Al-Mustaqbal University College
Department of Computer
Engineering Techniques

Information Theory and coding

Fourth stage

Lecture 5

Average information (entropy)

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Average information (entropy):

In information theory, **entropy** is the average amount of information contained in each message received. Here, *message* stands for an event, sample or character drawn from a distribution or data stream. Entropy thus characterizes our uncertainty about our source of information.

1. Source Entropy:

If the source produces not equal probability messages then $I(xi)$, i = 1, 2, 3,…n are different. Then the statistical average of $I(xi)$ over i will give the average amount of uncertainty associated with source X. This average is called source entropy and denoted by $H(X)$, given by:

$$H(x) = \sum_{i=1}^{n} P(xi)\, I(xi)$$

$$H(x) = -\sum_{i=1}^{n} P(xi) log_a P(xi) \qquad bit/symbol$$

Example: Find the entropy of the source producing the following messages:

$$Px_1 = 0.25, Px_2 = 0.1, Px_3 = 0.15, and\ Px_4 = 0.5$$

Solution:

$$H(x) = -\sum_{i=1}^{4} P(xi) log_2 P(xi)$$

$$= -\left[\frac{0.25ln0.25 + 0.1ln0.1 + 0.15ln0.15 + 0.5ln0.5}{ln2}\right]$$

$$H(x) = 1.742\ bit/symbol$$

2. Binary Source entropy:

In information theory, the **binary entropy function**, denoted or H(X) or Hb(X), is defined as the entropy of a Bernoulli process with probability *p* of one of two values. Mathematically, the Bernoulli trial is modeled as a random variable *X* that can take on only two values: 0 and 1:

$$P(0) + P(1) = 1$$

We have:

$$H(x) = -\sum_{i=1}^{n} P(xi) log_a P(xi)$$

$$H_b(x) = -\sum_{i=1}^{2} P(xi) log_a P(xi)$$

Then

$$H_b(x) = -[P(0)log_2 P(0) + P(1)log_2 P(1)] \quad bit/symbol$$

Example: Find the entropy for binary source if P(0)=0.2.

Solution:

$$P(1) = 1\text{-}P(0) = 1\text{-}0.2 = 0.8$$

Then

$$H_b(x) = -\sum_{i=1}^{2} P(xi) log_a P(xi)$$

$$H_b(x) = -[0.2 log_2 0.2 + 0.8 log_2 0.8]$$

$$= -\left[\frac{0.2ln0.2 + 0.8ln0.8}{ln2}\right]$$

$$= 0.7 \quad bit/symbol$$

3. Maximum Source Entropy:

For binary source, if $P(0) = P(1) = 0.5$, then the entropy is:

$$H_b(x) = -[0.5log_2 0.5 + 0.5log_2 0.5]$$

$$= -\left[log_2\left(\frac{1}{2}\right)\right] = [log_2(2)] = 1\ bit$$

Note that $H_b(X)$ is maximum equal to 1(bit) if: $P(0) = P(1) = 0.5$, the entropy of binary source or any source having only two value is distributed as shown in Figure 1:

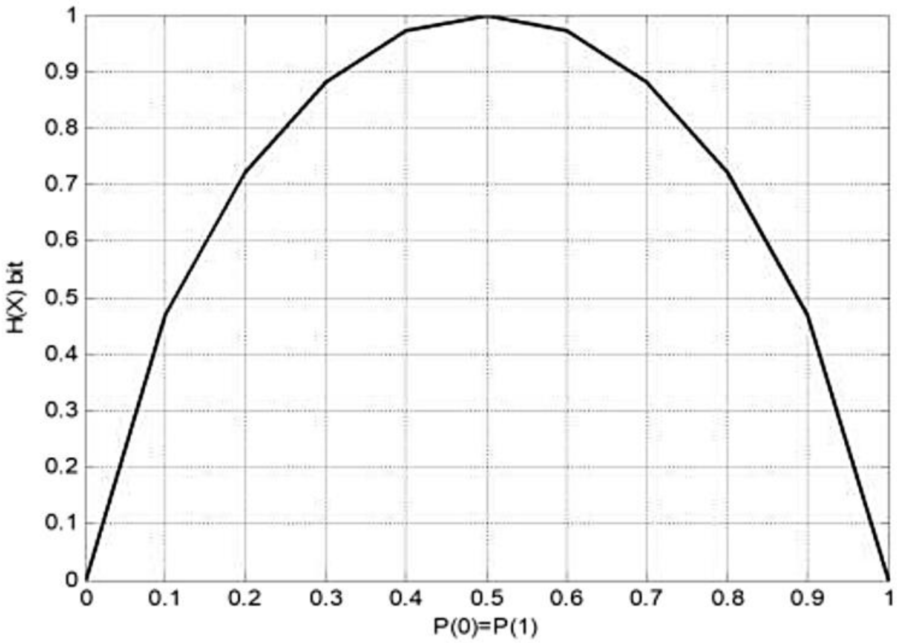

Figure 1: Entropy of binary source distribution

For any non-binary source, if all messages are equiprobable

Then $P(xi) = 1/n$

so that:

$$H(x) = H(x)_{max} = -\left[\frac{1}{n} log_a \left(\frac{1}{n}\right)\right] \times n$$
$$= -\left[log_a \left(\frac{1}{n}\right)\right]$$
$$= log_a n \qquad bits/symbol$$

Which is the maximum value of source entropy. Also, $H(X) = 0$ if one of the message has the probability of a certain event or p(x) =1.

Example: A source emits 8 characters with equal probability, Find the max entropy $H(x)_{max}$.

Solution:

$$H(x)_{max} = log_2 n = log_2 8 = 3\ bit/symbol$$

4. **Source Entropy Rate:**

It is the average rate of amount of information produced per second.

$$R(\mathrm{x}) = H(\mathrm{x}) \times rate\ of\ producing\ the\ symbols = \mathrm{bits/sec} = \mathrm{bps}$$

The unit of H(X) is bits/symbol and the rate of producing the symbols is symbol/sec, so that the unit of R(X) is bits/sec.

$$R(x) = \frac{H(x)}{\bar{\tau}}$$

Where

$$\bar{\tau} = \sum_{i=1}^{n} \tau_i P(xi)$$

$\bar{\tau}$ is the average time duration of symbols, τi is the time duration of the symbol xi.

Example: A source produces dots '.' And dashes '-' with P(dot)=0.65. If the time duration of dot is 200ms and that for a dash is 800ms. Find the average source entropy rate.

Solution:

$$P(dash) = 1 - P(dot) = 1 - 0.65 = 0.35$$

$$H(x) = -\sum_{i=1}^{n} P(xi) log_a P(xi)$$

$$H(x) = -[0.65 log_2 0.65 + 0.35 log_2 0.35]$$

$$=0.934 \quad bits/symbol$$

$$\bar{\tau} = \sum_{i=1}^{n} \tau_i P(xi)$$

$$\bar{\tau} = 0.2 \times 0.65 + 0.8 \times 0.35 = 0.41\ sec$$

$$R(x) = \frac{H(x)}{\bar{\tau}} = \frac{0.934}{0.41} = 2.278 \quad bps$$

Example: A discrete source emits one of five symbols once every millisecond. The symbol probabilities are 1/2, 1/4, 1/8, 1/16 and 1/16 respectively. Calculate the information rate.

Solution:

$$H(x) = -\sum_{i=1}^{5} P(xi) log_a P(xi)$$

$$= -\left(\frac{1}{2}\log_2\frac{1}{2} + \frac{1}{4}\log_2\frac{1}{4} + \frac{1}{8}log_2\frac{1}{8} + \frac{1}{16}log_2\frac{1}{16} + \frac{1}{16}log_2\frac{1}{16}\right)$$

$$= \left(\frac{1}{2}\log_2 2 + \frac{1}{4}\log_2 4 + \frac{1}{8}log_2 8 + \frac{1}{16}log_2 16 + \frac{1}{16}log_2 16\right)$$

$$= (0.5 + 0.5 + 0.375 + 0.25 + 0.25) = 1.875\ \ bit/symbol$$

$$R(x) = \frac{H(x)}{\bar{\tau}} = \frac{1.875}{10^{-3}} = 1.875 \quad Kbps$$