Homogenous Differential equations 1st order

$$\frac{dy}{dx} = f(x,y)$$

اذا كان بالإمكان كتابتها بالشكل ادناه فهي متجانسة

$$\boxed{\frac{dy}{dx} = f\left(\frac{x}{y}\right)}$$

يجب ان تعتمد على النسبة $\left(\frac{x}{y}\right)$ أو $\left(\frac{y}{x}\right)$

خطوات الحل

① Assume the variable called $v = \frac{y}{x}$

$$y = v \cdot x$$

نحولها الى دالة يمكن فصلها separable.

$$\therefore \boxed{\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}}$$

② Substitute by using variable separable to find the general solution of v

$$v = \checkmark$$

③ $\because v = \frac{y}{x}$, $\therefore$ substitute every v so we can get

$$y = \checkmark$$

Ex/ $\dfrac{dy}{dx} = \dfrac{x+y}{x}$, Is it Homogenous ?

sol

$$\frac{dy}{dx} = \frac{x}{x} + \frac{y}{x}$$

$$\frac{dy}{dx} = 1 + \frac{y}{x} = v = \frac{y}{x} \Rightarrow y = xv$$

product rule to solve

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$

$$\left[v + x\frac{dv}{dx} = 1 + v \right] - v$$

$$\left(x\frac{dv}{dx} = 1 \right] \div x$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$\int dv = \int \frac{1}{x}\,dx$$

$$v = \ln|x| + C$$

∴ substitue $\boxed{v = \frac{y}{x}}$

$$\frac{y}{x} = \ln|x| + C$$

$$\boxed{y = x\ln|x| + Cx}$$

لمعرفة قيمة (C) يجب ان نعرف Initial condition من اجل لجعله particular solution هذة المعادلة التفاضلية

③

Ex2/ $\dfrac{dy}{dx} = \dfrac{x^2+3y^2}{2xy}$

هل هي متجانسة ؟ لنحاول الحصول على دالة $\boxed{\dfrac{dy}{dx} = f\left(\dfrac{y}{x}\right)}$

$$\left(\frac{dy}{dx} = \frac{x^2+3y^2}{2xy}\right) \frac{\frac{1}{x^2}}{\frac{1}{x^2}}$$

$$= \frac{1 + 3\frac{y^2}{x^2}}{2\frac{y}{x}}$$

$$\frac{dy}{dx} = \frac{1+3\left(\frac{y}{x}\right)^2}{2\left(\frac{y}{x}\right)}$$ Yes it is Homogenous

$\boxed{v = \dfrac{y}{x}}$, $\boxed{y = xv}$, $\boxed{\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}}$

Sub.

$$v + xv' = \frac{1+3v^2}{2v} \longrightarrow$$ لنجعلها separable equation

$$\left(2v^2 + 2xvv' = 1+3v^2\right] - 2v^2$$

$$2xvv' = 1 + v^2$$

$$\left(\frac{2xvv'}{1+v^2} = 1\right) \div x$$

$$\frac{2v}{1+v^2}\frac{dv}{dx} = \frac{1}{x}$$

$$\int \frac{2v}{1+v^2}\,dv = \int \frac{1}{x}\,dx$$

$$\ln(1+v^2) = \ln|x| + \ln|C|$$

$$\ln(1+v^2) = \ln|Cx|$$

$$\boxed{1+v^2 = Cx}$$

Sub

$$\left.1+\left(\frac{y}{x}\right)^2 = Cx\right] * x^2$$

$$x^2+y^2 = Cx^3$$

$$\boxed{x^2+y^2-Cx^3=0}$$ Ans.

Ex3/ $\frac{dy}{dx} = \frac{3y^2 + xy}{x^2}$, is it Homogenous, Solve.

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$$\left(\frac{dy}{dx} = \frac{3y^2 + xy}{x^2} \right] \div x^2$$

$$= \frac{3\frac{y^2}{x^2} + \frac{y}{x}}{1}$$

$$\frac{dy}{dx} = 3\left(\frac{y}{x}\right)^2 + \frac{y}{x}$$, Yes, Is is D.E. Homogeneous

let $\boxed{v = \frac{y}{x}} \Rightarrow \boxed{y = v \cdot x} \Rightarrow \boxed{\frac{dy}{dx} = v + x \cdot \frac{dv}{dx}}$

$$\left[\cancel{v} + x\frac{dv}{dx} = 3v^2 + \cancel{v} \right] \cdot \frac{1}{x}$$

$$\frac{dv}{dx} = 3\frac{v^2}{x}$$

$$\int \frac{dv}{3v^2} = \int \frac{1}{x} dx$$

$$-\frac{1}{3} v^{-1} = \ln|x| + \ln|c|$$

Sub

$$-\frac{1}{3} \frac{x}{y} = \ln|xc|$$

$$\frac{x}{y} = -3\ln|xc|$$

$$\therefore \boxed{y = \frac{x}{-3\ln|xc|}}$$ Ans.

Ex 4/ Solve the Homogenous D.E.

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$$\frac{dy}{dx} = \frac{x^2+y^2}{xy}$$

Sol

let $\boxed{v=\frac{y}{x}}$, $\boxed{y = vx}$, $\boxed{\frac{dy}{dx} = v + x\frac{dv}{dx}}$

Sub

$$v + x\frac{dv}{dx} = \frac{x^2+(vx)^2}{x(vx)}$$

$$= \frac{\cancel{x^2}(1+(\frac{y}{x})^2)}{\cancel{x^2}v}$$

$$v + x\frac{dv}{dx} = \frac{1+v^2}{v}$$

$$x\frac{dv}{dx} = \frac{1+v^2}{v} - v$$

$$= \frac{1}{v} + \frac{v^{\cancel{2}}}{\cancel{v}} - v$$

$$x\frac{dv}{dx} = \frac{1}{v}$$

$$x \cdot v \cdot \frac{dv}{dx} = 1$$

$$\int v\,dv = \int \frac{1}{x}\,dx$$

$$\frac{v^2}{2} = \ln|xc|$$

$\therefore$ $\boxed{v=\frac{y}{x}}$ Sub

$$\frac{1}{2}\frac{y^2}{x^2} = \ln|xc|$$

$$y^2 = 2x^2\ln|xc|$$

$$\boxed{y = \sqrt{2x^2\ln|xc|}}$$

Ans.