BEARING CAPACITY OF FOUNDATIONS

INTRODUCTION

The soil must be capable of carrying the loads from any engineered structure placed upon it without *a shear failure* and with the *resulting settlements* being acceptable for that structure.
A soil shear failure can result in excessive building distortion and even collapse whereas excessive settlements can result in structural damage to a building frame.
It is necessary to investigate both base *shear resistance* and *settlements* for any structure.

The recommendation for the allowable bearing capacity q_a to be used for design is based on the *minimum* of either

1. Limiting the settlement to acceptable amount.
2. The ultimate bearing capacity, which considers soil strength, as computed in the following sections.

The allowable bearing capacity based on shear control q_a is obtained by reducing (or dividing) the ultimate bearing capacity q_{ult} (based on soil strength) by a safety factor **SF** that is deemed adequate to avoid a base shear failure to obtain

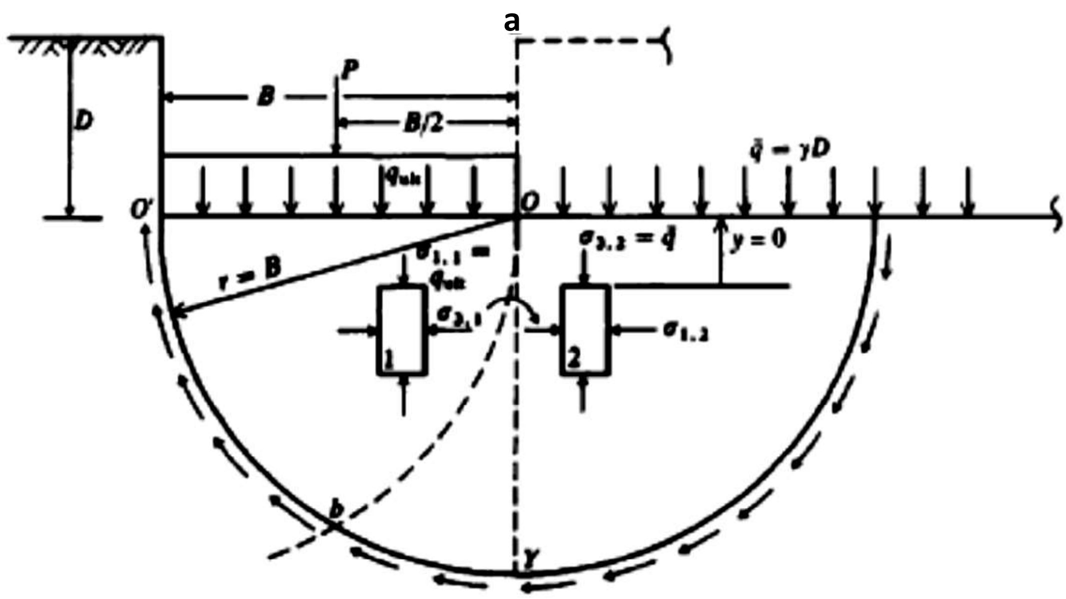
$$q_a = \frac{q_{ult}}{SF}$$

BEARING CAPACITY

From Fig. 4-1*a* and Fig. 4-2 it is evident we have two potential failure modes, where the footing, when loaded to produce the maximum bearing pressure q_{ult}, will do one or both of the following:
a. Rotate as in Fig. 4-*1a* about some center of rotation (probably along the vertical line *Oa)* with shear resistance developed along the perimeter of the slip zone shown as a circle.
b. Punch into the ground as the wedge *agb* of Fig. 4-2 or the approximate wedge *ObO'* of Fig. *4-1a.*

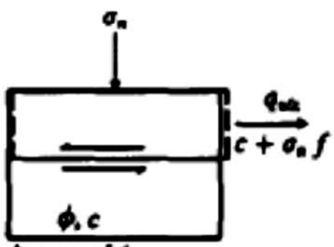
It should be apparent that both modes of potential failure develop the limiting soil shear strength along the slip path according to the shear strength equation given as

$s = c + \sigma \tan \phi$



(a) Footing on $\phi = 0°$ soil.

Note: $\bar{q} = p'_o = \gamma' D$, but use $\bar{q}$, since this is the accepted symbol for bearing capacity computations.

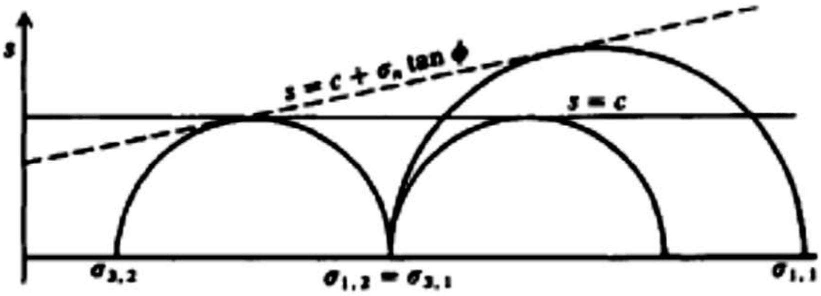


Area = dA

Friction = $\sigma_n \tan \phi = \sigma_n f$

Cohesion = $\dfrac{c\,dA}{dA}$

(b) Physical meaning of Eq. (2-52) for shear strength.



(c) Mohr's circle for (a) and for a ϕ–c soil.

Figure 4-1 Bearing capacity approximation on a $\phi = 0$ soil.

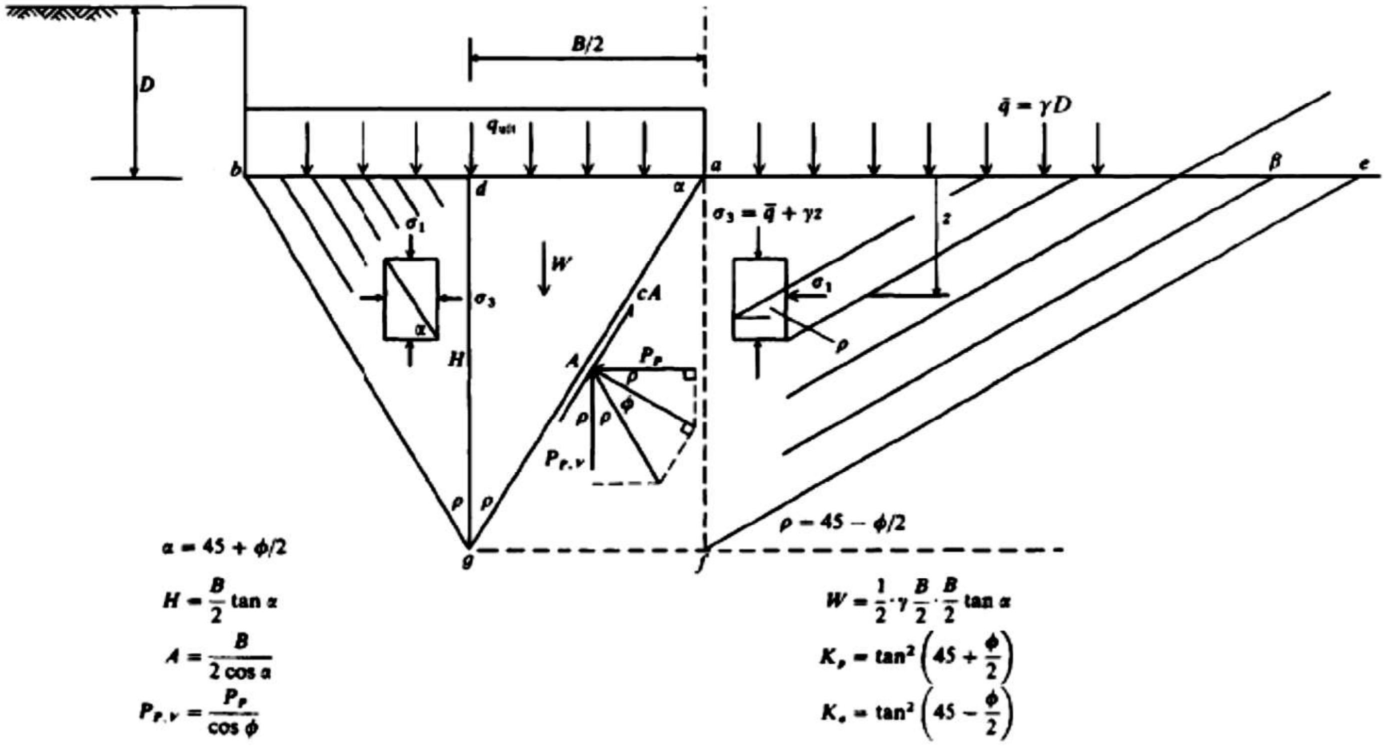


Figure 4-2 Simplified bearing capacity for a ϕ–c soil.

BEARING-CAPACITY EQUATIONS

There is currently no method of obtaining the ultimate bearing capacity of a foundation other than as an estimate.

The Terzaghi Bearing-Capacity Equation

One of the early sets of bearing-capacity equations was proposed by Terzaghi (1943) using the theory of plasticity to analyze the punching of a rigid base into a softer (soil) material as shown in Table 4-1.

Terzaghi's bearing-capacity equations were intended for "shallow" foundations where $D \le B$

Note that the original equation for ultimate bearing capacity is derived only for the plane-strain case (i.e., for continuous foundations).

Since the soil wedge beneath round and square bases is much closer to a triaxial than plane strain state, the adjustment of ϕ_{tr} to ϕ_{ps} is recommended only when *L/B > 2*

$\phi_{ps} = 1.50\phi_{tr} - 17°$ $\quad (\phi_{tr} > 34°)$

$\phi_{ps} = \phi_{tr}$ $\quad (\phi_{tr} \le 34°)$

TABLE 4-1

Bearing-capacity equations by the several authors indicated

Terzaghi (1943). See Table 4-2 for typical values and for $K_{p\gamma}$ values.

$$q_{ult} = cN_c s_c + \bar{q}N_q + 0.5\gamma BN_\gamma s_\gamma \qquad N_q = \frac{a^2}{a\cos^2(45 + \phi/2)}$$

$$a = e^{(0.75\pi - \phi/2)\tan\phi}$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = \frac{\tan\phi}{2}\left(\frac{K_{p\gamma}}{\cos^2\phi} - 1\right)$$

For:	strip	round	square
$s_c =$	1.0	1.3	1.3
$s_\gamma =$	1.0	0.6	0.8

Meyerhof (1963).* See Table 4-3 for shape, depth, and inclination factors.

$$q_{ult} = cN_c s_c d_c i_c + \bar{q}\, N_q s_q d_q i_q + 0.5\gamma BN_\gamma s_\gamma d_\gamma i_\gamma$$

$$N_q = e^{\pi\tan\phi}\tan^2\left(45 + \frac{\phi}{2}\right)$$

$$N_c = (N_q - 1)\cot\phi$$

$$N_\gamma = (N_q - 1)\tan(1.4\phi)$$

Hansen (1970).* See Table 4-5 for shape, depth, and other factors.

General:† $q_{ult} = cN_c s_c d_c i_c g_c b_c + \bar{q}N_q s_q d_q i_q g_q b_q + 0.5\gamma B'N_\gamma s_\gamma d_\gamma i_\gamma g_\gamma b_\gamma$

when $\phi = 0$

use $q_{ult} = 5.14s_u(1 + s'_c + d'_c - i'_c - b'_c - g'_c) + \bar{q}$

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 1.5(N_q - 1)\tan\phi$

Vesić (1973, 1975).* See Table 4-5 for shape, depth, and other factors.
Use Hansen's equations above.

N_q = same as Meyerhof above

N_c = same as Meyerhof above

$N_\gamma = 2(N_q + 1)\tan\phi$

*These methods require a trial process to obtain design base dimensions since width B and length L are needed to compute shape, depth, and influence factors.

†See Sec. 4-6 when $i_i < 1$.

Table 4-2 Terzaghi Bearing capacity factors —Eqs. (4.15), (4.13), and (4.11).[a]

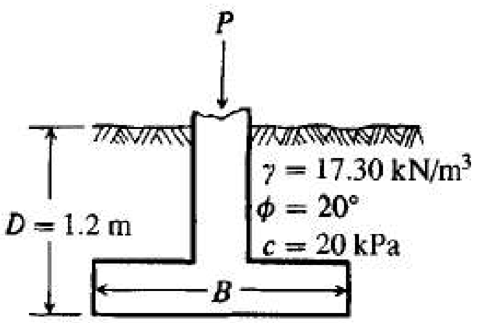
ϕ'	N_c	N_q	N_γ[a]	ϕ'	N_c	N_q	N_γ[a]
0	5.70	1.00	0.00	26	27.09	14.21	9.84
1	6.00	1.10	0.01	27	29.24	15.90	11.60
2	6.30	1.22	0.04	28	31.61	17.81	13.70
3	6.62	1.35	0.06	29	34.24	19.98	16.18
4	6.97	1.49	0.10	30	37.16	22.46	19.13
5	7.34	1.64	0.14	31	40.41	25.28	22.65
6	7.73	1.81	0.20	32	44.04	28.52	26.87
7	8.15	2.00	0.27	33	48.09	32.23	31.94
8	8.60	2.21	0.35	34	52.64	36.50	38.04
9	9.09	2.44	0.44	35	57.75	41.44	45.41
10	9.61	2.69	0.56	36	63.53	47.16	54.36
11	10.16	2.98	0.69	37	70.01	53.80	65.27
12	10.76	3.29	0.85	38	77.50	61.55	78.61
13	11.41	3.63	1.04	39	85.97	70.61	95.03
14	12.11	4.02	1.26	40	95.66	81.27	115.31
15	12.86	4.45	1.52	41	106.81	93.85	140.51
16	13.68	4.92	1.82	42	119.67	108.75	171.99
17	14.60	5.45	2.18	43	134.58	126.50	211.56
18	15.12	6.04	2.59	44	151.95	147.74	261.60
19	16.56	6.70	3.07	45	172.28	173.28	325.34
20	17.69	7.44	3.64	46	196.22	204.19	407.11
21	18.92	8.26	4.31	47	224.55	241.80	512.84
22	20.27	9.19	5.09	48	258.28	287.85	650.67
23	21.75	10.23	6.00	49	298.71	344.63	831.99
24	23.36	11.40	7.08	50	347.50	415.14	1072.80
25	25.13	12.72	8.34				

[a]From Kumbhojkar (1993)

The bearing capacity factors N_c, N_q, and N_γ are, respectively, the contributions of cohesion, surcharge, and unit weight of soil to the ultimate load-bearing capacity.

BEARING-CAPACITY EXAMPLES

Example 4-0. Compute the allowable bearing pressure using the Terzaghi equation for the square footing and soil parameters shown in Figure below. Use a safety factor of 3 to obtain q_a..



Solution.
Find the bearing capacity. Note that this value is usually what a geotechnical consultant would have to recommend *(B* not known but *D* is).
Since the footing is square (B=L), no adjustment of ϕ value is required.
From Table 4-2 obtain

$N_c = 17.7$ $N_q = 7.4$ $N_\gamma = 3.64$

$s_c = 1.3$ $s_\gamma = 0.8$ (from table 4-1, square footing)

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma B N_\gamma s_\gamma$

$= 20 (17.7) (1.3) + 1.2(17.3)(7.4) + 0.5 (17.3) (\boldsymbol{B})(3.64)(0.8)$
$= (613.8 + 25.2 \boldsymbol{B})$ kPa

The allowable pressure (a SF = 3 is commonly used when $c > 0$) is

$$q_a = \frac{q_{ult}}{SF}$$

$$= \frac{613.8+25.2B}{3} = (205 + 8.4B) \text{ kPa}$$

Since B is likely to range from 1.5 to 3 m

at B= 1.5 m $q_a = 205 + 8.4(1.5) = 218$ kPa (rounding)

at B = 3m $q_a = 205 + 8.4(3) = 230$ kPa

Recommend $q_a = 215{\sim}230$ kPa

Example 4.1

A square foundation is 2 m x 2 m in plan. The soil supporting the foundation has a friction angle of ϕ = 25^0 and c = 20 kN/m^2. The unit weight of soil, γ, is 16.5 kN/m^3.
Determine the allowable gross load on the foundation using Terzaghi Bearing Capacity Equations with a factor of safety (FS) of 3.
Assume that the depth of the foundation (D_f) is 1.5 m and that general shear failure occurs in the soil.

Solution

Since the footing is square (B=L), no adjustment of ϕ value is required.

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma BN_\gamma s_\gamma$

$s_c = 1.3$ $s_\gamma = 0.8$ (from table 4-1, square footing)

At B=2.0m

From Table 4.1, for $\phi' = 25°$,

$$N_c = 25.13$$

$$N_q = 12.72$$

$$N_\gamma = 8.34$$

Thus,

q_{ult} = **(20) (25.13) (1.3) + (1.5x16.5)(12.72) + (0.5)(16.5)(2)(8.34)(0.8)**

= 653.38 + 314.82 + 110.09 = 1078.29 kN/m²

So, the allowable load per unit area of the foundation is

$$q_a = \frac{q_{ult}}{FS} = \frac{1078.29}{3} = 359.5 \text{ kN/m}^2$$

Thus, the total allowable gross load is

$$Q = (359.5)B^2 = (359.5)(2 \times 2) = 1438 \text{ kN}$$

H.W: Resolve the same example assuming the foundation is circular with a diameter of 3m.

Example 4.2

Refer to Example 4.1. Assume that the shear-strength parameters of the soil are the same. A square foundation measuring $B \times B$ will be subjected to an allowable gross load of 1000 kN with FS = 3 and $D_f = 1$ m. Determine the size B of the foundation.

Solution

Allowable gross load Q =1000 kN with FS =3. Hence, the ultimate load $Q_{ult} = (Q_u)/(FS)$ = (1000)(3) = 3000 kN. So,

$$q_{ult} = \frac{Q_u}{B^2} = \frac{3000}{B^2}$$

$q_{ult} = cN_c s_c + \overline{q} N_q + 0.5\gamma BN_\gamma s_\gamma$

For $\phi' = 25°$, $N_c = 25.13$, $N_q = 12.72$, and $N_\gamma = 8.34$.
Also,

$$q = \gamma D_f = (16.5)(1) = 16.5 \text{ kN/m}^2$$

Now,

q_{ult} = (20) (25.13) (1.3) + (16.5)(12.72) + (0.5)(16.5)(B)(8.34)(0.8)
= 863.26 + 55.04 B **(b)**

Combining Eqs. (a) and (b),

$$\frac{3000}{B^2} = 863.26 + 55.04B \qquad \text{(c)}$$

B (m)	L.H.S	R.H.S
1.0	3000	918.3
1.5	1333	945.8
2.0	750	973.3
Try B=1.75m	979.6	959.6

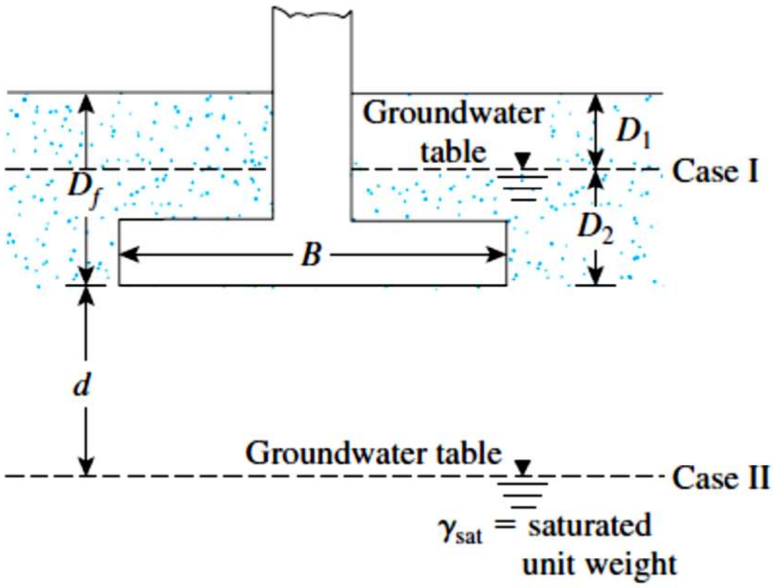
By trial and error, we have

$$B = 1.77 \text{ m} \approx \mathbf{1.8 \text{ m}}$$ ■

H.W.: Resolve the same example if the allowable gross load is 2500 kN.

Modification of Bearing Capacity Equations for Water Table

Equations in table 4.1 give the ultimate bearing capacity, based on the assumption that the water table is located well below the foundation. However, if the water table is close to the foundation, some modifications of the bearing capacity equations will be necessary. (See Figure below)



Case I. If the water table is located so that $0 \leq D1 \leq D_f$, the factor q in the bearing capacity equations takes the form

$\bar{q}$ = effective surcharge = $D_1 \gamma + D_2 \gamma'$

where $\gamma' = \gamma_{sat} - \gamma_w$

γ_{sat} = saturated unit weight of soil

γ_w = unit weight of water = 10 kN/m^3

Also, the value of γ in the last term of the equations has to be replaced by $\gamma' = \gamma_{sat} - \gamma_w$

Case II. For a water table located so that $0 \leq d \leq B$,

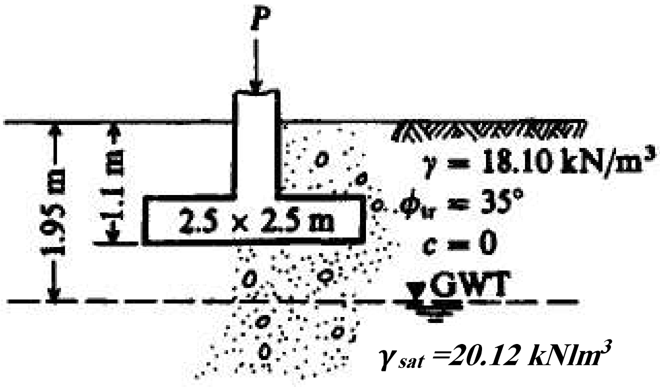
$\bar{q} = \gamma D_f$

In this case, the factor γ in the last term of the bearing capacity equations must be replaced by the factor

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

Case III. When the water table is located so that $d \geq B$, the water will have no effect on the ultimate bearing capacity.

Example 4-8. A square footing that is vertically and concentrically loaded is to be placed on a cohesionless soil as shown in Figure below. The soil and other data are as shown.



Required. What is the allowable bearing capacity using the Terzaghi equation and a SF = 2.5?

Solution:

Since the footing is square (B=L), no adjustment of ϕ value is required.

d = 1.95 – 1.1 = 0.85 m

B= 2.5m and d < B

$$\bar{\gamma} = \gamma' + \frac{d}{B}(\gamma - \gamma')$$

$\gamma = 18.1\ kN/m^3$ $\gamma_{sat} = 20.12\ kN/m^3$

$\acute{\gamma} = \gamma_{sat} - \gamma_w$

$\acute{\gamma} = 20.12 - 10 = 10.12\ kN/m^3$

$\bar{\gamma} = 10.12 + \frac{0.85}{2.5}(18.1 - 10.12) = 12.83\ kN/m^3$

$q_{ult} = cN_c\, s_c + \bar{q}\, N_q + 0.5\gamma B\, N_\gamma s_\gamma$

From table 4.2 $N_c = 57.75$ $N_q = 41.44$ $N_\gamma = 45.41$

$s_c = 1.3$ $s_\gamma = 0.8$ (from table 4-1, square footing)

for B = 2.5m

$q_{ult} = 0 + 1.1 \times 18.1 \times 41.44 + 0.5 \times 12.83 \times 2.5 \times 45.41 \times 0.8$

= 825.1 + 582.6 = 1407.7 kN/m²

q_a **= 1407.7 / 2.5 = 563 kN/m² = 563 kPa**

H.W: Resolve the same example assuming the water table is A: 0.5 m below ground level
B: 4.0 m below ground level

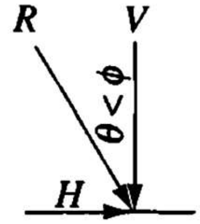
Meyerhof 's Bearing-Capacity Equation

Meyerhof (1951, 1963) proposed a bearing-capacity equation similar to that of Terzaghi but included a shape factor s_q with the depth term N_q. He also included depth factors d_i and inclination factors i_i for cases where the footing load is inclined from the vertical. These additions produce equations of the general form shown in Table 4-1, with select N factors computed in Table 4-4.

TABLE 4-3

Shape, depth, and inclination factors for the Meyerhof bearing-capacity equations of Table 4-1

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p\frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$



Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2

θ = angle of resultant R measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.

B, L, D = previously defined

TABLE 4-4
Bearing-capacity factors for the Meyerhof, Hansen, and Vesić bearing-capacity equations

Note that N_c and N_q are the same for all three methods; subscripts identify author for N_γ

ϕ	N_c	N_q	$N_{\gamma(H)}$	$N_{\gamma(M)}$	$N_{\gamma(V)}$	N_q/N_c	$2\tan\phi(1-\sin\phi)^2$
0	5.14*	1.0	0.0	0.0	0.0	0.195	0.000
5	6.49	1.6	0.1	0.1	0.4	0.242	0.146
10	8.34	2.5	0.4	0.4	1.2	0.296	0.241
15	10.97	3.9	1.2	1.1	2.6	0.359	0.294
20	14.83	6.4	2.9	2.9	5.4	0.431	0.315
25	20.71	10.7	6.8	6.8	10.9	0.514	0.311
26	22.25	11.8	7.9	8.0	12.5	0.533	0.308
28	25.79	14.7	10.9	11.2	16.7	0.570	0.299
30	30.13	18.4	15.1	15.7	22.4	0.610	0.289
32	35.47	23.2	20.8	22.0	30.2	0.653	0.276
34	42.14	29.4	28.7	31.1	41.0	0.698	0.262
36	50.55	37.7	40.0	44.4	56.2	0.746	0.247
38	61.31	48.9	56.1	64.0	77.9	0.797	0.231
40	75.25	64.1	79.4	93.6	109.3	0.852	0.214
45	133.73	134.7	200.5	262.3	271.3	1.007	0.172
50	266.50	318.5	567.4	871.7	761.3	1.195	0.131

* = $\pi + 2$ as limit when $\phi \rightarrow 0°$.

Slight differences in above table can be obtained using program BEARING.EXE on diskette depending on computer used and whether or not it has floating point.

Hansen's Bearing-Capacity Method

Hansen (1970) proposed the general bearing-capacity case and N factor equations shown in Table 4-1. Hansen's shape, depth, and other factors making up the general bearing capacity equation are given in Table 4-5. The extensions include base factors for situations in which the footing is tilted from the horizontal b_i and for the possibility of a slope β of the ground supporting the footing to give ground factors g_i.

Note that when the base is tilted, V and H are perpendicular and parallel, respectively, to the base, compared with when it is horizontal as shown in the sketch with Table 4-5c. The bearing capacity using N factors as given in Table 4-4.

The Hansen equation can be used for both shallow (footings) and deep (piles, drilled caissons) bases.

TABLE 4-5a

Shape and depth factors for use in either the Hansen (1970) or Vesić (1973, 1975*b*) bearing-capacity equations of Table 4-1. Use s'_c, d'_c when $\phi = 0$ only for Hansen equations. Subscripts *H, V* for Hansen, Vesić, respectively.

Shape factors	Depth factors
$s'_{c(H)} = 0.2\dfrac{B'}{L'} \quad (\phi = 0°)$ $s_{c(H)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B'}{L'}$ $s_{c(V)} = 1.0 + \dfrac{N_q}{N_c} \cdot \dfrac{B}{L}$ $s_c = 1.0$ for strip	$d'_c = 0.4k \quad (\phi = 0°)$ $d_c = 1.0 + 0.4k$ $k = D/B$ for $D/B \le 1$ $k = \tan^{-1}(D/B)$ for $D/B > 1$ k in radians
$s_{q(H)} = 1.0 + \dfrac{B'}{L'}\sin\phi$ $s_{q(V)} = 1.0 + \dfrac{B}{L}\tan\phi$ for all ϕ	$d_q = 1 + 2\tan\phi(1 - \sin\phi)^2 k$ k defined above
$s_{\gamma(H)} = 1.0 - 0.4\dfrac{B'}{L'} \quad \ge 0.6$ $s_{\gamma(V)} = 1.0 - 0.4\dfrac{B}{L} \quad \ge 0.6$	$d_\gamma = 1.00 \quad$ for all ϕ

Notes:

1. Note use of "effective" base dimensions B', L' by Hansen but not by Vesić.
2. The values above are consistent with either a vertical load or a vertical load accompanied by a horizontal load H_B.
3. With a vertical load and a load H_L (and either $H_B = 0$ or $H_B > 0$) you may have to compute two sets of shape s_i and d_i as $s_{i,B}$, $s_{i,L}$ and $d_{i,B}$, $d_{i,L}$. For i, L subscripts of Eq. (4-2), presented in Sec. 4-6, use ratio L'/B' or D/L'.

TABLE 4-5b

Table of inclination, ground, and base factors for the Hansen (1970) equations. See Table 4-5c for equivalent Vesić equations.

Inclination factors	Ground factors (base on slope)
$i'_c = 0.5 - 0.5\sqrt{1 - \frac{H_i}{A_f C_a}}$	$g'_c = \frac{\beta°}{147°}$
$i_c = i_q - \frac{1 - i_q}{N_q - 1}$	$g_c = 1.0 - \frac{\beta°}{147°}$
$i_q = \left[1 - \frac{0.5H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_1}$ $2 \le \alpha_1 \le 5$	$g_q = g_\gamma = (1 - 0.5\tan\beta)^5$
	Base factors (tilted base)
$i_\gamma = \left[1 - \frac{0.7H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$	$b'_c = \frac{\eta°}{147°}$ $(\phi = 0)$
$i_\gamma = \left[1 - \frac{(0.7 - \eta°/450°)H_i}{V + A_f c_a \cot\phi}\right]^{\alpha_2}$ $2 \le \alpha_2 \le 5$	$b_c = 1 - \frac{\eta°}{147°}$ $(\phi > 0)$ $b_q = \exp(-2\eta\tan\phi)$ $b_\gamma = \exp(-2.7\eta\tan\phi)$ η in radians

Notes:

1. Use H_i as either H_B or H_L, or both if $H_L > 0$.
2. Hansen (1970) did not give an i_c for $\phi > 0$. The value above is from Hansen (1961) and also used by Vesić.
3. Variable c_a = base adhesion, on the order of 0.6 to 1.0 × base cohesion.
4. Refer to sketch for identification of angles η and β, footing depth D, location of H_i (parallel and at top of base slab; usually also produces eccentricity). Especially note V = force *normal* to base and is not the resultant R from combining V and H_i.

TABLE 4-5c
Table of inclination, ground, and base factors for the Vesic (1973, 1975) bearing-capacity equations. See notes below and refer to sketch for identification of terms.

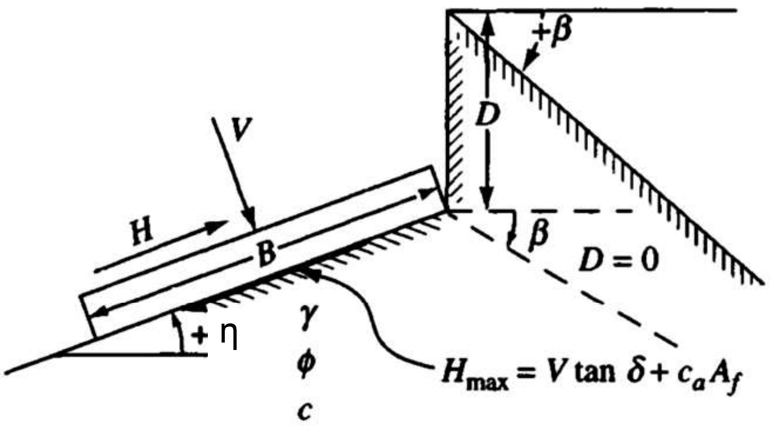
Inclination factors	Ground factors (base on slope)
$i'_c = 1 - \dfrac{mH_i}{A_f c_a N_c} \quad (\phi = 0)$	$g'_c = \dfrac{\beta}{5.14} \quad \beta$ in radians
$i_c = i_q - \dfrac{1 - i_q}{N_q - 1} \quad (\phi > 0)$	$g_c = i_q - \dfrac{1 - i_q}{5.14 \tan\phi} \quad \phi > 0$
i_q, and m defined below	i_q defined with i_c
$i_q = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^m$	$g_q = g_\gamma = (1.0 - \tan\beta)^2$
	Base factors (tilted base)
$i_\gamma = \left[1.0 - \dfrac{H_i}{V + A_f c_a \cot\phi}\right]^{m+1}$	$b'_c = g'_c \quad (\phi = 0)$
$m = m_B = \dfrac{2 + B/L}{1 + B/L}$	$b_c = 1 - \dfrac{2\beta}{5.14 \tan\phi}$
$m = m_L = \dfrac{2 + L/B}{1 + L/B}$	$b_q = b_\gamma = (1.0 - \eta \tan\phi)^2$

Notes:

1. When $\phi = 0$ (and $\beta \neq 0$) use $N_\gamma = -2\sin(\pm\beta)$ in N_γ term.
2. Compute $m = m_B$ when $H_i = H_B$ (H parallel to B) and $m = m_L$ when $H_i = H_L$ (H parallel to L). If you have both H_B and H_L use $m = \sqrt{m_B^2 + m_L^2}$. Note use of B and L, not B', L'.
3. Refer to Table sketch and Tables 4-5*a,b* for term identification.
4. Terms N_c, N_q, and N_γ are identified in Table 4-1.
5. Vesić always uses the bearing-capacity equation given in Table 4-1 (uses B' in the N_γ term even when $H_i = H_L$).
6. H_i term ≤ 1.0 for computing i_q, i_γ (always).

Notes: $\beta + \eta$ 90° (Both β and η have signs (+) shown.)

β ϕ



For: $L/B \le 2$ use ϕ_{tr}

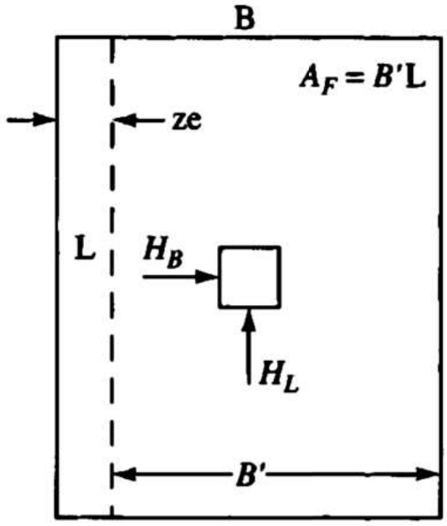
$L/B > 2$ use $\phi_{ps} = 1.5\,\phi_{tr} - 17°$

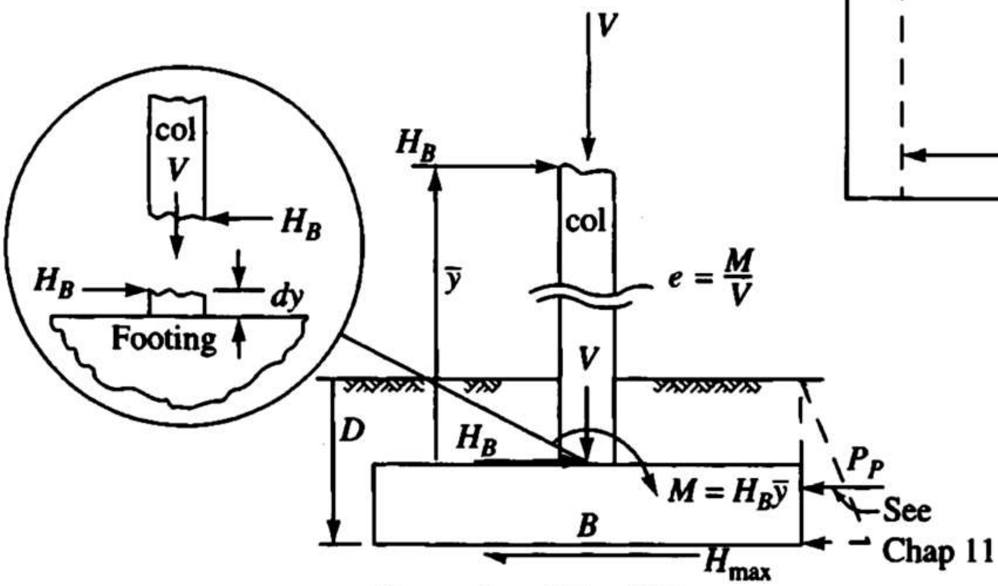
$\phi_{tr} \le 34°$ use $\phi_{tr} = \phi_{ps}$

δ = friction angle between base and soil ($.5\phi \le \delta \le \phi$)

$A_f = B'L'$ (effective area)

c_a = base adhesion (0.6 to 1.0c)





$H_{max} + P_p \ge SF \times (H_B)$

Example 4-2: A footing load test made produced the following data:

$D = 0.5$ m $B = 0.5$ m $L = 2.0$ m
$\gamma' = 9.31$ kN/m^3 $\phi_{triaxial} = 42.5°$ Cohesion $c = 0$
$P_{ult} = 1863$ kN (measured) $q_{ult} = \frac{P_{ult}}{BL} = \frac{1863}{0.5x2} = 1863$ kPa (computed)

Required: Compute the ultimate bearing capacity by both Hansen and Meyerhof equations and compare these values with the measured value.
Solution:
a. Since $c = 0$, any factors with subscript c do not need computing. All g_i and b_i factors are 1.00; with these factors identified, the Hansen equation simplifies to

$$q_{ult} = \gamma' D N_q s_q d_q + 0.5\gamma' B N_\gamma S_\gamma d_\gamma$$

$$L/B = \frac{2}{0.5} = 4 \rightarrow \phi_{ps} = 1.5(42.5) - 17 = 46.75°$$

$$\text{Use } \phi = 47°$$

From a table of ϕ in 1° increments (table not shown) obtain

$$N_q = 187 \qquad N_\gamma = 299$$

Using linear interpolation of Table 4-4 gives 208.2 and 347.2. Using Table 4-5*a* one obtains [get the $2\tan\phi(1-\sin\phi)^2$ part of d_q term from Table 4-4] the following:

$$s_{q(H)} = 1 + \frac{B'}{L'}\sin\phi = 1.18 \qquad s_{\gamma(H)} = 1 - 0.4\frac{B'}{L'} = 0.9$$

$$d_q = 1 + 2\tan\phi(1-\sin\phi)^2\frac{D}{B'} = 1 + 0.155\frac{D}{B'}$$

$$= 1 + 0.155\left(\frac{0.5}{0.5}\right) = 1.155 \qquad d_\gamma = 1.0$$

With these values we obtain

$q_{ult} = 9.31(0.5)(187)(1.18)(1.155) + 0.5(9.31)(0.5)(299)(0.9)(1)$
$= 1812$ kPa vs. 1863 kPa measured

b. By the Meyerhof equations of Table 4-1 and 4-3, and $\phi_{ps} = 47°$, we can proceed as follows:

Step 1. Obtain $N_q = 187$

$$N_\gamma = (N_q - 1)\tan(1.4\phi) = 413.6 \rightarrow 414$$

$$K_p = \tan^2\left(45 + \frac{\phi}{2}\right) = 6.44 \rightarrow \sqrt{K_p} = 2.54$$

$$s_q = s_\gamma = 1 + 0.1K_p\frac{B}{L} = 1 + 0.1(6.44)\frac{0.5}{2.0} = 1.16$$

$$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B} = 1 + 0.1(2.54)\frac{0.5}{0.5} = 1.25$$

Step 2. Substitute into the Meyerhof equation (ignoring any c subscripts):

$$\begin{aligned} q_{ult} &= \gamma' D N_q s_q d_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma \\ &= 9.31(0.5)(187)(1.16)(1.25) + 0.5(9.31)(0.5)(414)(1.16)(1.25) \\ &= 1262 + 1397 = \mathbf{2659}\ \text{kPa} \end{aligned}$$

Example 4-3:

A series of large-scale footing bearing-capacity tests were performed on soft saturated clay (ϕ =0). One of the tests consisted of a 1.05-m-square footing at a depth D = 1.5 m. At a 25 mm. settlement the load was approximately 16.1 tons from interpretation of the given load-settlement curve. Unconfined compression and shear tests gave values as follows:

q_u **= 3.0 ton/m²** c **= 1.92 ton/m² , the unit weight of soil is 17.5 kN/m³**

Required: Compute the ultimate bearing capacity by the Hansen equations and compare with the load-test value of 16.1 tons.

Solution: Obtain N, s'_i, and d'_i factors. Since ϕ= 0°, we have N_c = 5.14 and N_q= 1.0

$$s'_c = 0.2\frac{B}{L} = 0.2\frac{1}{1} = 0.2$$

$$d'_c = 0.4\tan^{-1}\frac{D}{B} = 0.4\tan^{-1}\frac{1.5}{1.05} = 0.38 \qquad (D > B)$$

q_{ult}**= 5.14s$_u$ (1 +** $s'_c + d'_c$**) +** $\overline{q}$ **Table 4-1 for** ϕ**= 0 case**

c **= 1.92 x 10 = 19.2 kN/m²** **(10 converts ton to kN)**

q_{ult} **= 5.14(19.2)(1 + 0.2 + 0.38) + 17.5 x1.5 = 182.2 kN/m²**

From load test, q_{actual} = 16.1/1.05^2 = 14.6 ton/m² = 146 kN/m²

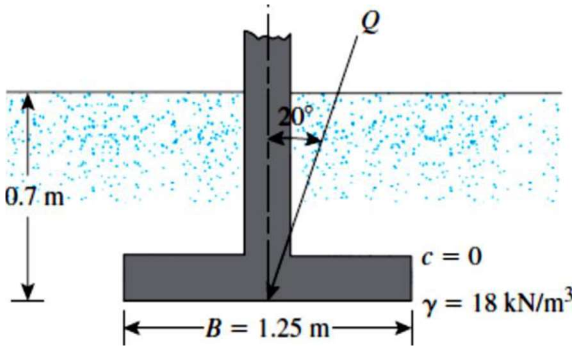
If we use the unconfined compression tests and take $c = q_u/2$, we obtain

q_{ult} **= (1.5/ 1.92) x182.2 = 142.4 kN/m²**

Example 4.5

A square column foundation (see figure below) is to be constructed on a fine sand deposit. The allowable load *Q* will be inclined at an angle *β* = 20° with the vertical. The standard penetration numbers N_{70} obtained from the field are as follows.

Depth (m)	N_{70}
1.5	5
3.0	4
4.5	9
6.0	7
7.5	8
9	8



Determine Q using Meyerhof bearing capacity equations, use F.S =3

Solution:

The average SPT number is (5 + 4 + 9 + 7 + 8 + 8) / 6 = 6.83

From table 3-4, the soil can be classified as medium density fine sand and the angle of internal friction (ϕ) is estimated to be = 30^0

Since the footing is square (B=L), no adjustment of ϕ value is required

The general form of Meyerhof B.C equation is:

$$q_{ult} = cN_c s_c d_c i_c + \overline{q}\, N_q s_q d_q i_q + 0.5\gamma B N_\gamma s_\gamma d_\gamma i_\gamma$$

From table 4-4 and for ϕ = 30^0, we have $N_c = 30.13$, $N_q = 18.4$ and $N_\gamma = 15.7$

Since *c* = 0, any factors with subscript *c* do not need computing.

Factors	Value	For
Shape:	$s_c = 1 + 0.2K_p \frac{B}{L}$	Any ϕ
	$s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$	$\phi > 10°$
	$s_q = s_\gamma = 1$	$\phi = 0$
Depth:	$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$	Any ϕ
	$d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$	$\phi > 10$
	$d_q = d_\gamma = 1$	$\phi = 0$
Inclination:	$i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$	Any ϕ
R V θ H	$i_\gamma = \left(1 - \frac{\theta°}{\phi°}\right)^2$	$\phi > 0$
	$i_\gamma = 0$ for $\theta > 0$	$\phi = 0$

Where $K_p = \tan^2(45 + \phi/2)$ as in Fig. 4-2
θ = angle of resultant *R* measured from vertical without a sign; if $\theta = 0$ all $i_i = 1.0$.
B, L, D = previously defined

for ϕ > 10^0 $\quad s_q = s_\gamma = 1 + 0.1K_p \frac{B}{L}$

where $K_p = tan^2\,(45 + \phi/2) = tan^2\,(45 + 30/2) = 3.0$

$\therefore\; s_q = s_\gamma = 1 + 0.1 \times 3 \times \frac{1.25}{1.25} = 1.3$

for ϕ > 10^0 $\quad d_q = d_\gamma = 1 + 0.1\sqrt{K_p}\frac{D}{B}$

$\therefore\; d_q = d_\gamma = \mathbf{1 + 0.1\sqrt{3}\,\frac{0.7}{1.25} = 1.097 \approx 1.1}$

for any ϕ $\quad i_c = i_q = \left(1 - \frac{\theta°}{90°}\right)^2$

$\therefore \quad i_q = (1 - \frac{20}{90})^2 = 0.605$

$$i_\gamma = \left(1 - \frac{\theta^\circ}{\phi^\circ}\right)^2 \qquad \text{for } \phi > 0$$

$\therefore \quad i_\gamma = (1 - \frac{20}{30})^2 = 0.111$

$\bar{q} = D \times \gamma = 0.7 \times 18 = 12.6\ kN/m^2$

$q_{ult} = 12.6 \times 18.4 \times 1.3 \times 1.1 \times 0.605 + 0.5 \times 18 \times 1.25 \times 15.7 \times 1.3 \times 1.1 \times 0.111 = 200.5 + 28.03$
$= 228.3\ kN/m^2$

$q_a = 228.3/3 = 76.2\ kN/m^2$

$Q = q_a \times B \times L = 76.2 \times 1.25^2 = 119\ kN$

Example 4-4:

Given: A series of unconfined compression tests in the zone of interest (from SPT samples) from a boring-log give an average $q_u = 200$ kPa. The soil is fully saturated ($\phi = 0$)

Required: Estimate the allowable bearing capacity for square footings located at somewhat uncertain depths (let D =0 m) and *B* dimensions unknown using both the Meyerhof and Terzaghi bearing-capacity equations. Use safety factor SF = 3.0.

Solution: (The reader should note this is the most common procedure for obtaining the allowable bearing capacity for cohesive soils with limited data.)

a: By Meyerhof equations,
from table 4.1

$$q_{ult} = cN_c s_c d_c + \bar{q} N_q s_q d_q + 0.5\gamma B' N_\gamma s_\gamma d_\gamma$$

$c = q_u/2$ (for both equations)

from table 4.3 $\quad s_c = 1 + 0.2\, K_p \frac{B}{L}$

$K_p = \tan^2 (45 + \emptyset / 2) = \tan^2 (45) = 1.0$

$s_c = 1.2$

$$d_c = 1 + 0.2\sqrt{K_p}\frac{D}{B}$$

$d_c = 1. + 0 = 1.0$

$$s_q = s_\gamma = 1 \qquad \phi = 0$$

$$d_q = d_\gamma = 1 \qquad \phi = 0$$

$$q_{ult} = 1.2cN_c + \bar{q}N_q$$

$$q_a = \frac{q_{ult}}{3} = 1.2\frac{q_u}{2}(5.14)\frac{1}{3} + \frac{\bar{q}}{3} = 1.03q_u + 0.3\bar{q}$$

b. By Terzaghi equations, we can take $s_c = 1.3$ for $\phi = 0$.

$$q_a = \frac{q_{ult}}{3} = \frac{q_u}{2}(5.7)(1.3)\frac{1}{3} + \frac{\bar{q}}{3} = 1.24q_u + 0.3\bar{q}$$

FOOTINGS WITH ECCENTRIC OR INCLINED LOADINGS

A footing may be eccentrically loaded from a concentric column with an axial load and moments about one or both axes as in Fig. 4-4. The eccentricity may result also from a column that is initially not centrally located.

Footings with Eccentricity

Research and observation [Meyerhof and Hansen] indicate that *effective* footing dimensions obtained (refer to Fig. 4-4) as

$L' = L - 2e_x \qquad B' = B - 2e_y$

should be used in bearing-capacity analyses to obtain an effective footing area defined as

$A_f = B'L'$

and the center of pressure when using a rectangular pressure distribution of q' is the center of area $B'L'$ at point A'; i.e., from Fig *4-4a:*

$$2e_x + L' = L$$
$$e_x + c = L/2$$

Substitute for L and obtain $c = L'/2$. If there is no eccentricity about either axis, use the actual footing dimension for that B' *or* L'.

For design the minimum dimensions (to satisfy ACI 318 code) of a rectangular footing with a central column of dimensions $w_x \times w_y$ are required to be

$B_{min} = 4e_y + w_y \qquad B' = 2e_y + w_y$

$L_{min} = 4e_x + w_x \qquad L' = 2e_x + w_x$

Final dimensions may be larger than B_{min} or L_{min} based on obtaining the required allowable bearing capacity.
The ultimate bearing capacity for footings with eccentricity, using Hansen/Vesic equations, is found by either the Hansen or Vesic bearing-capacity equation given in Table 4-1 with the following adjustments:

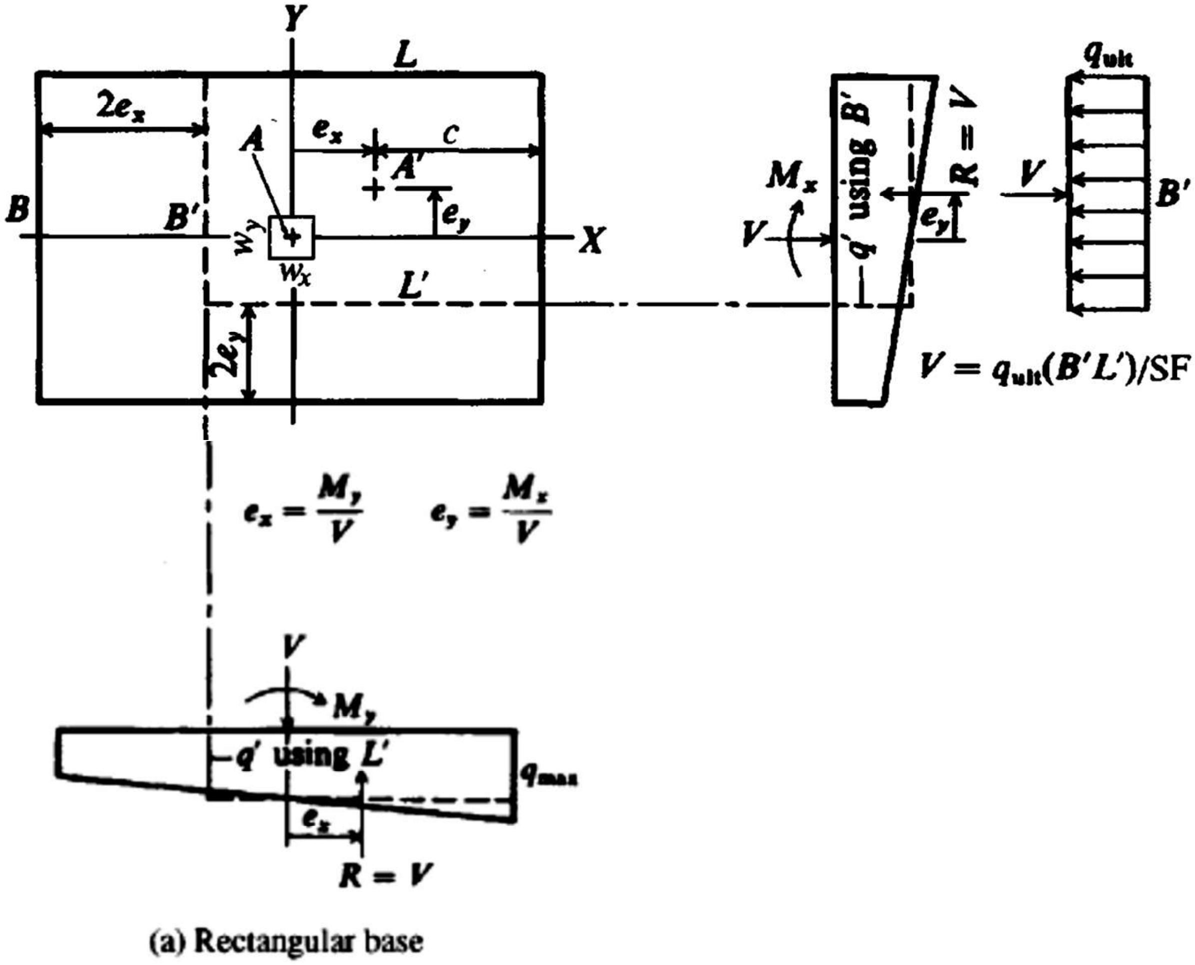


Figure 4-4. Method of computing effective footing dimensions when footing is eccentrically loaded for rectangular bases.

a. Use B' in the $\gamma B N_\gamma$ term.
b. Use B' and L' in computing the shape factors.
c. Use actual B and L for all depth factors.
The computed ultimate bearing capacity q_{ult} is then reduced to an allowable value q_a with an appropriate safety factor **SF** as

$q_a = q_{ult}/\mathrm{SF}$ (and $P_a = q_a B'L'$)

Example 4-5. A square footing is 1.8 X 1.8 m with a 0.4 X 0.4 m square column. It is loaded with an axial load of 1800 kN and M_x = 450 kN • m; M_y = 360 kN • m. Undrained triaxial tests (soil not saturated) give ϕ = 36° and c = 20 kPa. The footing depth D = 1.8 m; the soil unit weight γ = 18.00 kN/m^3; the water table is at a depth of 6.1 m from the ground surface.

Required: What is the allowable soil pressure, if SF = 3.0, using the Hansen bearing-capacity equation with *B', L'?*

Solution. See Fig. E4-5.
e_y = 450/1800 = 0.25 m $\qquad$ e_x = 360/1800 = 0.20 m

Both values of e are < $B/6$ = 1.8/6 = 0.30 m. Also
B_{min} = 4(0.25) + 0.4 = 1.4 < 1.8 m given
L_{min} = 4(0.20) + 0.4 = 1.2 < 1.8 m given

Now find

$B' = B - 2e_y$ = 1.8 - 2(0.25) = 1.3 m
$L' = L - 2e_x$ = 1.8 - 2(0.20) = 1.4 m (L' > B')

By Hansen's equation.

From Table 4-4 at ϕ = 36° and rounding to integers, we obtain
$N_c = 51 \quad N_q = 38 \quad N_\gamma = 40$
$N_q/N_c = 0.746 \qquad 2\tan\phi\,(1-\sin\phi)^2 = 0.247$

Compute D/B = 1.8/1.8 = 1.0
Now compute

$s_c = 1 + (N_q/N_C)(B'/L') = 1 + 0.746(1.3/1.4) = 1.69$
$d_c = 1 + 0.4D/B = 1 + 0.4(1.8/1.8) = 1.40$
$s_q = 1 + (B'/L')\sin\phi = 1 + (1.3/1.4)\sin 36° = 1.55$
$d_q = 1 + 2\tan\phi\,(1 - \sin\phi)^2 D/B = 1 + 0.247(1.0) = 1.25$
$s_\gamma = 1 - 0.4\,B'/L' = 1 - 0.4 \times 1.3/1.4 = 0.62 > 0.60$ (O.K.)
$d_\gamma = 1.0$
All $i_i = g_i = b_i = 1.0$ (not 0.0)
The Hansen equation is given in Table 4-1 as
$q_{ult} = c\,N_c s_c d_c + \bar{q}\,N_q s_q d_q + 0.5\gamma B'\,N_\gamma s_\gamma d_\gamma$

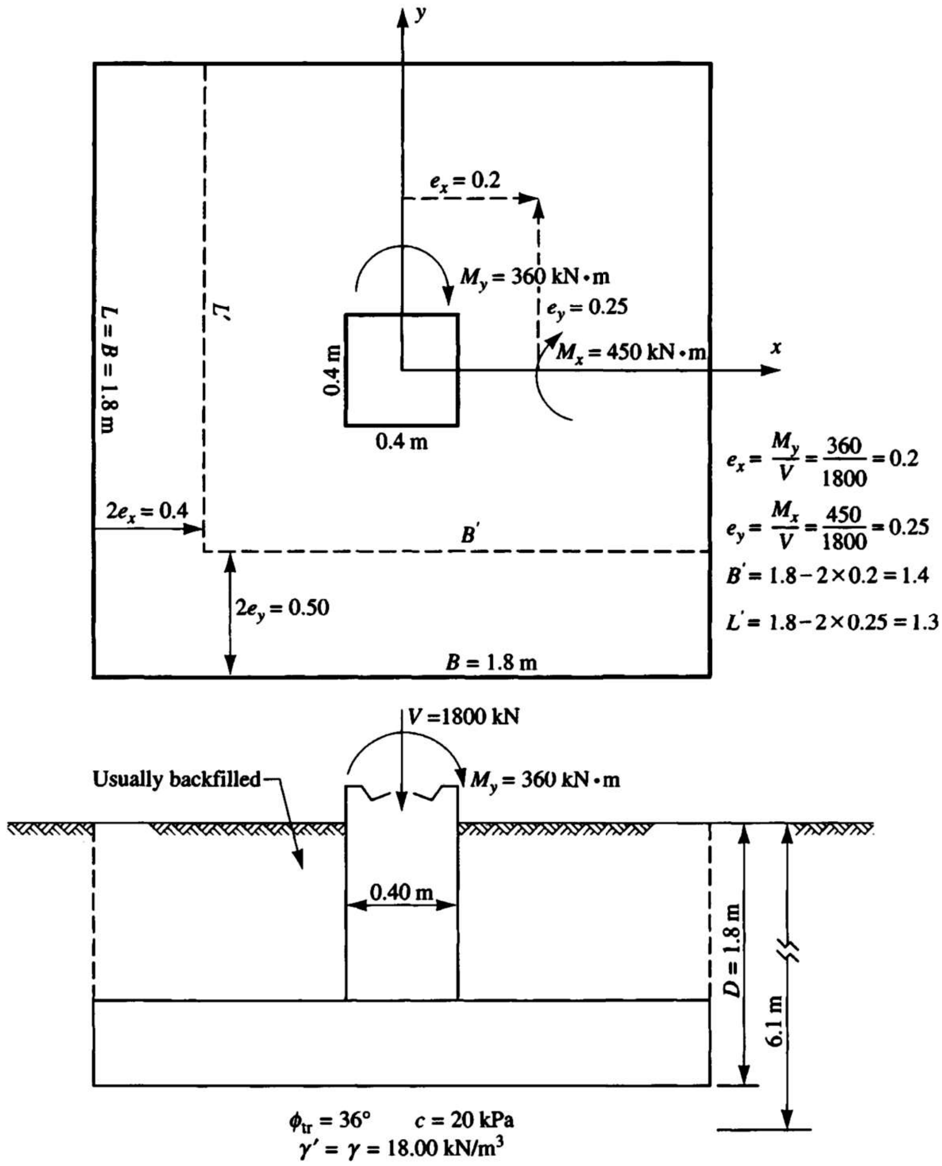


Figure E4-5

Inserting values computed above with terms of value 1.0 not shown (except d_γ) and using $B' = 1.3$, we obtain

$$q_{ult} = 20(51)(1.69)(1.4) + 1.8(18.0)(38)(1.55)(1.25) + 0.5(18.0)(1.3)(40)(0.62)(1.0)$$

$$= 2413 + 2385 + 290 = 5088 \text{ kPa}$$

For SF = 3.0 the allowable soil pressure q_a is

$q_{all} = 5088/3 = 1696$ kPa → **1700** kPa

The actual soil pressure is

$$q_{act} = \frac{1800}{B'\ L'} = \frac{1800}{1.3x1.4} = 989\ kPa$$

Note that the allowable pressure q_{all} is very large, and the actual soil pressure q_{act} is also large. With this large actual soil pressure, settlement may be the limiting factor. Some geotechnical consultants routinely limit the maximum allowable soil pressure to around 500 kPa in recommendations to clients for design whether settlement is a factor or not. Small footings with large column loads are visually not very confidence-inspiring during construction.

BEARING CAPACITY FROM SPT

The SPT is widely used to obtain the bearing capacity of soils directly. One of the earliest published relationships was that of Terzaghi and Peck. This has been widely used, but these curves were overly conservative. Meyerhof published equations for computing the allowable bearing capacity for a 25-mm settlement. These were also very conservative.
Joseph E. Bowels adjusted the equations to obtain the following:

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B \leq 1.22 \text{ m})$$

and

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08} \left(\frac{B + 0.3}{B}\right)^2 F_d \left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m})$$

where

q_{net} = allowable bearing pressure for ΔH_0 = 25-mm, kPa

$F_d = 1 + 0.33 \frac{D_f}{B} < 1.33$ [as suggested by Meyerhof]

B = foundation width, in meters

S_e = settlement, in mm.

In these equations the allowable soil pressure is proportional to settlement. In general the allowable pressure for any settlement ΔH_j is

$$q_a' = \frac{\Delta H_j}{\Delta H_o} q_a$$

where ΔH_0 = 25 mm.

Example 4-12

Given. The average N_{60} blow count = 6 in the effective zone for a footing located at D = 1.6 m (blow count average in range from 1- to 4-m depth).

Required. What is the allowable bearing capacity for a 40-mm settlement? Present data as a table of q_a versus ***B.***

Solution. From Figure 3.17 we can see D_r is small, soil is "loose," and settlement may be a problem.
Should one put a footing on loose sand or should it be densified first?
(including F_d) on a programmable calculator or personal computer and obtain the table, which can be plotted as required.

for B = 1 m $F_d = 1 + 0.33\frac{1.6}{1}$ =1.528> 1.33 take $F_d = 1.33$ O.K

∵ B < 1.2 mm

∴ $$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.05} F_d\left(\frac{S_e}{25}\right)$$

$$q_{net} = \frac{6}{0.05} \text{ x } 1.33 \text{ x } \left(\frac{40}{25}\right) = 255.36 \text{ kN/m}^2$$

For example for B = 2 m $F_d = 1 + 0.33\frac{1.6}{2}$ =1.264 < 1.33 O.K

$$q_{net}(\text{kN/m}^2) = \frac{N_{60}}{0.08}\left(\frac{B+0.3}{B}\right)^2 F_d\left(\frac{S_e}{25}\right) \quad (\text{for } B > 1.22 \text{ m})$$

$$q_{net} = \frac{6}{0.08}\, x\left(\frac{2+0.3}{2}\right)^2 x\ 1.264x\ \left(\frac{40}{25}\right) = 200.6 \text{ kN/m}^2$$

for B = 3 m $F_d = 1 + 0.33\frac{1.6}{3}$ =1.176 < 1.33 O.K

$$q_{net} = \frac{6}{0.08}\, x\left(\frac{3+0.3}{3}\right)^2 x\ 1.176x\ \left(\frac{40}{25}\right) = 170.72 \text{ kN/m}^2$$

for B = 4 m $F_d = 1.32$ and $q_{net} = 157$ kN/m^2

B (m)	1	2	3	4
q_{net} (kPa)	255.4	200.6	170.7	157