AL- MUSTAQBAL UNIVERSITY COLLEGE
DEPARTMENT OF BIOMEDICAL ENGINEERING

Digital Signal Processing (DSP)

BME 312

Lecture 5

- Convolution I -

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Convolution

Convolution is a mathematical **operation on two functions** producing a third function that is affected at any time by all previous input values.

Convolution is the most **important** and fundamental concept in signal processing and analysis. By using convolution, we can **construct the output of any LTI system** for any arbitrary input signal, if we know the **impulse response** of that system.

The **impulse response** goes by a different name in some applications. **If the system** being considered **is a *filter***, the impulse response is called the **filter kernel**, the **convolution kernel**, **or** simply, the **kernel**. In image processing, the impulse response is called the **point spread function**.

The convolution is performed by sliding the kernel along the input signal

Convolution of discrete-time signals

- The convolution of two discrete-time signals $x[n]$ and $h[n]$ to produce a new signal $y[n]$ is denoted by:

$$y[n] = x[n] * h[n]$$

and defined by

$$y[n] = x[n] * h[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

- This equation referred to as the *convolution sum*

i. Commutative property:

$$y[n] = h[n] * x[n] = x[n] * h[n]$$

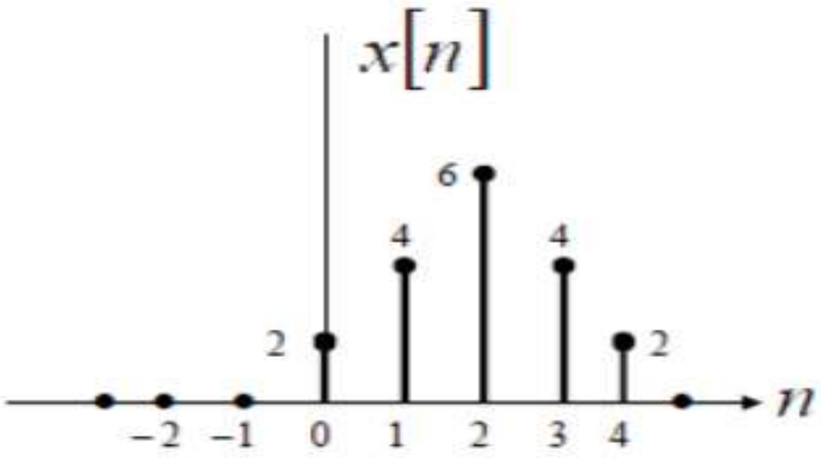
ii. Associative property

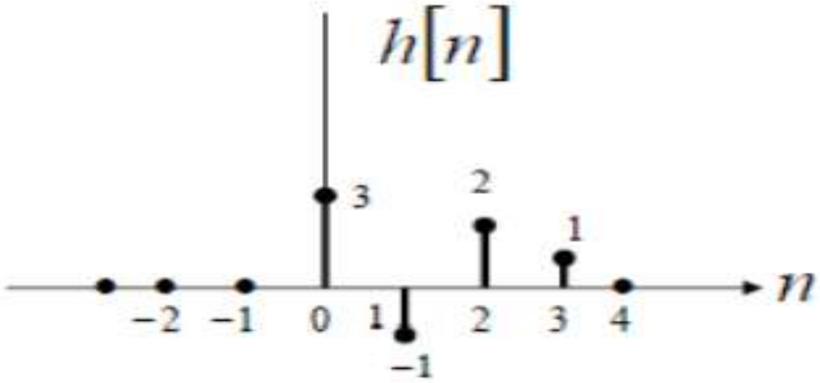
$$(x[n] * h_1[n]) * h_2[n] = x[n] * (h_1[n] * h_2[n])$$

iii. Distributive property

$$x[n] * (h_1[n] + h_2[n]) = x[n] * h_1[n] + x[n] * h_2[n]$$

Ex.1 Find $y[n]=x[n]*h[n]$

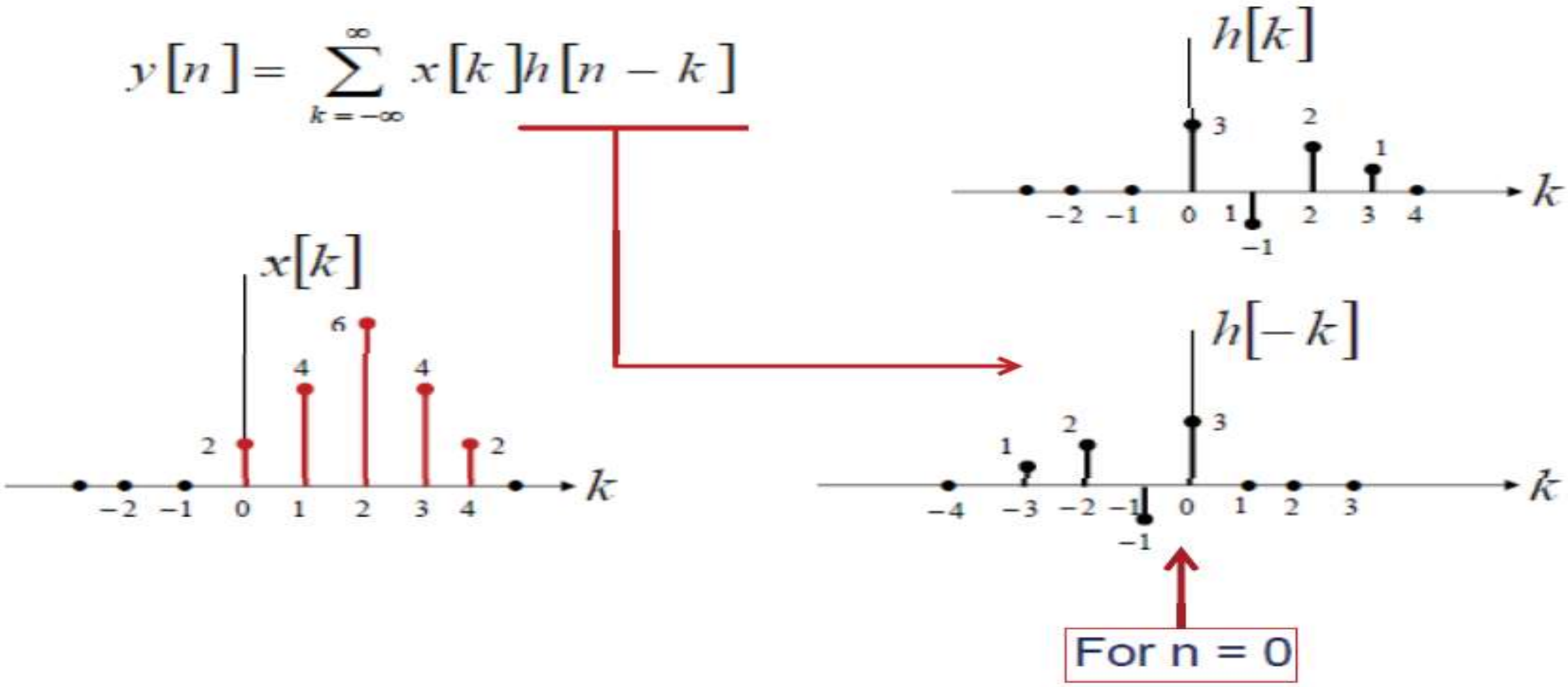




$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

Solution : Ex.1 (Method 1 - Graphical convolution)

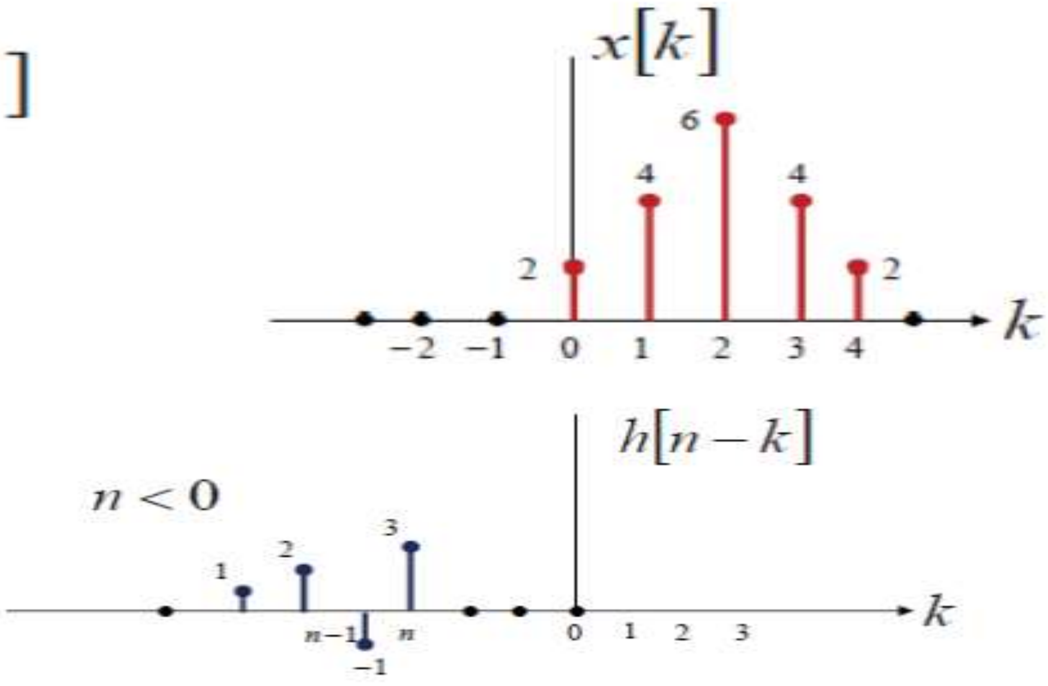
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



Solution : Ex.1 (Method 1 - Graphical convolution)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

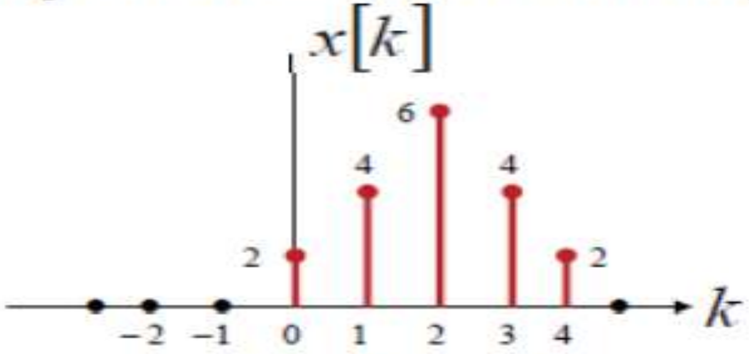
- For n < 0:



$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k] = 0$$

Solution : Ex.1 (Method 1 - Graphical convolution)

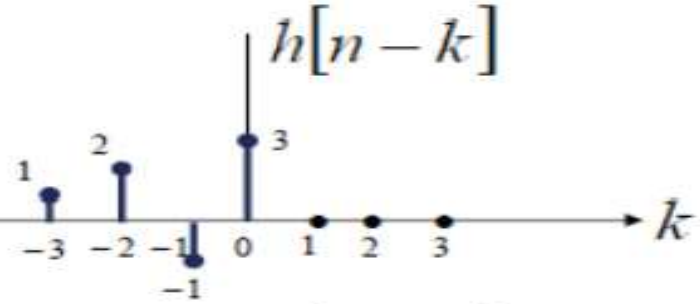
$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



- For n = 0:

$$y[n] = (2)(3) = 6$$

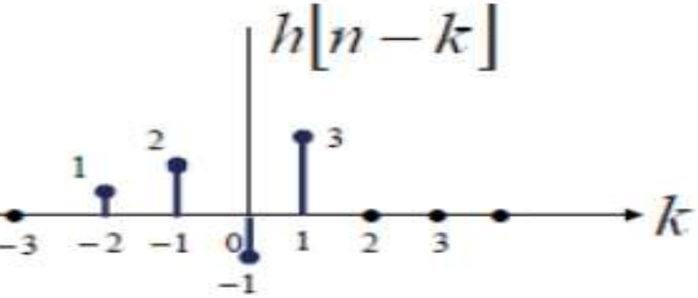
$n = 0$



- For n = 1:

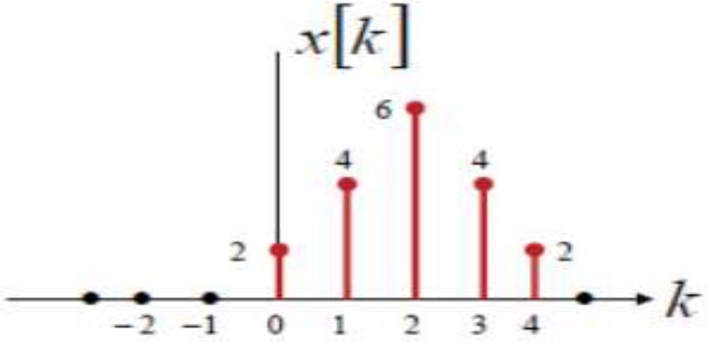
$$y[n] = (2)(-1) + (4)(3)$$
$$= 10$$

$n = 1$



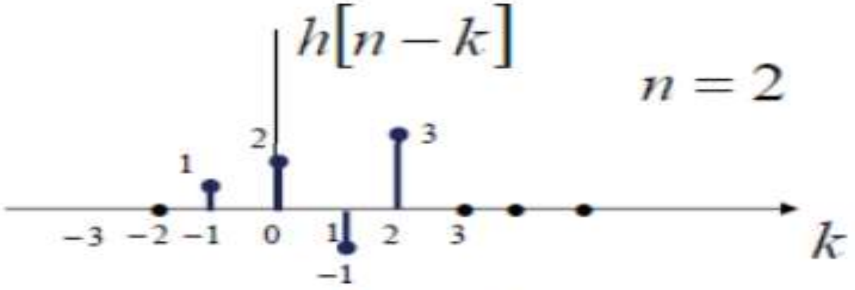
Solution : Ex.1 **(Method 1 - Graphical convolution)**

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



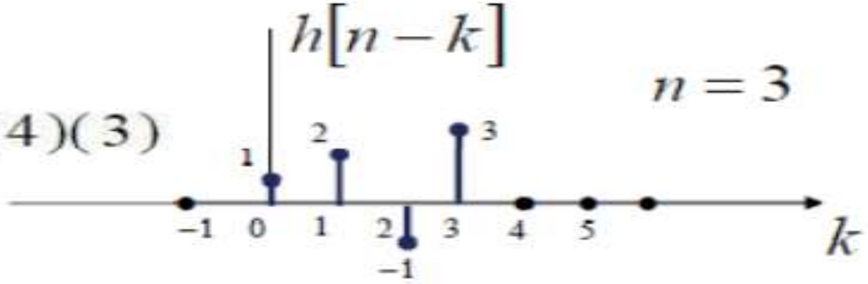
- For n = 2:

$$y[n] = (2)(2) + (4)(-1) + (6)(3) = 18$$



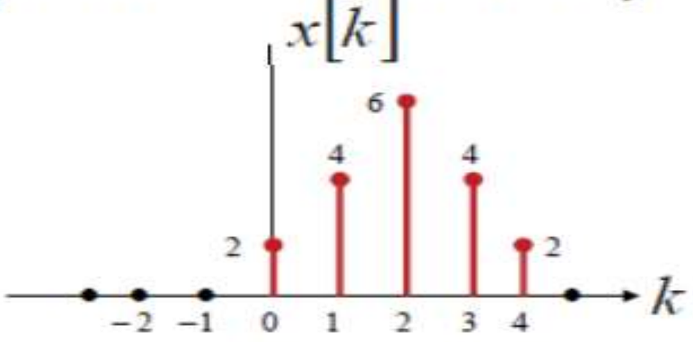
- For n = 3:

$$y[n] = (2)(1) + (4)(2) + (6)(-1) + (4)(3) = 16$$



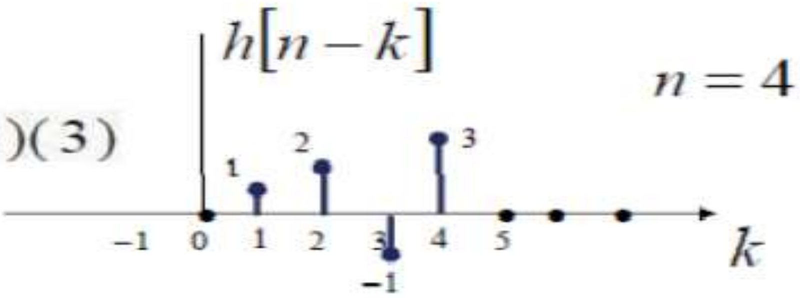
Solution : Ex.1 (Method 1 - Graphical convolution)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



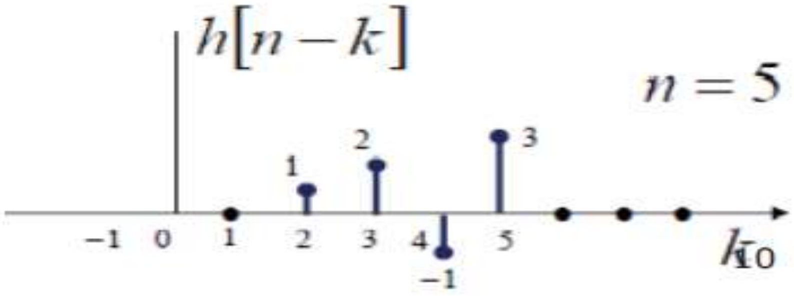
- For n = 4:

$$y[n] = (4)(1) + (6)(2) + (4)(-1) + (2)(3)$$
$$= 18$$



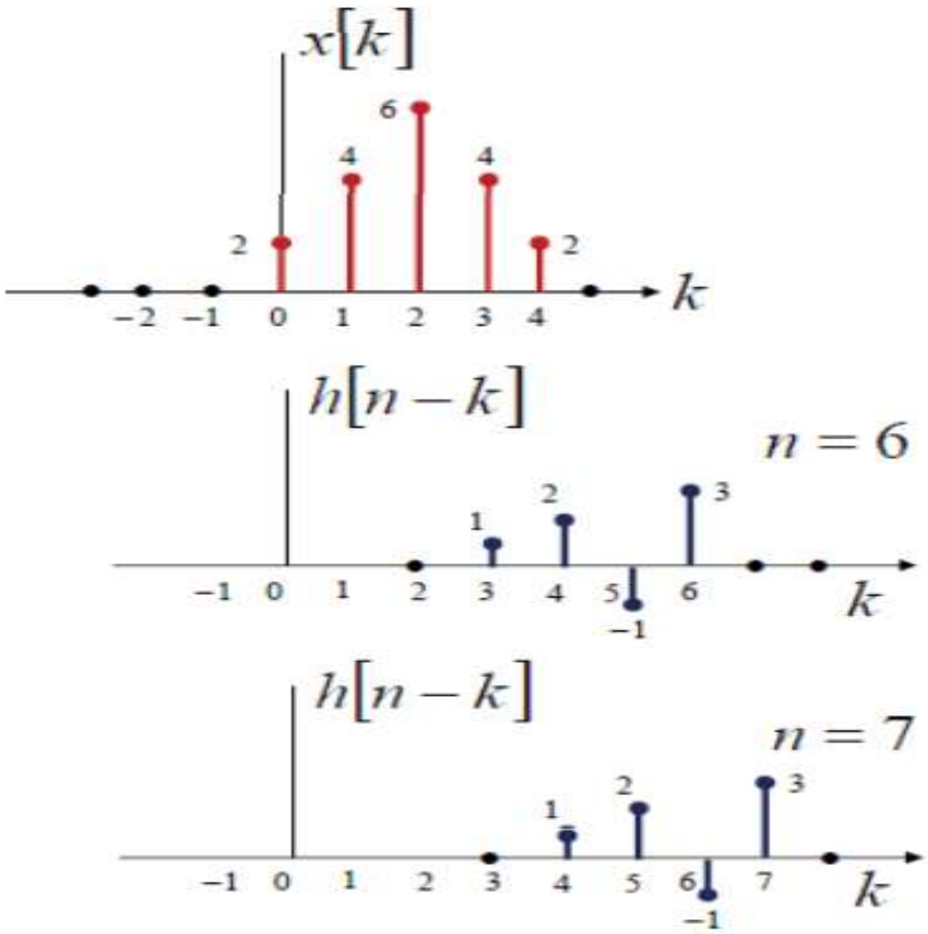
- For n = 5:

$$y[n] = (6)(1) + (4)(2) + (2)(-1)$$
$$= 12$$



Solution : Ex.1 (Method 1 - Graphical convolution)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$



- For n = 6:

$$y[n] = (4)(1) + (2)(2) = 8$$

- For n = 7:

$$y[n] = (2)(1) = 2$$

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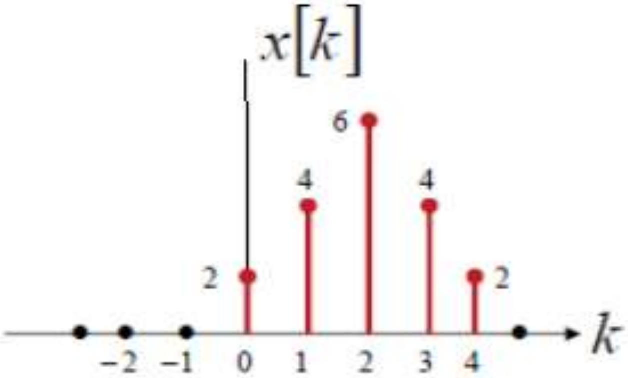
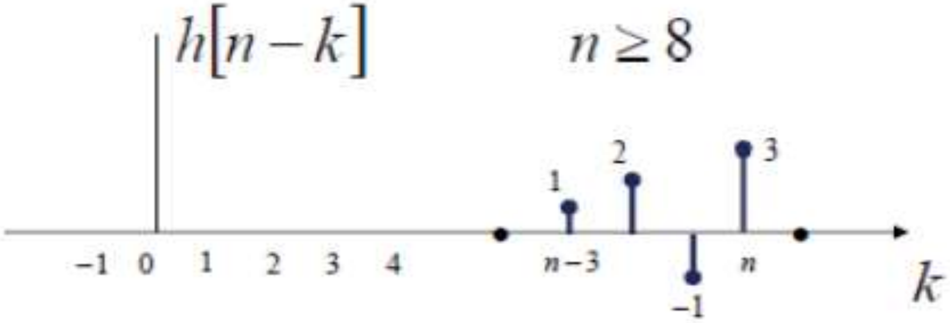
Solution : Ex.1 (Method 1 - Graphical convolution)

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

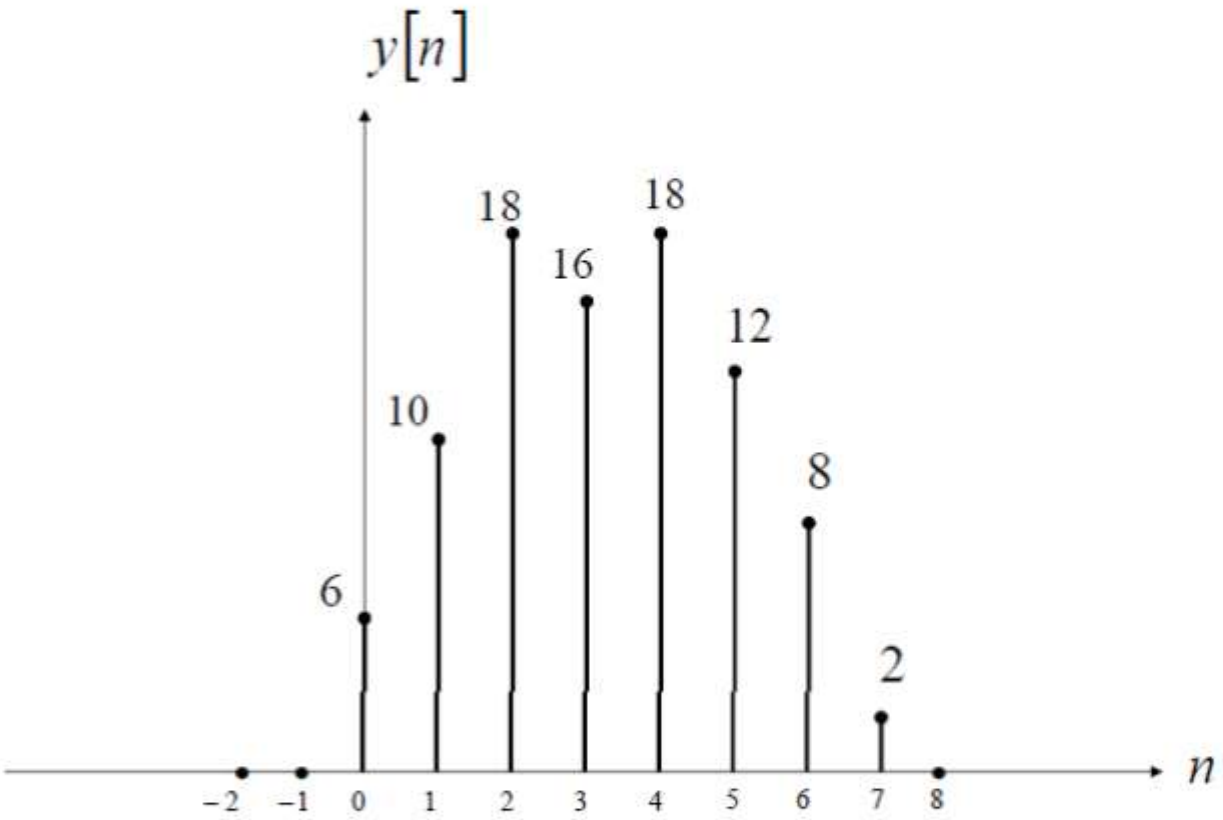


- For n ≥ 8:

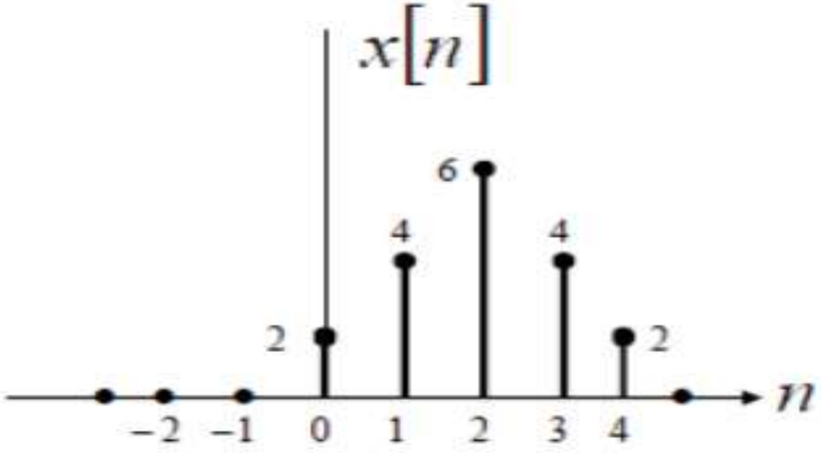
$$y[n] = 0$$

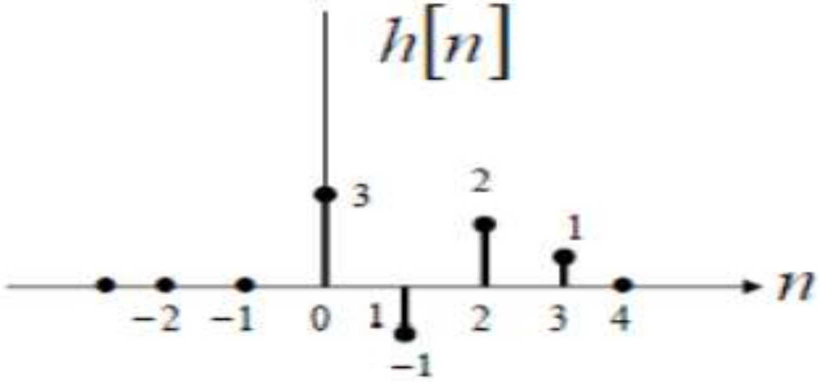


Solution : Ex.1 (Method 1 - Graphical convolution)



Ex.1 Find $y[n]=x[n]*h[n]$



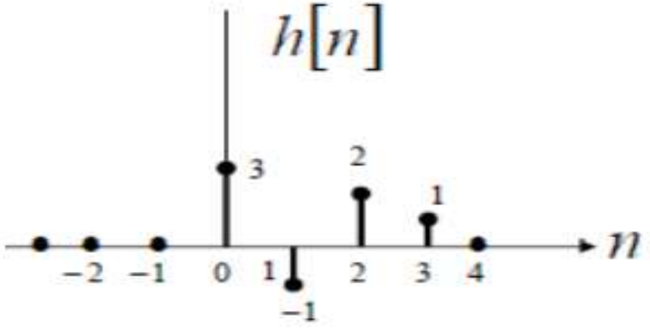


$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

Solution : Ex.1 (Method 2 - Tabular method)

First: we denote **the nonzero terms** of the impulse response h[n] as the convolution mask

$$h[k]=\{3\ ,-1\ ,2,1\ \}$$



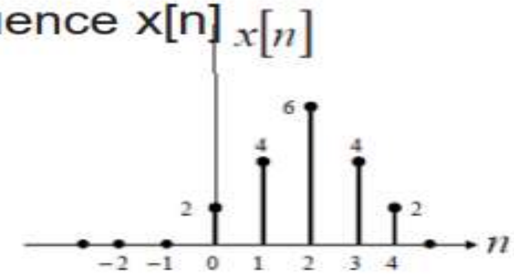
Second: we reverse the order of this convolution mask

$$h[-k]=\{1\ ,2\ ,-1\ ,3\ \}$$

Third: we slide the reversed convolution mask along the sequence x[n] and take the **dot product** between them for all n.

This process is illustrated in the following table for

$$n\chi\{0,1,\ldots,8\}$$



x=[0 0 2 4 6 4 2 0];

Solution : Ex.1 (Method 2 - Tabular method)

x=[00246420 0]

h=[3 -1 2 1]

$$y[n]=\sum_{k=-\infty}^{\infty} x[k]h[n-k]$$

n	k	-4	-3	-2	-1	0	1	2	3	4	5	6	7	8
	x[k]	0	0	0	0	2	4	6	4	2	0	0	0	0
0	h[0−k]		1	2	-1	3								
1	h[1−k]			1	2	-1	3							
2	h[2−k]				1	2	-1	3						
3	h[3−k]					1	2	-1	3					
4	h[4−k]						1	2	-1	3				
5	h[5−k]							1	2	-1	3			
6	h[6−k]								1	2	-1	3		
7	h[7−k]									1	2	-1	3	
	h[8−k]										1	2	-1	3

y[0]=[0 0 0 2].[1 2 -1 3] = 6

y[1]=[0 0 2 4].[1 2 -1 3] = 10

y[2]=[0 2 4 6].[1 2 -1 3] = 18

y[3]=[2 4 6 4].[1 2 -1 3] = 16

y[4]=[4 6 4 2].[1 2 -1 3] = 18

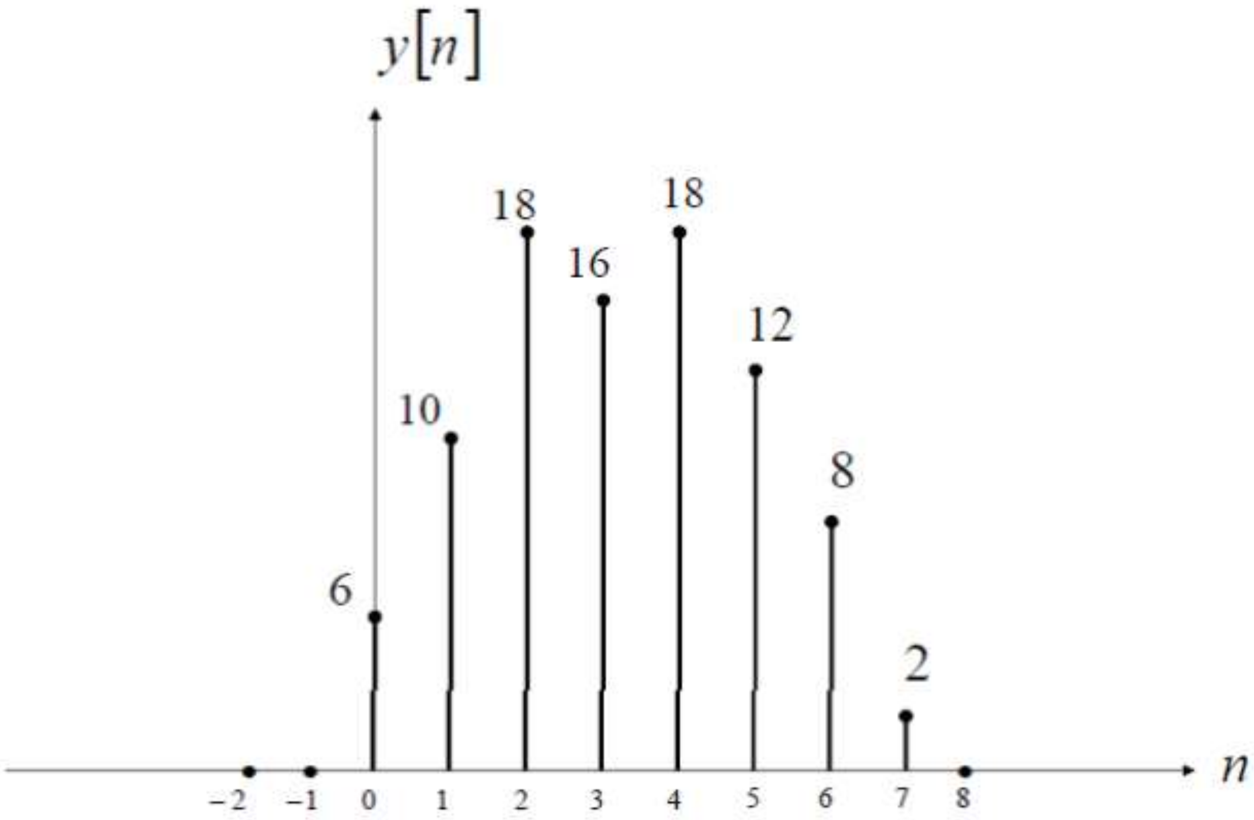
y[5]=[6 4 2 0].[1 2 -1 3] = 12

y[6]=[4 2 0 0].[1 2 -1 3] = 8

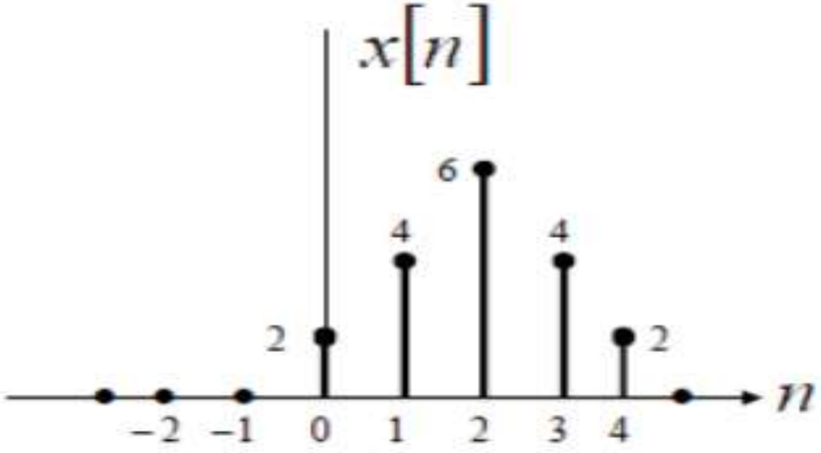
y[7]=[2 0 0 0].[1 2 -1 3] = 2

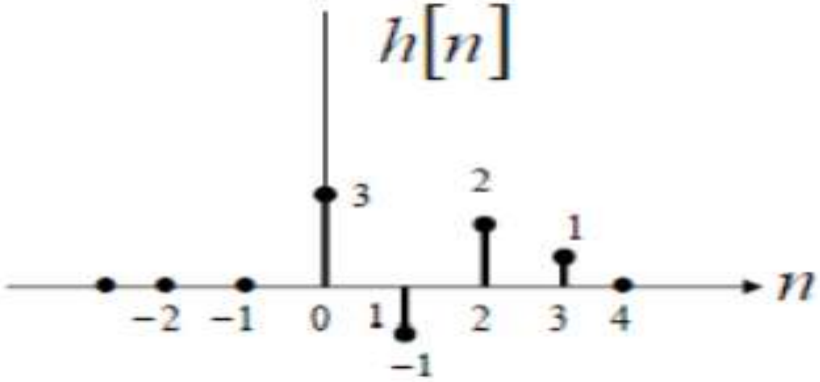
y[8]=[0 0 0 0].[1 2 -1 3] = 0

Solution : Ex.1 (Method 2 - Tabular method)



Ex.1 Find $y[n]=x[n]*h[n]$





$$y[n]=x[n]*h[n]=\sum_{k=-\infty}^{\infty}x[k]h[n-k]$$

Solution : Ex.1 (Method 3 – using MATLAB)

The convolution of two discrete-time signals can be carried out with the **MATLAB** M-file *conv*.

```
x=[0 2 4 6 4 2 0];
h=[0 3 -1 2 1 0];
y = conv(x,h);
```