Ordinary differential Equations of 1^{st} order (1-1)

An equation involves 1 or 2, more derivatives or differentials.

Ordinary derivatives

occurs when dependent variable (y) is a function of 1 independent variable (X) $y = f(x)$

partial derivatives

when the dependent variable (y) is a function of two or more independent variables $y = f(x, t)$

Order → highest derivative

Degree → (power of highest derivative)

Ex1 $$x^2 \bar{\bar{y}} + \bar{y} + (x^2 - 4)\, y = 0$$

ordinary , 2^{nd} order , 1^{st} degree

Ex2 $$\left(\frac{d^3y}{dx^3}\right)^2 + \left(\frac{d^2y}{dx^2}\right)^5 + \frac{y}{x^2+1} = e^x$$

ordinary , 3^{rd} order , 2^{nd} degree

Ex3/ $$\frac{\partial^4 u}{\partial x^4} + 2\frac{\partial^4 u}{\partial x^2 \partial y^2} + \frac{\partial^4 u}{\partial y^4} = 0$$

partial, 4^{th} order, 1^{st} degree

Ex4/ $$\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2}$$

partial, 2^{nd} order, 1^{st} degree

linear equations/ No y * its derivatives / 1^{st} degree

$$a_0 y + a_1 \bar{y} + a_2 \bar{\bar{y}} + a_3 \bar{\bar{\bar{y}}} + \cdots + a_n y^{(n)} = b$$

linear equation form

A non-linear D.E, cannot be put if the above form.

Ex/ $\bar{\bar{y}} + 4x\bar{y} + 2y = \cos x$ linear

$\bar{y} + 4y\bar{y} + 2y = \cos x$ Non-linear because $(y\bar{y})$

$\bar{\bar{y}} + \sin y = 0$ Non-linear because $(\sin y)$

A solution of D.E / It is a relation between the dependent and independent variables. 1-3

Ex/ $$\frac{d^2y}{dx^2} + y = 0$$

its solution will be $\boxed{y = a\cos x + b\sin x}$

General Solution

when n^{th} order contains n constants which cannot be replaced by a smaller number.

Ex/ $a\cos^2 x + b\sin^2 x + c\cos 2x$

3 constants and can be reduced

$$a\cos^2 x + b\sin^2 x + c(\cos^2 x - \sin^2 x)$$

$$(a+b)\cos^2 x + (b-c)\sin^2 x$$

$$d\cos^2 x + e\sin^2 x$$

However, $\left|\frac{dy}{dx}\right| + |y| = 0$ (only 1 solution $y=0$)

$\left|\frac{dy}{dx}\right| + 1 = 0$ (No solution at all)

1-4

particular solution

$y = a \cos x + b \sin x$ general solution

$a = 1$ & $b = 0$

$y = \cos x$ particular solution

Singular solution

cannot be obtained from a general solution by assigning values to the constants

Complete solution

If a general solution has the property that every solution of the D.E. can be obtained from it by assighing values to its constants

Ex/ show $y = ae^{-x} + b e^{2x}$ is a solution of $y'' - y' - 2y$ for all values of a & b

Sol $y'' - y' - 2y$

$(ae^{-x} + 4be^{2x}) - (-ae^{-x} + 2be^{2x}) - 2(ae^{-x} + be^{2x})$

$= (e^{-x} + e^{-x} - 2e^{-x})a + (4e^{2x} - 2e^{2x} - 2e^{2x})b$

$= 0a + 0b = 0$ O.K.

~~$y_1 = ae^{-x}$ & $y_2 = be^{2x}$~~

~~So $y = y_1 + y_2 = ae^{-x} + be^{2x} \rightarrow$ is not a solution~~