CHAPTER TWO
Introduction to Conduction

Conduction refers to the transport of energy in a medium due to a temperature gradient, and the physical mechanism is one of random atomic or molecular activity.

2.1 The Conduction Equation of Rectangular Coordinate:

Consider an element of small control volume $dV = dx\ dy\ dz$ as shown in Figure (2.1) and the temperature distribution $T(x,\ y,\ z,\ t)$ is expressed in Cartesian coordinates where (t) is the time.

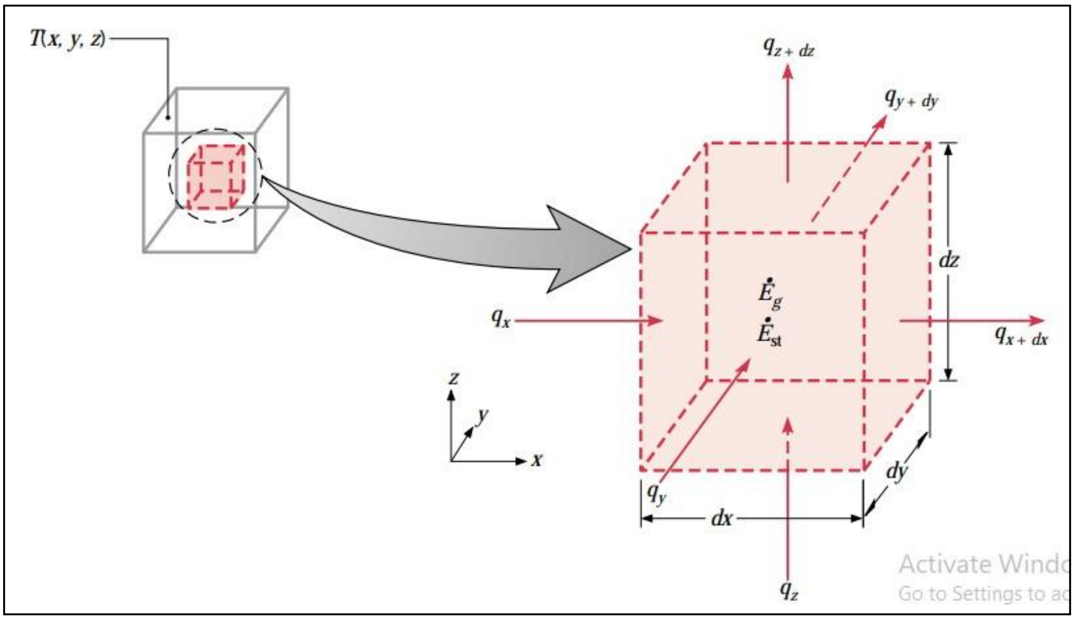

Figure (2.1) Differential Control Volume ($dx\,dy\,dz$) for Conduction Analysis in Cartesian Coordinates.

The conduction heat rates perpendicular to each of the control surfaces at the *x*, *y*, and *z* coordinate locations are indicated by the terms q_x, q_y, and q_z, respectively.

$$q_x = -kA\frac{\partial T}{\partial x} = -kdydz\frac{\partial T}{\partial x} \qquad (2.1a)$$

$$q_y = -kA\frac{\partial T}{\partial y} = -kdxdz\frac{\partial T}{\partial y} \qquad (2.1b)$$

$$q_z = -kA\frac{\partial T}{\partial z} = -kdxdy\frac{\partial T}{\partial z} \qquad (2.1c)$$

The conduction heat rates at the opposite surfaces can then be expressed as a Taylor series expansion where, neglecting higher order terms,

$$q_{x+dx} = q_x + \frac{\partial q_x}{\partial x} dx \tag{2.2a}$$

$$q_{y+dy} = q_y + \frac{\partial q_y}{\partial y} dy \tag{2.2b}$$

$$q_{z+dz} = q_z + \frac{\partial q_z}{\partial z} dz \tag{2.2c}$$

Within the medium there may also be an energy source term associated with the rate of thermal energy generation. This term is represented as

$$E_g = \dot{q}V = \dot{q}\, dx\, dy\, dz \tag{2.3}$$

where $\dot{q}$ is the rate at which energy is generated per unit volume of the medium (W/m^3).

In addition, there may occur changes in the amount of the internal thermal energy stored by the material in the control volume. If the material is not experiencing a change in phase, latent energy effects are not pertinent, and the energy storage term may be expressed as

$$E_{st} = \dot{m} C_p \frac{\partial T}{\partial t} = \rho C_p V \frac{\partial T}{\partial t} = \rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} \tag{2.4}$$

Where

$\rho C_p \frac{\partial T}{\partial t}$ is the time rate of change of the sensible (thermal) energy of the medium per unit volume.

C_p is specific heat capacity (J/kg. °C).

$\frac{\partial T}{\partial t}$ is the temperature change with time (K/s)

V is the volume (m^3).

ρ is the density (kg/m^3).

The general form of the conservation of energy requirement is

$$E_{in} + E_g - E_{out} = E_{st} \tag{2.5}$$

Substitute Eq. (2.1) to (2.4) in Eq. (2.5) get

$$(q_x + q_y + q_z) + \dot{q}\, dx\, dy\, dz - (q_{x+dx} + q_{y+dy} + q_{z+dz}) = C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} \tag{2.6}$$

$$\dot{q}\, dx\, dy\, dz - \frac{\partial q_x}{\partial x} dx - \frac{\partial q_y}{\partial y} dy - \frac{\partial q_z}{\partial z} dz \partial t = \rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} \tag{2.7}$$

$$\dot{q}\, dx\, dy\, dz - \frac{\partial}{\partial x}\left(-k dy dz \frac{\partial T}{\partial x}\right) dx - \frac{\partial}{\partial y}\left(-k dx dz \frac{\partial T}{\partial y}\right) dy - \frac{\partial}{\partial z}\left(-k dx dy \frac{\partial T}{\partial z}\right) dz \partial t = \rho C_p\, dx\, dy\, dz \frac{\partial T}{\partial t} \tag{2.8}$$

Divided Eq. (2.8) by ($dx\, dy\, dz$) get the general equation for conduction in a rectangular coordinate system

$$\frac{\partial}{\partial x}\left(k \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k \frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(k \frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p \frac{\partial T}{\partial t} \tag{2.9}$$

The following forms under the specific condition:

CASE (1): For homogenous material (isotropic material)

$k = constant$

$$\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{\partial T}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{\partial T}{\partial z}\right) + \frac{q}{k} = \frac{\rho C_p}{k} \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Where $\alpha = \frac{k}{\rho C_p} = Thermal\ diffusivity\ (m^2/s)$

CASE (2): For steady-state ($\partial/\partial t = 0$), homogenous material ($k = constant$)

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = 0$$

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CASE (3): For steady-state, without heat generation and homogenous material

$\frac{\partial}{\partial t} = 0$ $\quad k = constant$ $\quad \dot{q} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

CASE (4): 2-Dimension, steady-state, homogenous material and without heat generation

$\frac{\partial}{\partial t} = 0$ $\quad \dot{q} = 0$ $\quad k = constant$ $\quad \frac{\partial}{\partial z} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0$$

CASE (5): One-Dimension, steady-state, homogenous material and without heat generation

$\frac{\partial}{\partial t} = 0,$ $\quad \dot{q} = 0,$ $\quad \frac{\partial}{\partial y} = 0$ $\quad \frac{\partial}{\partial z} = 0$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

$$\frac{\partial^2 T}{\partial x^2} = 0$$

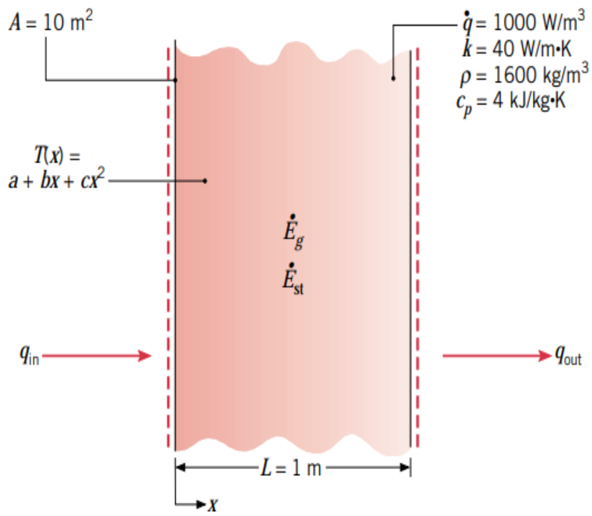
We may use Fourier's law to determine the conduction heat transfer rate. That is

$$q_x = -kA\frac{\partial T}{\partial x} = \frac{kA}{L}(T_1 - T_2)$$

Example (2.1): The temperature distribution across a wall (1m) thick at a certain instant of time is given as $T(x) = a + bx + cx^2$ where T is in degrees Celsius and x is in meters, while $a = 900\ °C, b = -300\ °C/m\ and\ c = -50\ °C/m^2$. A uniform heat generation $\dot{q} = 1000\ W/m^3$ is present in the wall of area 10 m^2 having the properties $\rho = 1600\ kg/m^3$, $k = 40\ W/m.K$ and $C_p = 4\ kJ/kg.K$.

1. Determine the rate of heat transfer entering the wall $(x = 0)$ and leaving the wall $(x = 1\ m)$.
2. Determine the rate of change of energy storage in the wall.
3. Determine the time rate of temperature change at $x = 0, 0.25, and\ 0.5\ m$.

Solution:



Assumptions:

1- One-dimensional conduction in the x-direction.
2- Isotropic medium with constant properties.
3- Uniform internal heat generation, $(\dot{q} = 1000\ W/m^3)$.

1- Heat rates entering $q_{in}\ (x = 0)$ and leaving $q_{out}\ (x = 1\ m)$ the wall.

$$q_{in} = -kA\frac{dT}{dx}\Big|_{x=0} = -kA(b + 2cx)_{x=0}$$

$$q_{in} = -kAb$$

$$q_{in} = -40 * 10 * -300 = 120000\ W = 120KW$$

$$q_{out} = -kA\frac{dT}{dx}\Big|_{x=L} = -kA(b + 2cx)_{x=L}$$

$$q_{out} = -kA(b + 2cL)$$

$$q_{out} = 40 * 10 * (-300 + 2 * -50 * 1)$$

$$q_{out} = 160000\ W = 160\ KW$$

2- Rate of change of energy storage in the wall E_{st}.

$E_{in} + E_g - E_{out} = E_{st}$

Where $E_g = \dot{q}V = E_g = \dot{q}AL$

$E_{st} = E_{in} + E_g - E_{out} = E_{in} + \dot{q}AL - E_{out}$

$E_{st} = 120000 + 1000 * 10 * 1 - 160000$

$E_{st} = -30000\ W = -30\ KW$

3- Time rate of temperature change at $x = 0, 0.25, and\ 0.5\ m$.

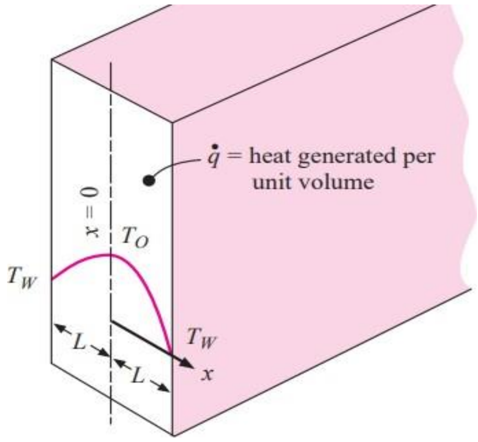
$$\frac{\partial T}{\partial t} = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial x^2} + \frac{q}{\rho C_p}$$

$$\frac{\partial^2 T}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) = \frac{\partial}{\partial x}(b + 2cx) = 2c = 2 * -50 = -100\ °C/m^2$$

$$\frac{\partial T}{\partial t} = \frac{40}{1600 * 4} * (-100) + \frac{1000}{1600 * 4}$$

$$\frac{\partial T}{\partial t} = -0.625 + 0.156 = -0.468\ °C/s \quad for\ x = 0, 0.25\ and\ 0.5$$

Example (2.2): Consider the plane wall with uniformly distributed heat sources shown in Figure. The thickness of the wall in the x direction is 2L, and it is assumed that the dimensions in the other directions are sufficiently large that the heat flow may be considered as one dimensional. The heat generated per unit volume is $\dot{q}$, and assume that the thermal conductivity does not vary with temperature. Derive an expression of the temperature distribution.



Solution:

Assumption:

1- One-Dimension ($\frac{\partial}{\partial y} = 0, \frac{\partial}{\partial z} = 0$).

2- Steady state ($\frac{\partial}{\partial t} = 0$)

3- Uniform heat generation ($\dot{q}$).

4- Homogeneous ($k = constant$).

$$\frac{\partial^2 T}{\partial x^2} + \frac{q}{k} = 0 \qquad integrate$$

$$\frac{\partial T}{\partial x} + \frac{q}{k} x = C_1 \qquad (1) \quad integrate\ again$$

$$T + \frac{\dot{q}}{k} x^2 = C_1 x + C_2$$

$$T = -\frac{\dot{q}}{2k} x^2 + C_1 x + C_2 \qquad (2)$$

B.C1: $at\ x = 0$ $\quad T = T_0$ Sub. in Eq. (2)

$$T_0 = -\frac{\dot{q}}{2k} (0)^2 + C_1 * 0 + C_2$$

$C_2 = T_0$ Sub. in Eq. (2)

B.C2: $at\ x = \pm L$ $\quad T = T_w$ Sub. in Eq. (2)

$$T_w = -\frac{\dot{q}}{2k} L^2 + C_1 L + T_0 \qquad (3)$$

$$T_w = -\frac{\dot{q}}{2k} L^2 - C_1 L + T_0 \qquad (4)$$

Subtract

$$0 = 0 + 2LC_1 + 0$$

$C_1 = 0$ Sub. in Eq. (2)

$$T = -\frac{\dot{q}}{2k} x^2 + T_0$$

$$T - T_0 = -\frac{\dot{q}}{2k} x^2 \qquad (5)$$

2.2 The Conduction Equation of Cylindrical Coordinates:

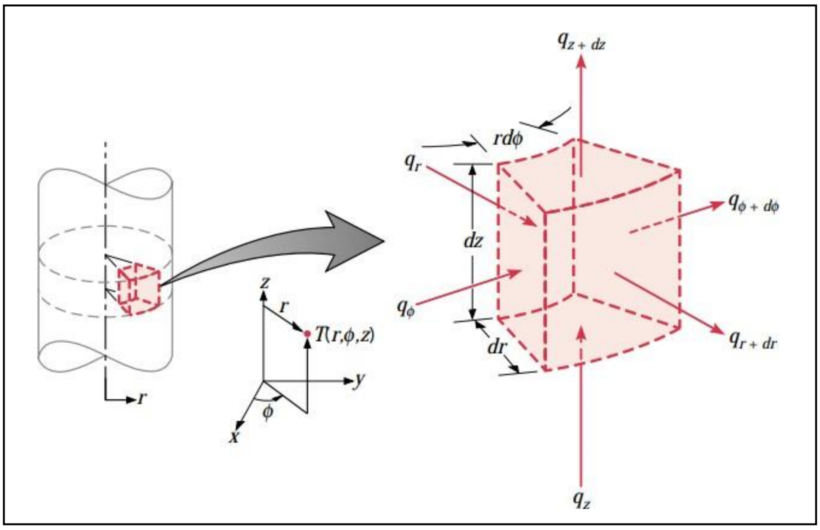


Figure (2.2) Differential Control Volume $(dr.\, r\, d\emptyset.\, dz)$ for Conduction Analysis in Cylindrical Coordinates $(r, \emptyset, z)$.

The general form of the heat flux vector, and hence of Fourier's law, is

$$q''_r = -k\frac{\partial T}{\partial r} \qquad (2.10a)$$

$$q''_\emptyset = -\frac{k}{r}\frac{\partial T}{\partial \emptyset} \qquad (2.10b)$$

$$q''_z = -k\frac{\partial T}{\partial z} \qquad (2.10c)$$

Applying an energy balance to the differential control volume of Figure (2.2) the following general form of the heat equation is obtained:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \frac{1}{r^2}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right) + \frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t} \qquad (2.11)$$

The following forms under the specific condition:

CASE (1): One dimension and homogenous material (isotropic material).

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

CASE (2): one dimension, steady state ($\partial/\partial t = 0$), homogenous material (isotropic material) and with heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{q}{k} = 0$$

CASE (3): one dimension, unsteady state, homogenous material (isotropic material) and without heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

CASE (4): one dimension, steady-state ($\partial/\partial t = 0$), homogenous material (isotropic material) and without heat generation.

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0$$

The solution of the heat equation is:

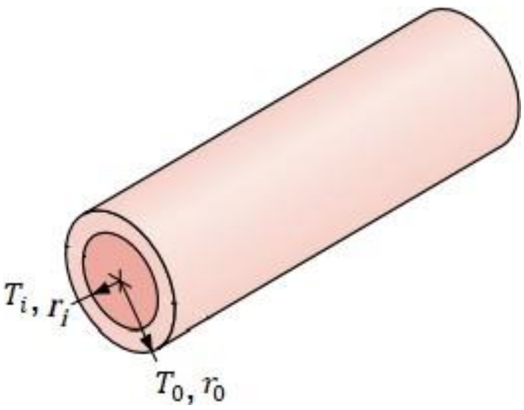
$$q = -kA\frac{\partial T}{\partial r} = -2\pi rLk\,\frac{(T_i - T_0)}{\ln\frac{r_i}{r_0}} = 2\pi rLk\,\frac{(T_i - T_0)}{\ln\frac{r_0}{r_i}} \qquad (2.12)$$

Example (2.3): consider a steam pipe of length (L), inner radius (r_i), outer radius (r_0) and thermal conductivity (k). The inner and outer surface of pipe are maintained at average temperature of (T_i) and (T_0) respectively. Obtain a general relation for the temperature distribution inside the pipe under steady conditions and determine the rate of heat loss from the steam through the pipe.

Solution:

Assumption:

1- Steady-state ($\partial/\partial t = 0$).
2- Homogenous material (isotropic material).
3- With heat generation.



$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad \textit{multiply by } r$$

$$\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0 \qquad \textit{integrate}$$

$$r\frac{\partial T}{\partial r} = C_1 \quad \rightarrow \quad \frac{\partial T}{\partial r} = \frac{C_1}{r} \qquad (1) \qquad integrate\ again$$

$$T = C_1 \ln r + C_2 \qquad (2)$$

B.C1: $at\ r = r_i \qquad T = T_i \qquad sub.\ in\ Eq.(2)$

$$T_i = C_1 \ln r_i + C_2 \qquad (3)$$

B.C2: $at\ r = r_0 \qquad T = T_0 \qquad sub.\ in\ Eq.(2)$

$$T_0 = C_1 \ln r_0 + C_2 \qquad (4)$$

Subtract Eq. (3) and Eq. (4)

$$T_i - T_0 = C_1 \ln\frac{r_i}{r_0}$$

$$C_1 = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \qquad sub.\ in\ Eq.(3)$$

$$T_i = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i + C_2$$

$$C_2 = T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

Sub. $C_1\ and\ C_2$ in Eq. (2)

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r + T_i - \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln r_i$$

$$T = \frac{T_i - T_0}{\ln\frac{r_i}{r_0}} \ln\frac{r}{r_i} + T_i$$

$$q = -kA\frac{\partial T}{\partial r} = -k(2\pi r L)\frac{C_1}{r} = -\frac{(2\pi r L k)}{r}\frac{T_i - T_0}{\ln\frac{r_i}{r_0}}$$

$$q = -2\pi L k\frac{(T_i - T_0)}{\ln\frac{r_i}{r_0}}$$

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Example(2.4): Uniform internal heat generation $\dot{q} = 5 \times 10^7\ W/m^3$ is occurring in a cylindrical nuclear reactor fuel rod of (50 mm) diameter, and under steady-state conditions, the temperature distribution is of the form $T(r) = a + br^2$, where T is in degrees Celsius and r is in meters, while $a = 800\ °C$ and $b = -4.167 \times 10^5\ °C/m^2$. The fuel rod properties are $k = 30\ W/m.K,\ \rho = 1100\ kg/m^3\ and\ C_p = 800\ J/kg\ .K$.

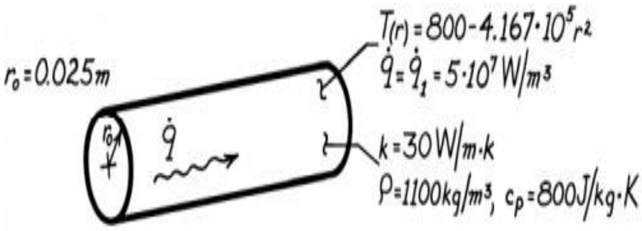
(a) What is the rate of heat transfer per unit length of the rod at $r = 0$ (the centerline) and at $r = 25\ mm$ (the surface)?

(b) If the reactor power level is suddenly increased to $\dot{q}_2 = 10^8\ W/m^3$, what is the initial time rate of temperature change at $r = 0$ and $r = 25\ mm$?

Solution:

Assumptions:

1- One-dimensional conduction in the r direction.
2- Uniform generation.
3- Steady-state for $\dot{q}_1 = 5 \times 10^7\ W/m^3$



(a) Steady-state centerline and surface heat transfer rates per unit length, (q_r').

$$q_r'' = -k\frac{\partial T}{\partial r} \qquad q_r = -kA_r\frac{\partial T}{\partial r}$$

$$q_r = -k(2\pi rL)\frac{\partial T}{\partial r}$$

$$q_r' = -k(2\pi r)\frac{\partial T}{\partial r}$$

at $r = 0$

$$\left.\frac{\partial T}{\partial r}\right]_{r=0} = 2br = o$$

$$\therefore q_r' = 0$$

at $r = r_0$

$$\frac{\partial T}{\partial r}\Big]_{r=r_0} = -2 \times 4.167 \times 10^5 r = -2 \times 4.167 \times 10^5 r_0$$

$$\frac{\partial T}{\partial r}\Big]_{r=r_0} = -2 \times 4.167 \times 10^5 \times 0.025$$

$$\frac{\partial T}{\partial r}\Big]_{r=r_0} = -0.208 \times 10^5\ K/m$$

$$q'_r = -k(2\pi r_0)\frac{\partial T}{\partial r} = -30 * 2\pi * 0.025 * -0.208 \times 10^5 = 0.98 \times 10^5 W/m$$

(b) The initial time rate of change of the centerline and surface temperatures in response to a change in the generation rate from $q_1 to\, q_2 = 10^8 \frac{W}{m^3}$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q}_2 = \rho C_p \frac{\partial T}{\partial t}$$

$$\frac{\partial T}{\partial t} = \frac{1}{\rho C_p}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) + \dot{q}_2\right] \qquad (1)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = \frac{k}{r}\frac{\partial}{\partial r}[r(-8.334 \times 10^5 r)] = \frac{k}{r}\frac{\partial}{\partial r}[-8.334 \times 10^5 r^2]$$

$$= \frac{k}{r}[-8.334 \times 10^5 \times 2r] = -16.668 \times 10^5 k = -16.668 \times 10^5 \times 30$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(kr\frac{\partial T}{\partial r}\right) = -5 \times 10^7 \frac{W}{m^3} \qquad sub.\ in\ Eq.(1)$$

$$\frac{\partial T}{\partial t} = \frac{1}{1100 \times 800}[-5 \times 10^7 + 10^8]$$

$$\frac{\partial T}{\partial t} = 56.82\ K/s$$

2.3 The Conduction Equation of Spherical Coordinates:

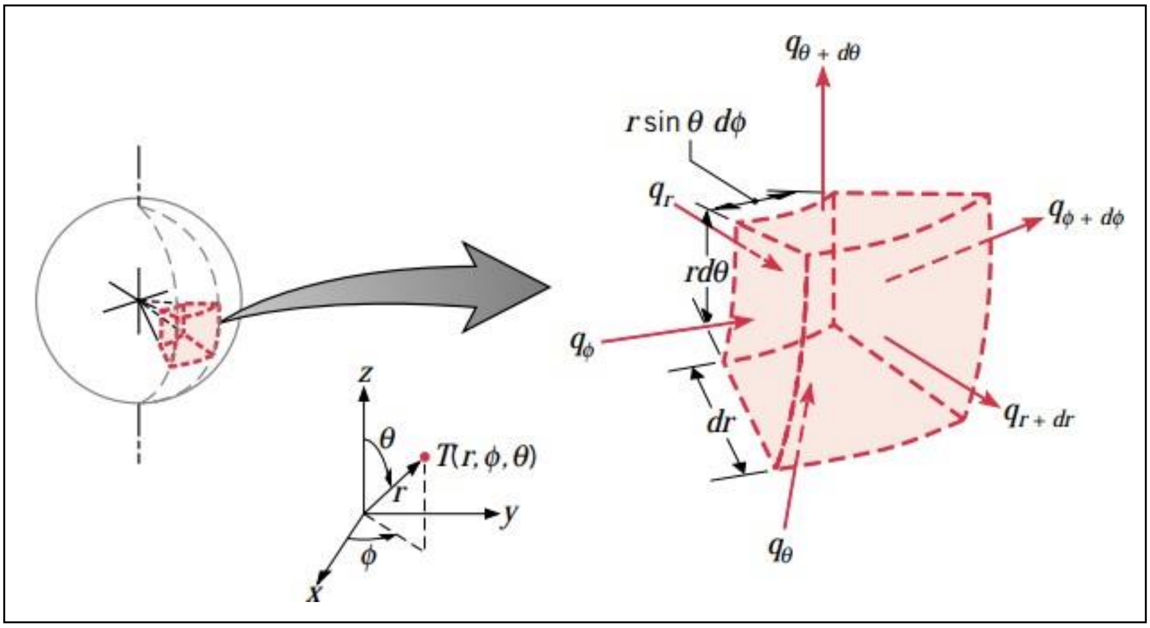


Figure (2.3) Differential Control Volume ($dr.\,rsin\theta d\emptyset.\,rd\theta$) for Conduction Analysis in Spherical Coordinates ($r,\theta,\emptyset$)

In spherical coordinates the general form of the heat flux vector and Fourier's law is

$$q''_r = -k\frac{\partial T}{\partial r} \qquad (2.13a)$$

$$q''_\theta = -\frac{k}{r}\frac{\partial T}{\partial \theta} \qquad (2.13b)$$

$$q''_z = -\frac{1}{rsin\theta}\frac{\partial T}{\partial \emptyset} \qquad (2.13c)$$

Applying an energy balance to the differential control volume of Figure 2.3 the following general form of the heat equation is obtained:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(kr^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin^2\theta}\frac{\partial}{\partial \emptyset}\left(k\frac{\partial T}{\partial \emptyset}\right) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial \theta}\left(ksin\theta\frac{\partial T}{\partial \theta}\right) + \dot{q} = \rho C_p\frac{\partial T}{\partial t} \qquad (2.14)$$

The following forms under the specific condition:

CASE (1): One dimension, unsteady-state and homogenous material (isotropic material).

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) + \frac{q}{k} = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

CASE (2): one dimension, steady-state ($\partial/\partial t = 0$), homogenous material (isotropic material) and with heat generation.

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) + \frac{q}{k} = 0$$

CASE (3): one dimension, unsteady state, homogenous material (isotropic material) and without heat generation.

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = \frac{1}{\alpha}\frac{\partial T}{\partial t}$$

CASE (4): one dimension, steady state ($\partial/\partial t = 0$), homogenous material (isotropic material) and without heat generation.

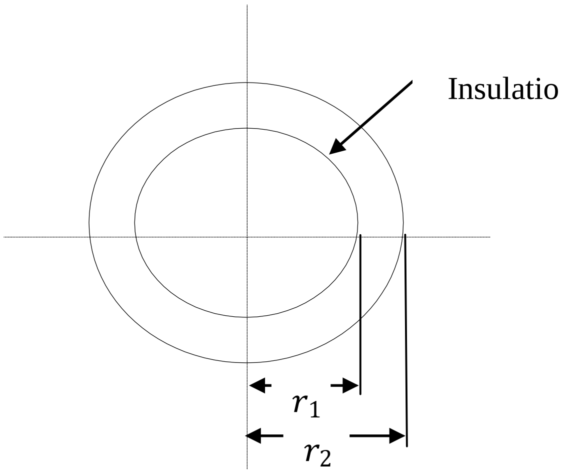
$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = 0$$

The solution of the heat equation is:

$$q_r = \frac{4\pi k(T_1 - T_2)}{(\frac{1}{r_1} - \frac{1}{r_2})} \qquad (2.15)$$

Example (2.5): A spherical container having outer diameter (500 mm) is insulated by (100 mm) thick layer of material with thermal conductivity ($k = 0.03(1 + 0.006T)$) W/m. °C, where T in °C. If the surface temperature of sphere is (-200 °C) and temperature of outer surface is (30 °C) determine the heat flow.

Solution:



$$q = \frac{4\pi k(T_1 - T_2)}{(\frac{1}{r_1} - \frac{1}{r_2})}$$

$$r_1 = \frac{D}{2} = \frac{500}{2} = 250\,mm$$

$$r_2 = r_1 + 100 = 350\,mm$$

$$k = 0.3(1 + 0.006T) = 0.3(1 + 0.006\,(\frac{-200 + 30}{2}))$$

$$k = 0.147\,W$$

$$q = \frac{4\pi * 0.147(-200 - 30)}{(\frac{1}{0.025} - \frac{1}{0.035})} = -37.14\,W$$

Example (2.6): A hollow sphere the inner and outer shell radius are (r_i) and (r_0) with uniform temperature at the inner and outer surface (T_i) and (T_0). Find the temperature distribution equation as a function of radius r without heat generation in the steady state.

Solution:

Assumption:

1- Steady state ($\partial/\partial t = 0$).
2- Homogenous material (isotropic material).
3- Without heat generation.

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = 0 \qquad multiply\ by\ r^2$$

$$\frac{\partial}{\partial r}(r^2\frac{\partial T}{\partial r}) = 0 \qquad integrate$$

$$r^2\frac{\partial T}{\partial r} = C_1$$

$$\frac{\partial T}{\partial r} = \frac{C_1}{r^2} \qquad (1) \quad integrate\ again$$

$$T = -\frac{C_1}{r} + C_2 \qquad (2)$$

B.C1: $at\ r = r_i \qquad T = T_i \qquad sub.\ in\ Eq.(2)$

$$T_i = -\frac{C_1}{r_i} + C_2 \qquad (3)$$

B.C2: $at\ r = r_0 \qquad T = T_0 \qquad sub.\ in\ Eq.(2)$

$$T_0 = -\frac{C_1}{r_0} + C_2 \qquad (4)$$

Subtract Eq. (3) and Eq. (4)

$$T_i - T_0 = C_1\left(\frac{1}{r_0} - \frac{1}{r_i}\right)$$

$$C_1 = \frac{T_i - T_0}{\frac{1}{r_0} - \frac{1}{r_i}} \qquad sub.\ in\ Eq.(3)$$

$$T_i = \frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right) r_i} + C_2$$

$$C_2 = T_i - \frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right) r_i}$$

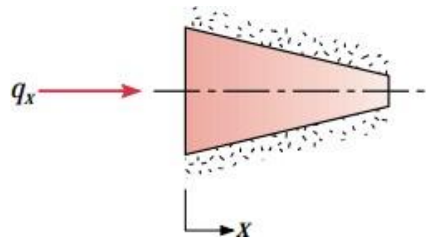
Sub. C_1 and C_2 in Eq. (2)

$$T = -\frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right) r} + T_i - \frac{T_i - T_0}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right) r_i}$$

$$T = \frac{T_0 - T_i}{\left(\frac{1}{r_0} - \frac{1}{r_i}\right)}\left(\frac{1}{r} + \frac{1}{r_i}\right) + T_i$$

Home Work (2):

1- Assume steady-state, one-dimensional heat conduction through the symmetric shape shown. Assuming that there is no internal heat generation, derive an expression for the thermal conductivity $k(x)$ for these conditions: $A(x) = (1 - x)$, $T(x) = 300(1 - 2x - x^3)$, and $q = 6000\ W$, where A is in square meters, T in kelvins, and x in meters.

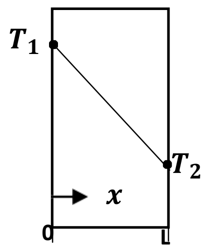


2- A plane wall is constructed of a material having a thermal conductivity that varies as the square of the temperature according to the relation $k = k_0(1 + \beta T^2)$. Derive an expression for the heat transfer in such a wall.

3- The temperature distribution across a wall 0.3 m thick at a certain instant of time is as $T(x) = a + bx + cx^2$ where T is in degrees Celsius and x is in meters, while $= 200\ °C$, $b = -200\ °C/m\ and\ c = 30\ °C/m^2$. The wall has a thermal conductivity of $k = 1\ W/m.\ K$. On a unit surface area basis, determine the rate of heat transfer into and out of the wall and the rate of change of energy stored by the wall.

4- A large thin concrete slab of thickness L. selling is an exothermic process that releases $\dot{q}(\frac{W}{m^3})$. The outside surfaces are kept at the ambient temperature so $T_\infty = T_w$. What is the maximum internal temperature?

5- A slab shown in figure is at steady state with dissimilar temperature on either side or no internal heat generation. Derive an expression for the temperature distribution and heat flux through it.



6- Consider a long solid tube, insulated at the outer radius (r_2) and cooled at the inner radius (r_1), with uniform heat generation ($\dot{q}(W/m^3)$) within the solid.

a- Obtain the general solution for the temperature distribution in the tube.
b- Determine the heat removal rate per unit length of tube.

7- Derive an expression for the temperature distribution in a sphere of radius (r_0) with uniform heat generation ($\dot{q}(W/m^3)$) and constant surface temperature (T_0).

8- A hollow sphere is constructed of aluminum (k=204 W/m. °C) with an inner diameter of (4 cm) and an outer diameter of 8 cm. The inside temperature is (100 °C) and the outer temperature is (50 °C). Calculate the heat transfer.

9- Derive an expression for the temperature distribution in a hollow cylinder with heat sources that vary according to the linear relation $\dot{q} = a + br$ with $\dot{q}_i$ the generation rate per unit volume at $r = r_i$. The inside and outside temperatures are $T = T_i$ at $r = r_i$ and $T = T_0$ at $r = r_0$.