Republic of Iraq

Ministry of Higher Education

and Scientific Research

Al-Mustaqbal University College

Computer Engineering Techniques Department

Subject: Digital Signal Processing

Third Class

Lecture Seven

By

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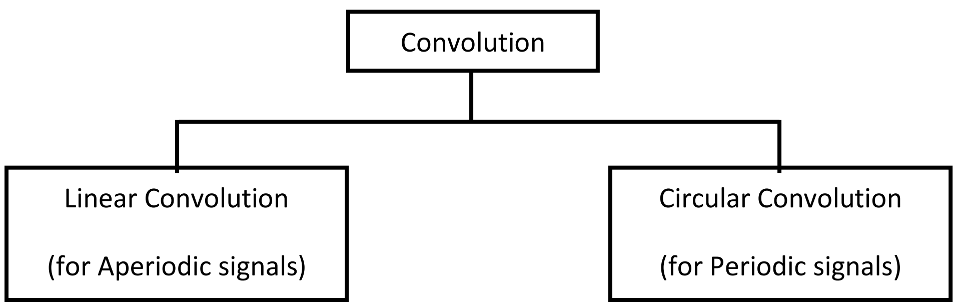
Convolution

Convolution is an operation between the input signal to a system, and its impulse response, resulting in the output signal. Convolution of two signals involves summing the product of the two signals – where one of the signals is “flipped and shifted”. It doesn't matter which signal is flipped and shifted . Convolution can be classified into two categories according to the signals that will convolved as shown in the figure.



Linear Convolution

Linear convolution is used when the convolved signals are aperiodic. The mathematical expression of linear convolution is:

$$\boldsymbol{y(n) = x(n) * h(n)}$$

$$\boldsymbol{y(n) = \sum_{k=-\infty}^{\infty} x(k)h(n-k)}$$

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A useful fact to remember in performing the convolution of two finite-length sequences is that if $x(n)$ is of length N_1 and $h(n)$ is of length N_2. Then, the output $y(n)$ will be of length :

$$N = N_1 + N_2 - 1$$

Methods of Calculation Convolutions

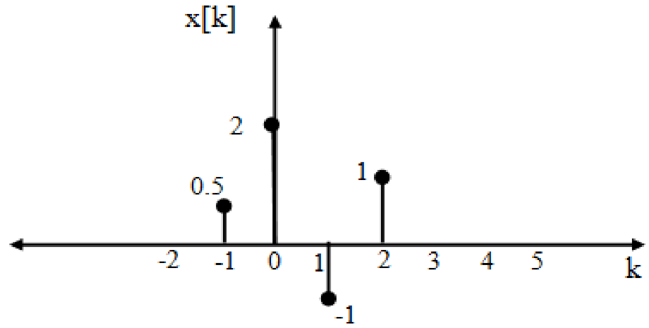
There are several different approaches that may be used, and the one that is the easiest will depend upon the form and type of sequences that are to be convolved.

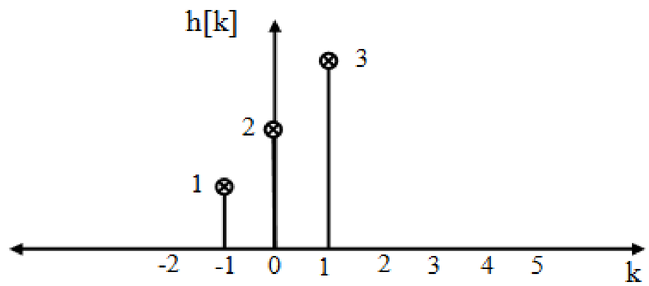
1. Graphical Method

In addition to the direct method, convolutions may also be performed graphically. The steps involved in using the graphical approach are as follows:

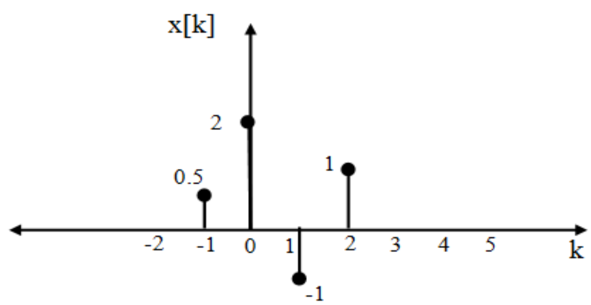
1) Plot both sequences, $x(k)$ and $h(k)$, as functions of k.
2) Choose one of the sequences, say $h(k)$, and time-reverse it to form the sequence $h(-k)$.
3) Shift the time-reversed sequence by n. [if $n > 0$, this corresponds to a shift to the right (delay), whereas if $n < 0$, this corresponds to a shift to the left (advance)].
4) Multiply the two sequences $x(k)$ and $h(n-k)$ and sum the product for all values of k. the resulting value will be equal to $y(k)$. This process is repeated for all possible shifts, n.

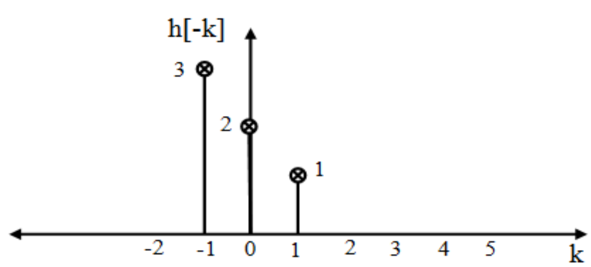
Example 2: Evaluate $y(n) = x(k) * h(k)$, where $x(k) = \{0.5, \underline{2}, -1, 1\}$ and $h(k) = \{1, \underline{2}, 3\}$.





Solution:





$$y(0) = \sum_{k=-\infty}^{\infty} x(k)h(-k) = 0.5 * 3 + 2 * 2 + (-1) * 1 = 4.5$$

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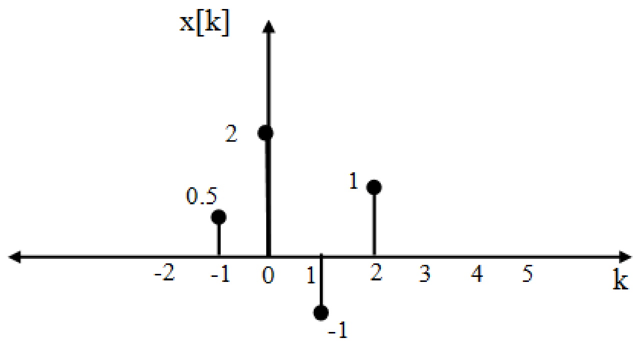
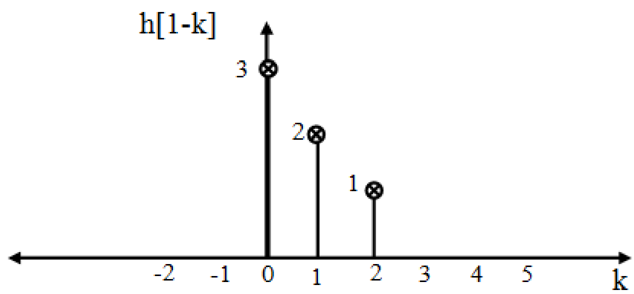
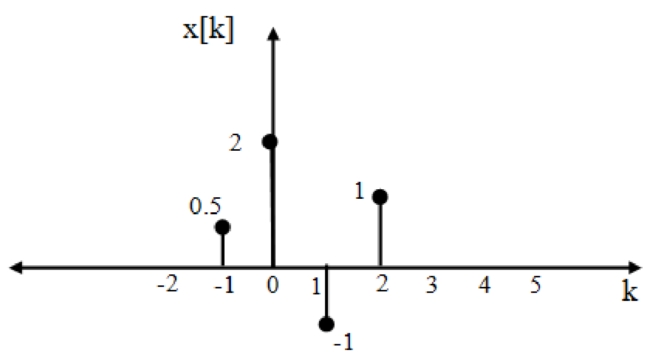
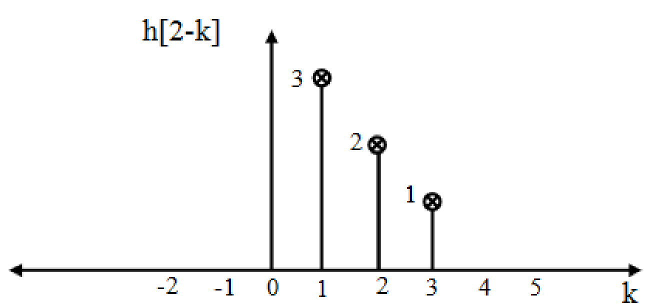


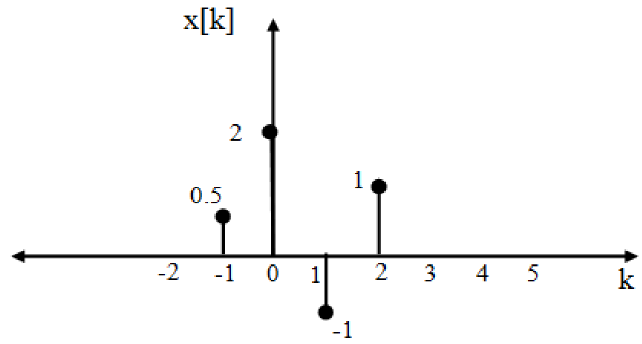


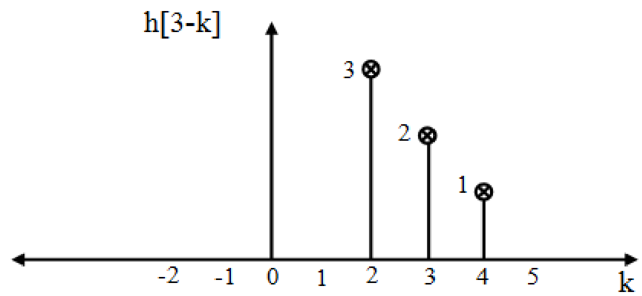
$$y(1) = \sum_{k=-\infty}^{\infty} x(k)h(1-k) = 2*3 + (-1)*2 + 1*1 = 5$$



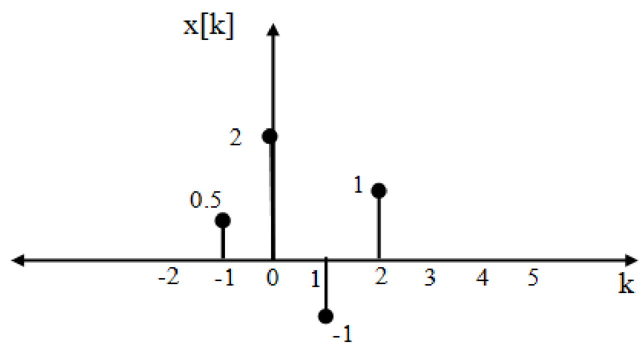


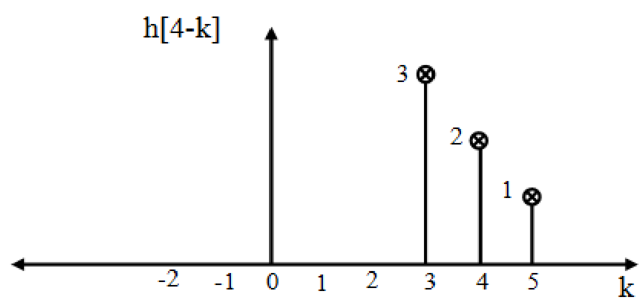
$$y(2) = \sum_{k=-\infty}^{\infty} x(k)h(2-k) = (-1)*3 + 1*2 = -1$$





$$y(3) = \sum_{k=-\infty}^{\infty} x(k)h(3-k) = 1 * 3 = 3$$





$$y(4) = \sum_{k=-\infty}^{\infty} x(k)h(4-k) = 0$$

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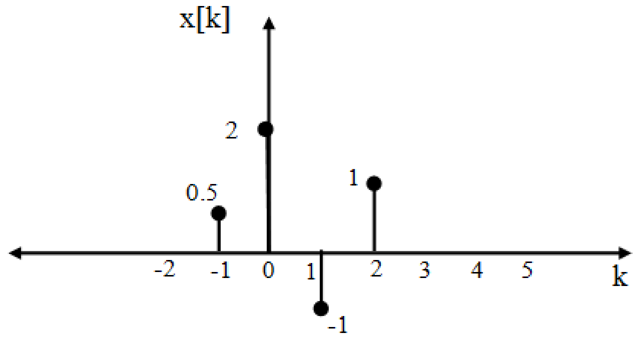
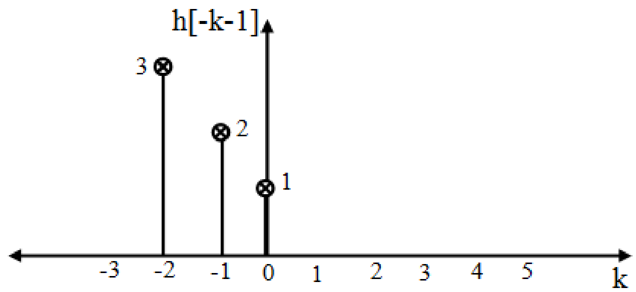
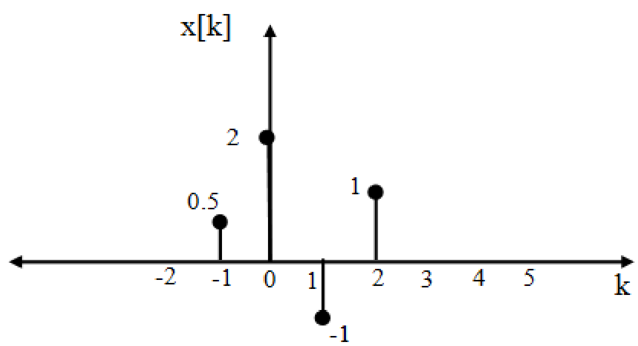
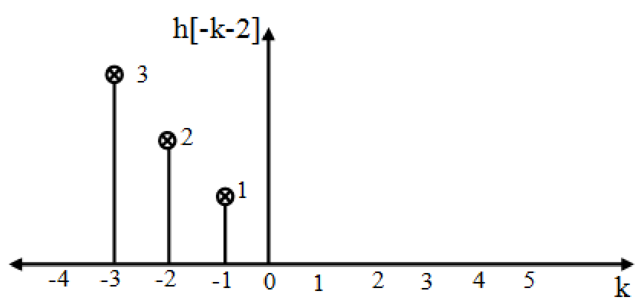


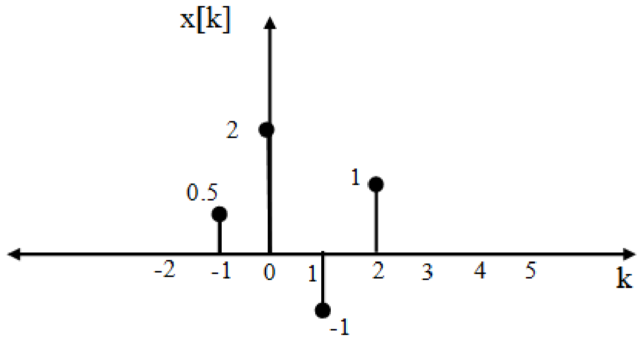


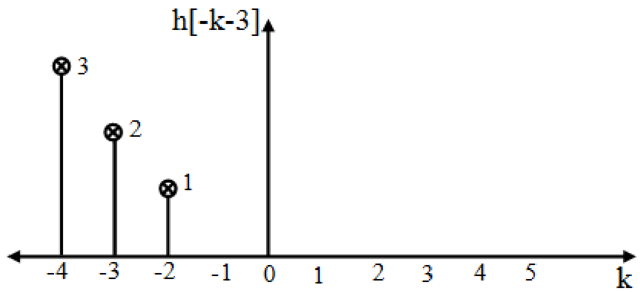
$$y(-1) = \sum_{k=-\infty}^{\infty} x(k)h(-k-1) = 0.5 * 2 + 2 * 1 = 3$$



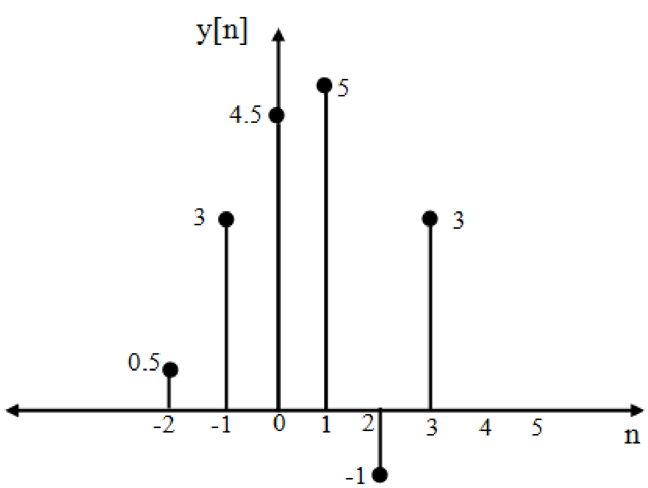


$$y(-2) = \sum_{k=-\infty}^{\infty} x(k)h(-k-2) = 0.5 * 1 = 0.5$$





$$y(-3) = \sum_{k=-\infty}^{\infty} x(k)h(-k-3) = 0$$



$$y(n) = \{\, 0.5, \quad 3, \quad \underline{4.5}, \quad 5, \quad -1, \quad 3\}$$