① Variable separable

①

Variable separable
homogenous
Linear
exact

$$\frac{dy}{dx} = F(x,y)$$

$$F(x,y) = f(x)\, g(y)$$

known functions

Ex/ Consider the DE

$$\frac{dy}{dx} - x^2y^2 = x^2$$

Sol/

$$\frac{dy}{dx} = x^2 + x^2y^2$$

$$\frac{dy}{dx} = x^2(1+y^2)$$

$f(x) = x^2$ $\qquad$ $g(y) = 1+y^2$

$\therefore$ It is separable DE

Ex/

$$\frac{dy}{dx} - x^2y^2 = 4$$

Sol/

$$dy = \cancel{\ } x^2y^2 + 4$$

~~we can't~~

we can't separate them, so, it is not separable

كل هذا النوع من المعادلات تكامل الطرفين

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EX/

$y' = xy$, initial condition $y(0) = 1$

Sol/

$f(x) = x$

$g(y) = y$

$\frac{dy}{y} = x\,dx$ Restriction $y \neq 0$

Integrating both sides

$$\int \frac{dy}{y} = \int x\,dx$$

$$\ln(y) = \frac{x^2}{2} + C$$

$$y = e^{\frac{x^2}{2} + C} = K e^{\frac{x^2}{2}}$$

If $y(0) = 1$, then $K = 1$

$$\boxed{\begin{matrix} y = e^{0} \\ y = 1 \end{matrix}}$$

$\therefore y \geq 1$, so, $y \neq 0$ is true

Ex/ $\dfrac{dy}{dx} = \dfrac{x+3}{y+4}$, initial condition $y(0)=0$

Sol

$f(x) = x+3$, $g(y) = \dfrac{1}{y+4}$

So, the restriction $y \neq -4$

$(y+4)\,dy = (x+3)\,dx$

Integration for both sides

$$\frac{y^2}{2} + 4y = \frac{x^2}{2} + 3x + C$$

$$y^2 + 8y = x^2 + 6x + C$$

لكل (y) نعمل اكمال مربع

$$y^2 + 8y + 16 - 16 = x^2 + 6x + 9 - 9 + C$$

$$(y+4)^2 - 16 = (x+3)^2 - 9 + C$$

$$(y+4)^2 = (x+3)^2 + 7 + C$$

$$y = \pm\sqrt{(x+3)^2 + 7 + C} - 4$$

using the $y(0)=0$

we get $y = 0 = \sqrt{3^2 + 7 + C} - 4$, so that $C = 0$

$\therefore$ $y = \sqrt{(x+3)^2 + 7} - 4$, Note that $y \geq (\sqrt{7} - 4) \approx -1.35$

So, the restriction is met