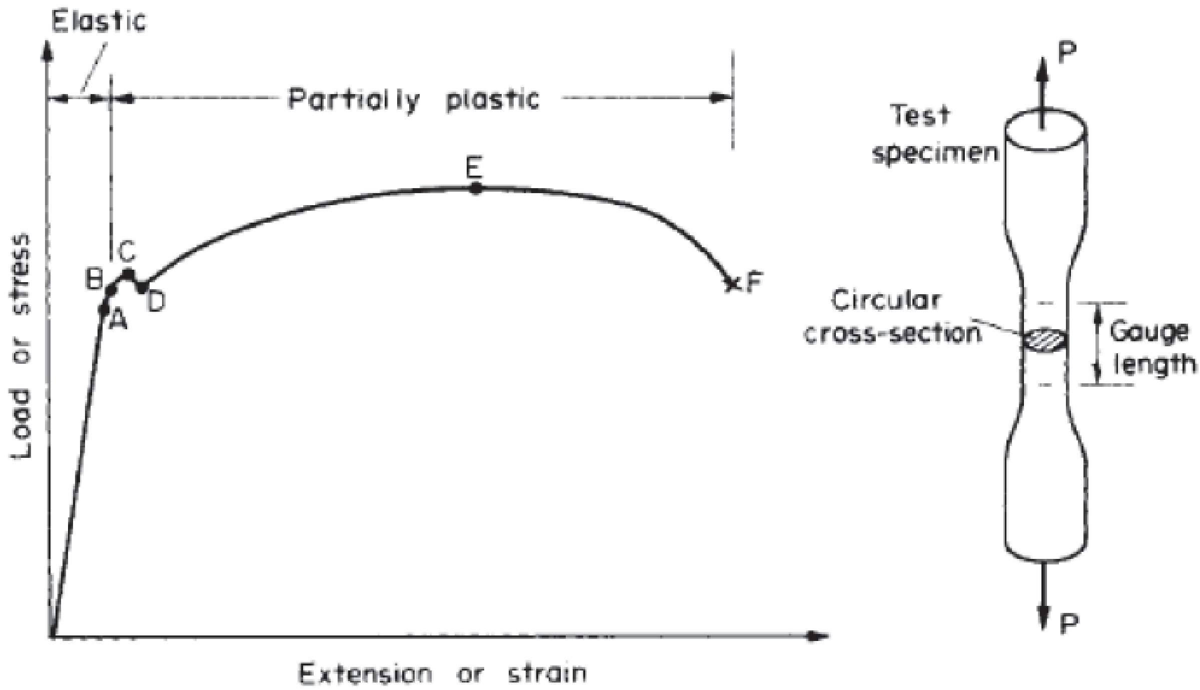
$$strain = \frac{\text{change in length}}{\text{orginal length}}$$

$$\varepsilon = \frac{\Delta L}{L}$$



1.5 Stress – Strain Diagram

The strength of material is not the only criterion that must be considered in designing structures the stiffness of material is frequently of equal importance. If a specimen of structural steel is gripped between the jaws of testing machine and the unit load or stress was plotted against unit elongation (or strain). The resulted diagram shown is called a stress-strain diagram



Notes:

1. The material behaves elastically till *elastic limit B.*

2. Point *A* is called *proportionality limit* where stress is proportional to strain.

3. The material beyond elastic limit is *plastically deformed*.

4. *C* is called *upper yield point*, and *D* is the *lower yield point*.

5. *E* is where the *ultimate stress* occurs, in this point necking occur.

6. *F* is the *fracture point*.

1.6 Hooke's Law

A material is said to be elastic if it returns to its original, unloaded dimension, when load is removed. In most engineering materials this elastic behavior is linear, i.e. the stress is directly proportional with strain, **Hooke's law** states that:

$$\text{Stress } (\sigma) \propto \text{Strain } (\varepsilon)$$

$$\frac{stress}{strain} = constant$$

$$\sigma = E * \varepsilon$$

Where E is called the modulus of elasticity or Young's modulus.

Note:

In most common engineering applications strain is rarely exceeded 0.001 or 0.1%.

$$\sigma = E\varepsilon, \quad \text{since } \sigma = \frac{F}{A} \quad \text{and} \quad \varepsilon = \frac{\delta}{L}$$

$$\frac{F}{A} = E\frac{\delta}{L}$$

$$\delta = \frac{FL}{AE}$$

1.7 Poisson's Ratio

When a specimen subjected to axial tensile loading a *reduction or lateral contraction induces* to the specimen's cross-sectional area. Similarly, a contraction owing to an axial compressive load is accompanied by a lateral extension. In the linearly elastic region, it is found experimentally that lateral strains, say in the *y* and *z* directions, are related by a constant of proportionality ν , to the axial strain caused by *uniaxial stress only* $\varepsilon_x = \sigma_y / E$, in the *x* direction:

Alternatively, the definition of *v* may be stated as:

$$\nu = -\frac{lateral\ strain}{axial\ strain}$$

Here ν is known as Poisson's ratio, after S. D. Poisson (1781-1840). The values of Poisson's ratio are 0.25 to 0.35 for most metals. Extreme cases range from a low of 0.1 (for some concretes) to a high of 0.5 (for rubber).

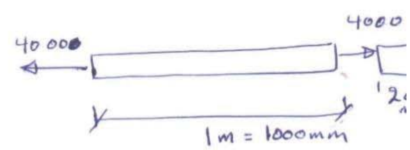
Example 5

A steel rod 1m long and 20 mm *20 mm in cross-section is subjected to a tensile force of 40 kN. Determine the elongation of the rod, if modulus of elasticity for the rod material is 200 GPa.

Solution/

$L = 1m = 1000mm$; $A = 20*20 = 400\ mm$; $P = 40*10^3\ N$

$E = 200*10^3\ MPa$; $\delta L = ?$



$$E = \frac{\sigma}{\epsilon} = \frac{\frac{P}{A}}{\frac{\delta L}{L}}$$

$$E = \frac{P L}{\delta L * A} \Rightarrow \delta L = \frac{PL}{AE}$$

$$= \frac{40*10^3 * 1000}{400 * 200*10^3}$$

$$= 0.5\ mm \longrightarrow Ans.$$

Example 6

A hollow cylinder 2 m long has an outside diameter of 50 mm and inside diameter of 30 mm. If the cylinder is carrying a load of 25 kN, find the stress in the cylinder Also find the deformation of the cylinder, if the value of modulus of elasticity for the cylinder material is 100 GPa.

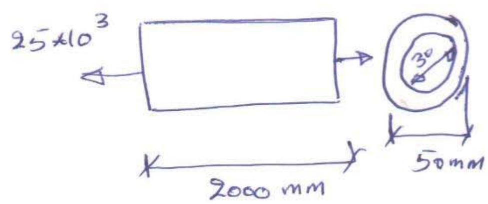
Solution/ $L = 2 * 10^3$ mm ; $d_o = 50$ mm ; $d_i = 30$ mm

$P = 25 * 10^3$ N

$E = 100 * 10^3$ MPa

① $\sigma = ?$

② $\delta L = ?$



① $\sigma = \dfrac{P}{A} = \dfrac{P}{\frac{\pi}{4}(d_o^2 - d_i^2)}$

$= \dfrac{25 * 10^3}{\frac{\pi}{4}(50^2 - 25^2)} = 19.8$ MPa $\rightarrow$ Ans (1)

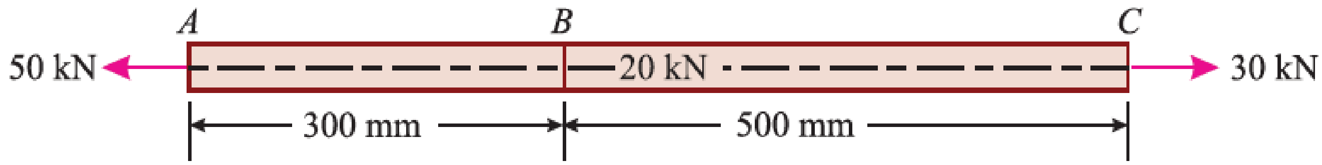
② $E = \dfrac{\sigma}{\epsilon} = \dfrac{\frac{P}{A}}{\frac{\delta L}{L}} \Rightarrow \delta L = \dfrac{PL}{AE}$

$= \dfrac{25 * 10^3 * 2000}{\frac{\pi}{4}(50^2 - 30^2) * 100 * 10^3}$

$= 0.3978$ mm $\rightarrow$ Ans (2)

Example 7

A steel bar of cross-sectional area 200 mm^2 is loaded as shown in Fig. Find the change in length of the bar. Take E as 200 GPa.

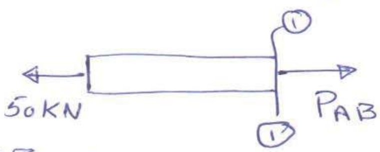


Solution/ $A = 200\ mm^2$

$L_{AB} = 300\ mm$

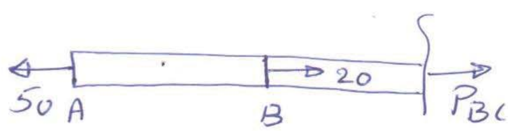
$L_{BC} = 500\ mm$

By using superposition method, the forces in each section calculated:-



$\Sigma F_x = 0$

$P_{AB} = 50\ KN$ (tensile)



$\Sigma F x = 0$

$-50 + 20 + P_{BC} = 0$

$P_{BC} = 30\ KN$ (tensile)

$$\delta L_t = \delta L_{1} + \delta L_{2}$$

$$= \frac{P_{AB}\, L_{AB}}{A_{AB}\, E} + \frac{P_{BC}\, L_{BC}}{A_{BC}\, E}$$

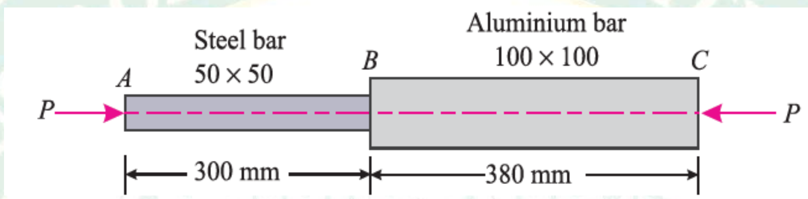
$$= \frac{50*10^3*300}{200*200*10^3} + \frac{30*10^3*500}{200*200*10^3}$$

$$\delta L_t = 0.75\ mm$$

$\longrightarrow$ Ans

Example 8

A member formed by connecting a steel bar to an aluminum bar is shown in Fig. Calculate the magnitude of force P, that will cause the total length of the member to decrease by 0.25 mm. The values of elastic modulus for steel and aluminum are 210 GPa and 70 GPa respectively.

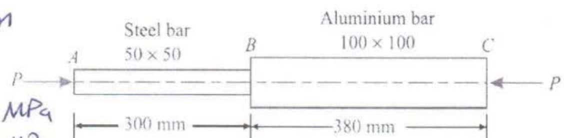


$\delta L_t = -0.25\,mm$

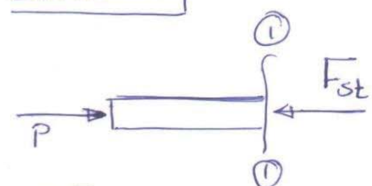
$P = ?$

$E_{st} = 200 \times 10^3\ MPa$

$E_{al} = 70 \times 10^3\ MPa$

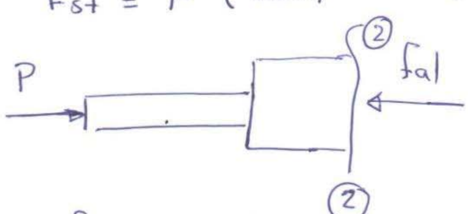


Solution



$\Sigma F_x = 0$

$F_{st} = P$ (compressive)



$\Sigma F_x = 0$

$F_{al} = P$ (compressive)

$$\delta_t = \delta_{st} + \delta_{al}$$

$$= \frac{F_{st} L_{st}}{A_s E_{st}} + \frac{F_{al} L_{al}}{A_{al} E_{al}}$$

$$= \frac{-P * 300}{50 \times 50 * 200 \times 10^3} + \frac{-P * 380}{100 \times 100 * 70 \times 10^3}$$

$$-0.25 = -6 \times 10^{-7} P - 5.4 \times 10^{-7} P$$

$$+0.25 = +11.4 \times 10^{-7} P$$

$P = 219298.24\ N$ → Ans.